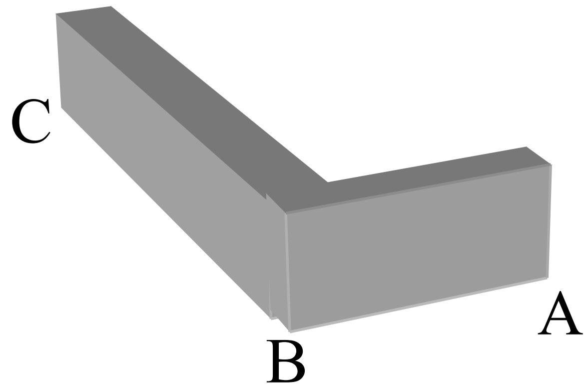
Equilibrium in 3-D

Consider supporting the member in the orientation shown with one finger under each of the points A, B and C.

Can the body be supported this way?

Yes **Gr**

No **Pi**



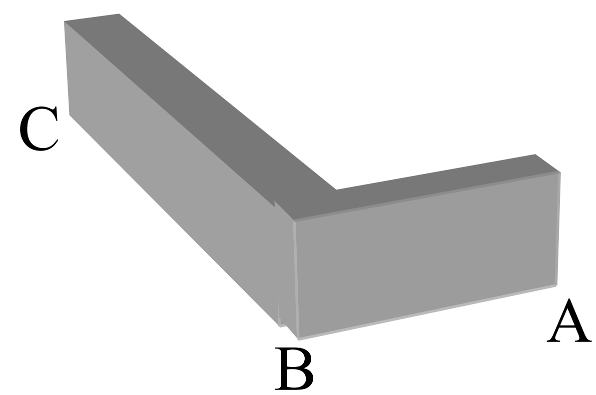
Equilibrium in 3-D

Consider supporting the member in the orientation shown with one finger under each of the points A, B and C.

What are the force magnitudes at A, B and C in order from smallest to largest?

- AB is 5 cm long and weighs 5 N, BC is 7 cm long and weighs 7 N
- the mass is uniformly distributed.

ABC	**Gr**
BCA	**Pi**
CAB	**Bl**
ACB	**Ye**
CBA	**Wh**

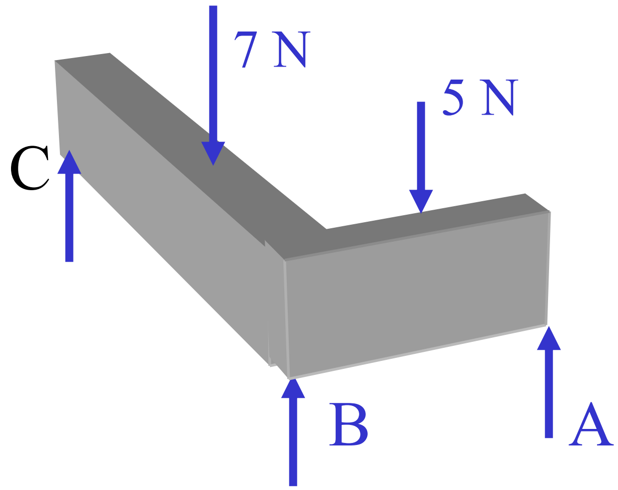


Equilibrium in 3-D

What are the force magnitudes at A, B and C in order from smallest to largest?

Hint: Think of the forces required to sustain the weight of AB by itself, then the forces required to sustain the weight of BC by itself, and then combine.

ABC	**Gr**
BCA	**Pi**
CAB	**Bl**
ACB	**Ye**
CBA	**Wh**

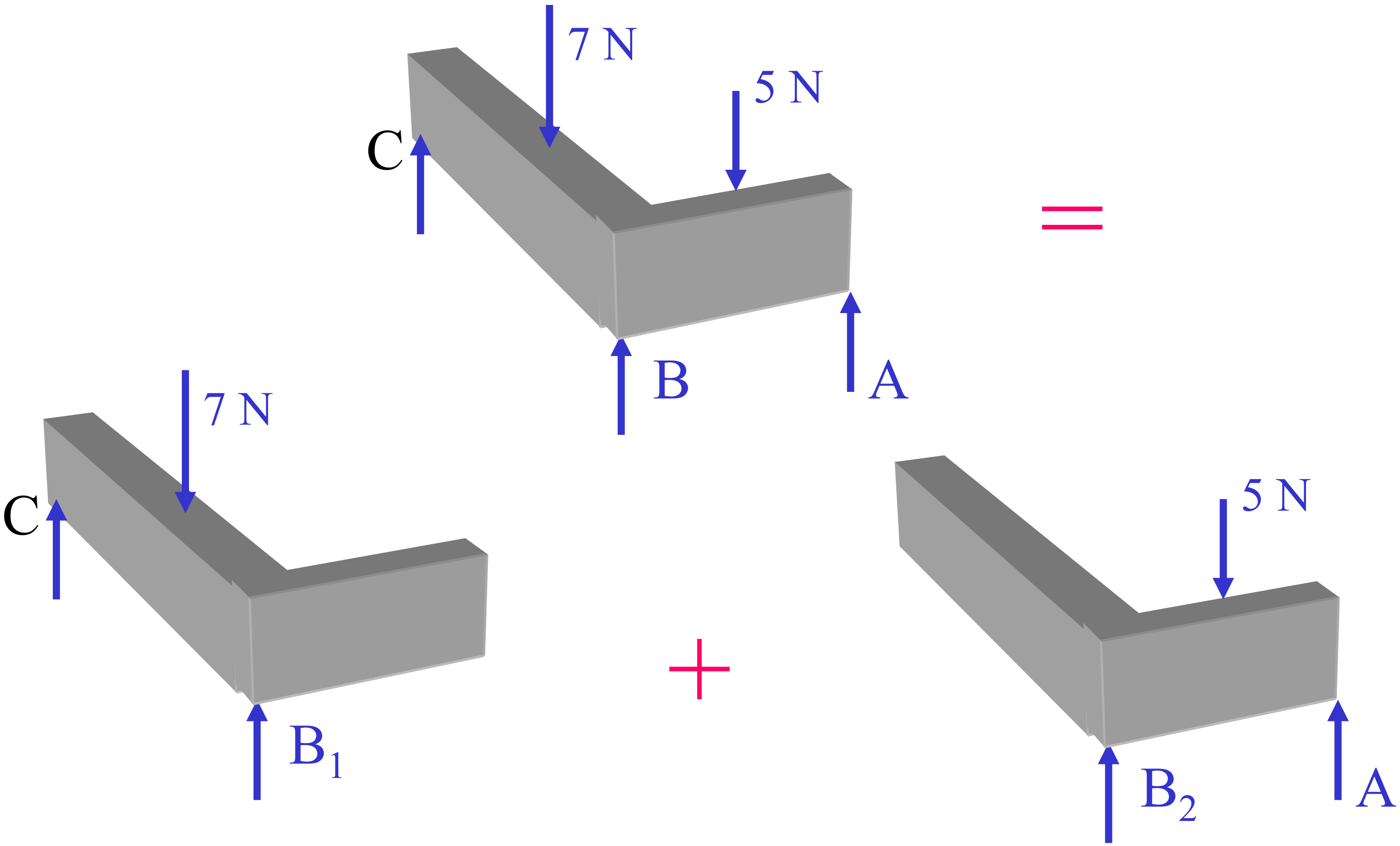


Equilibrium in 3-D

Intuitive approach

Combine:

- forces required to balance weight of AB alone
- forces required to balance weight of BC alone

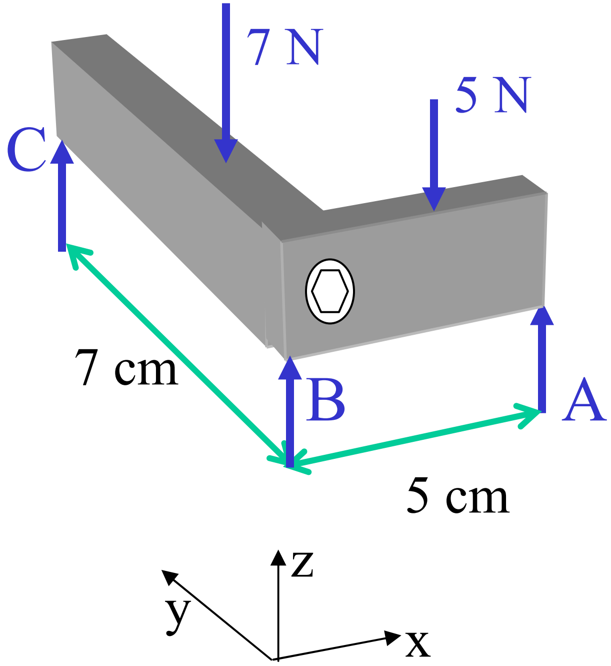


Equilibrium in 3-D

Which forces contribute to the moment summation $\Sigma M|_{Ax} = 0$?

(Neglect thickness compared with long dimensions)

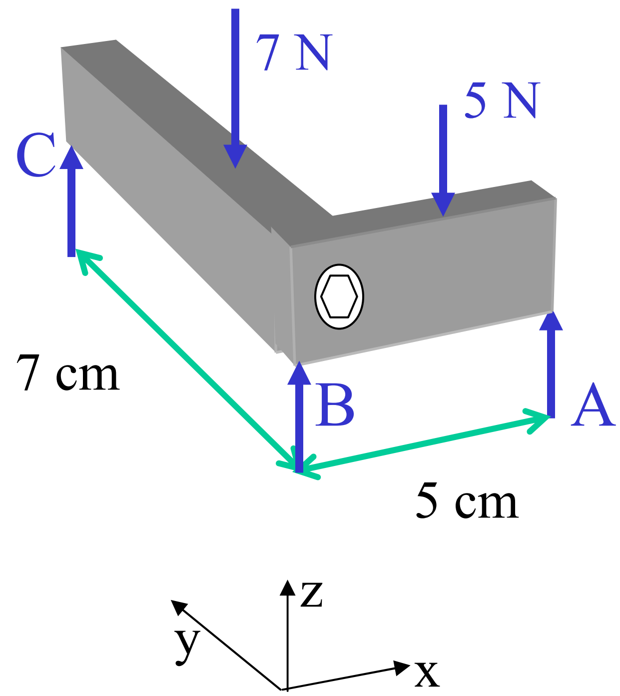
A	**Yes**	**Gr**	**No**	**Pi**
B	**Yes**	**Gr**	**No**	**Pi**
C	**Yes**	**Gr**	**No**	**Pi**
5 N	**Yes**	**Gr**	**No**	**Pi**
7 N	**Yes**	**Gr**	**No**	**Pi**



Equilibrium in 3-D

In the moment summation $\Sigma M|_{Ax} = 0$, the moment arm for C is:

7	**Gr**
5	**Pi**
12	**Bl**
$[5^2+7^2]^{1/2}$	**Ye**



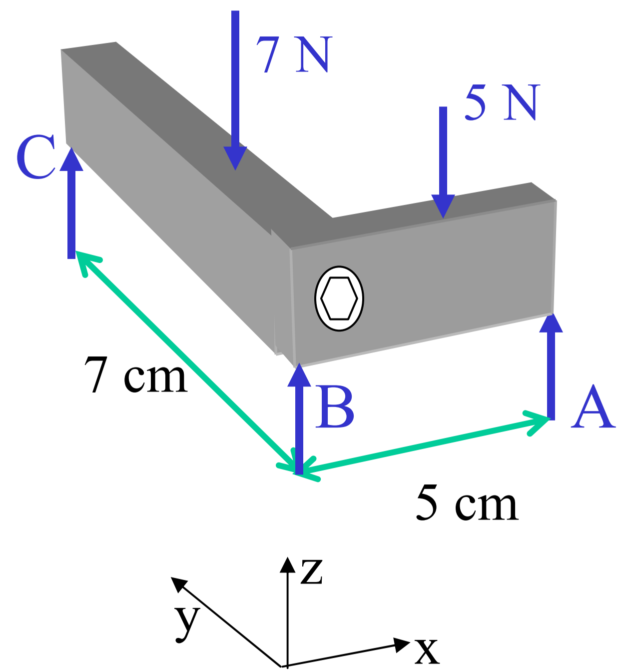
Equilibrium in 3-D

Imposition of Equilibrium Equations

In the moment summation $\Sigma M|_{Ax} = 0$, force C contributes a moment which is:

$M|_{Ax} > 0$ **Gr**

$M|_{Ax} < 0$ **Pi**

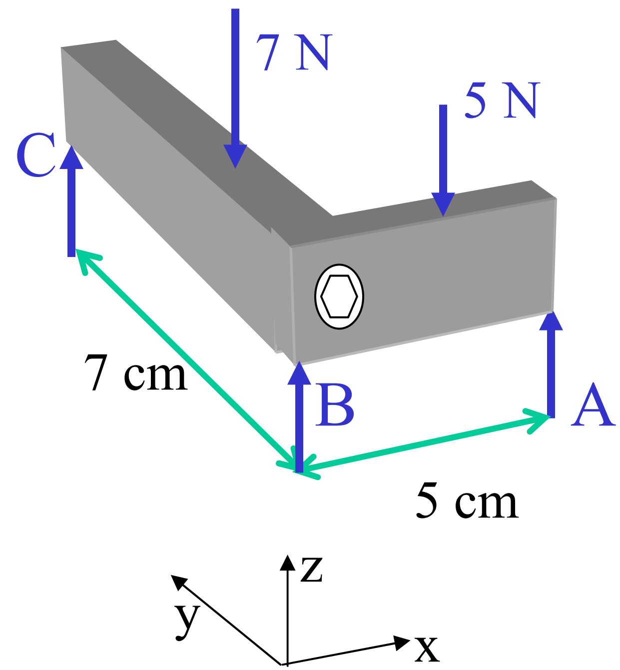


Equilibrium in 3-D

Imposition of Equilibrium Equations

$$\Sigma M|_{Ax} = C(7) - 7(7/2) = 0$$

$$C = 7/2 \text{ N}$$

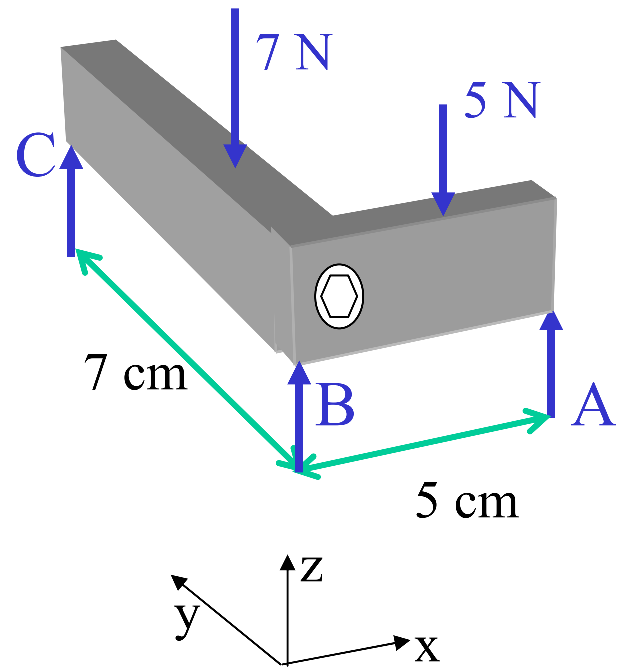


Equilibrium in 3-D

Imposition of Equilibrium Equations

Which forces contribute to the moment summation $\Sigma M|_{By} = 0$?

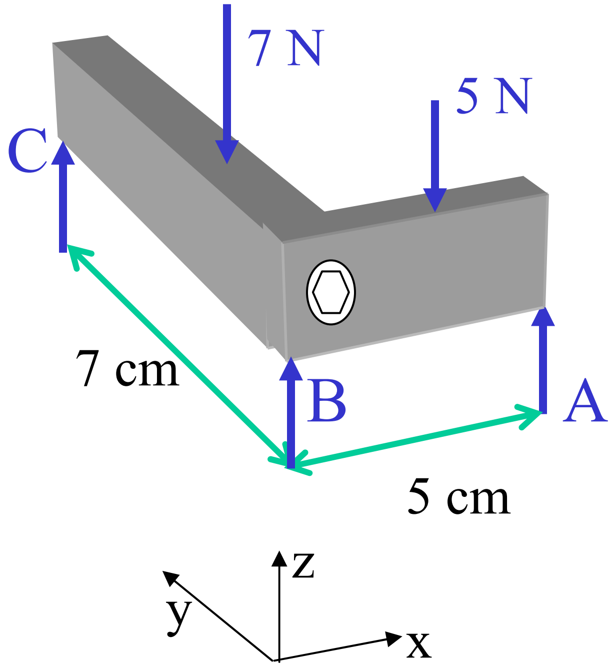
A	Yes Gr	No Pi
B	Yes Gr	No Pi
C	Yes Gr	No Pi
5 N	Yes Gr	No Pi
7 N	Yes Gr	No Pi



Equilibrium in 3-D

In the moment summation $\Sigma M|_{By} = 0$, the moment arm for A is:

7	**Gr**
5	**Pi**
12	**Bl**
$[5^2+7^2]^{1/2}$	**Ye**

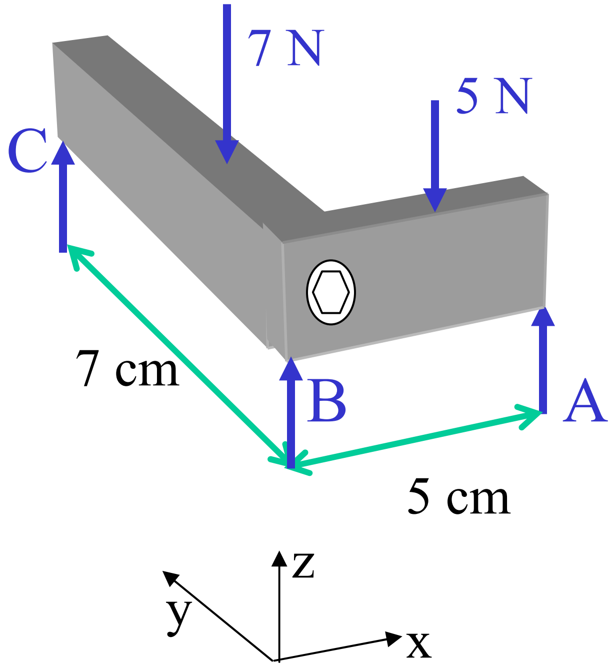


Equilibrium in 3-D

In the moment summation $\Sigma M|_{By} = 0$, force A contributes a moment which is:

$M|_{By} > 0$

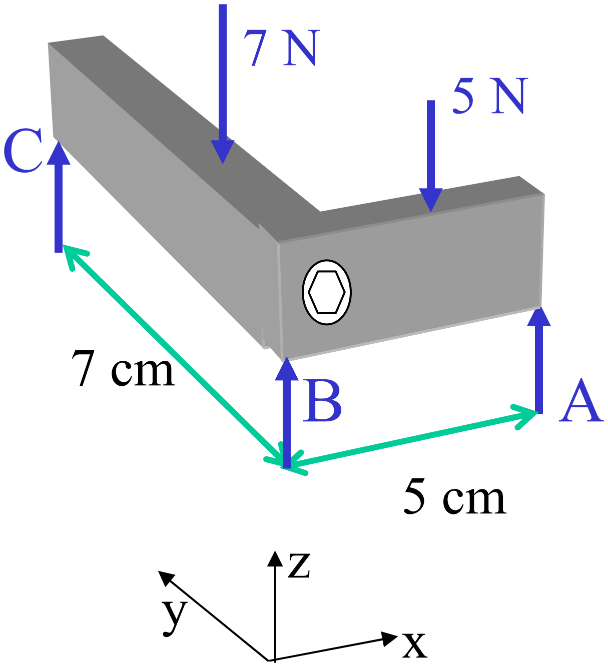
$M|_{By} < 0$



Equilibrium in 3-D

$$\Sigma M|_{By} = -A(5) + 5(5/2) = 0$$

$$A = 5/2\ N$$



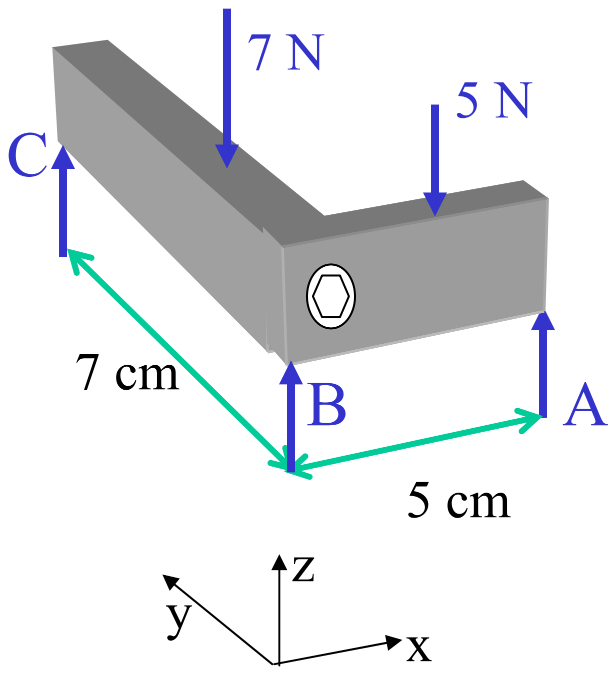
Equilibrium in 3-D

$A = 5/2$ N, $C = 7/2$ N

$$\Sigma F_z = A + B + C - 7 - 5 = 0$$

$B = 6$ N

$B > C > A$

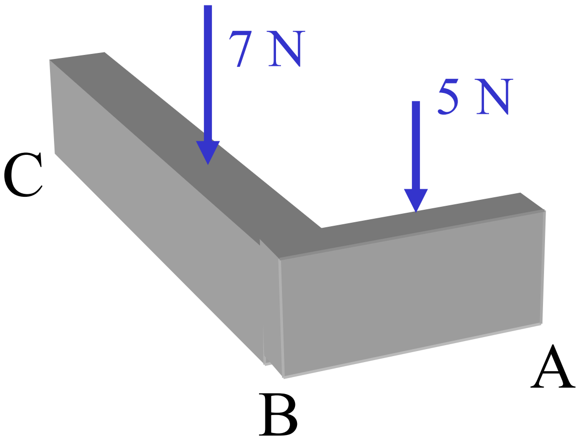


Equilibrium in 3-D

Can the member be supported by one finger under some point of the body?

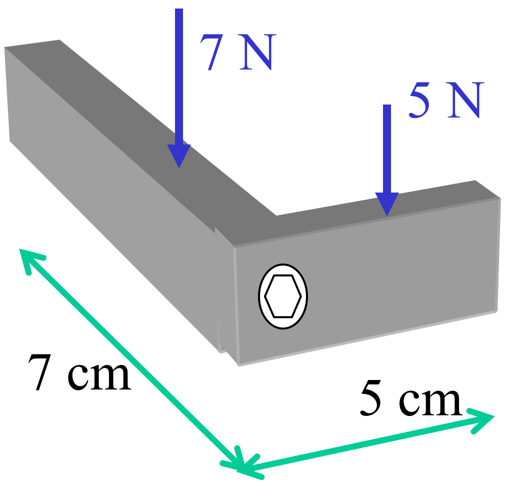
Yes **Gr**

No **Pi**



Equilibrium in 3-D: Center of Gravity

Locate single force which is statically equivalent to the 7 N and 5 N forces



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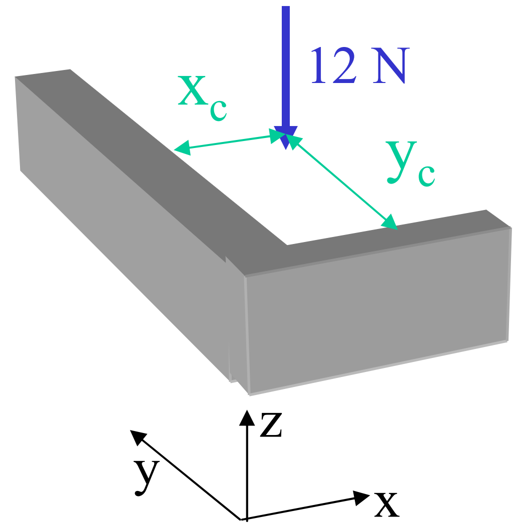
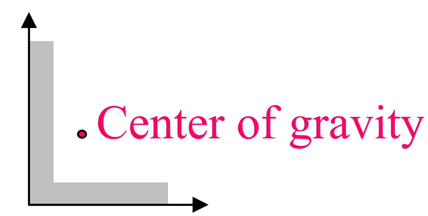


12 N force must produce the same moments $M|_x$ and $M|_y$ as the original forces:

$\Sigma M|_{By} = -5(5/2) = -12\ x_c$ ⇒ $x_c = 25/24$ cm

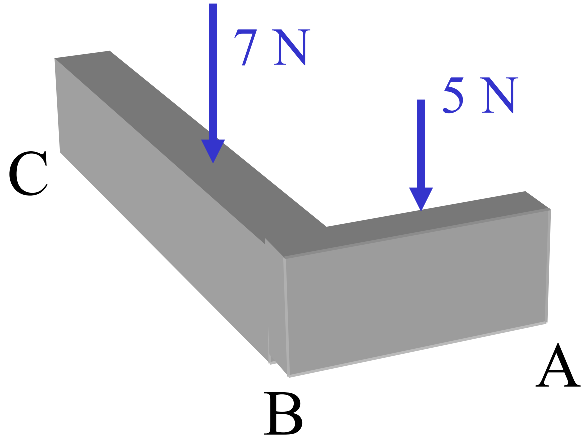
$\Sigma M|_{Bx} = -7(7/2) = -12\ y_c$ ⇒ $y_c = 49/24$ cm



"L" member cannot be supported by one finger under some point of the body, because the "L" center of gravity is located outside its boundary

Equilibrium in 3-D

Can the member be supported by two fingers, each under a different point of the body?



No **Gr**

Yes, in one way **Pi**

Yes, in many ways **Ye**

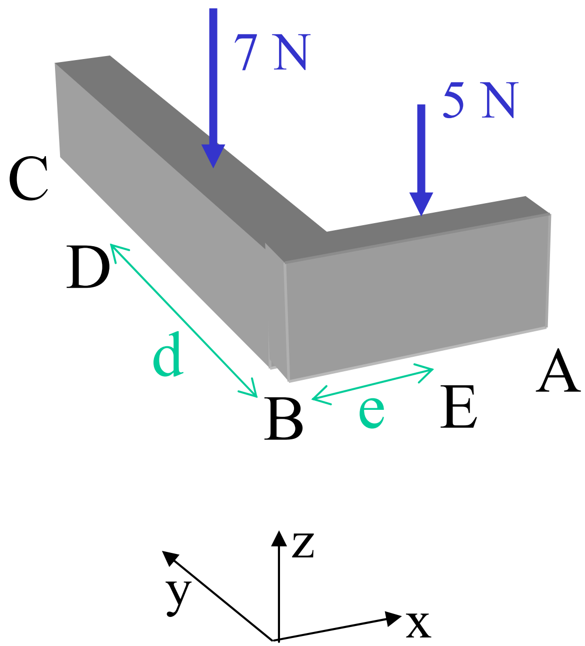
Equilibrium in 3-D

Select one support point D at a distance d from B along BC.

Can you always balance the member with a second support point E at a distance e from B along BC (no matter where D is between B and C)?

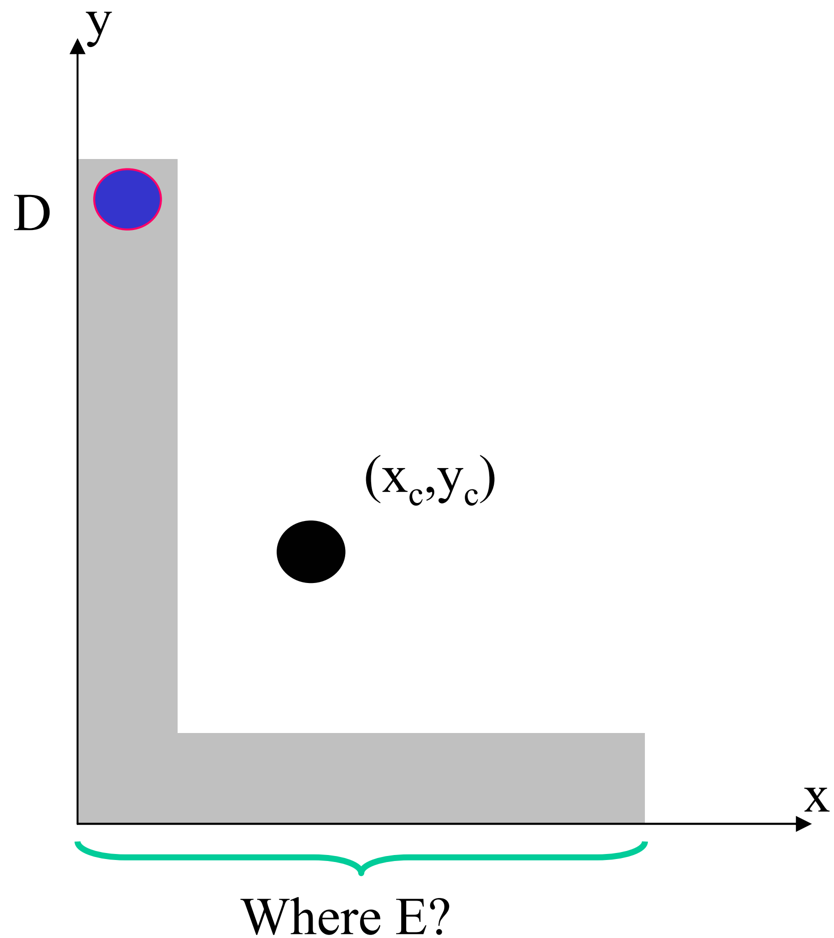
Yes **Gr**

No **Pi**



Equilibrium in 3-D

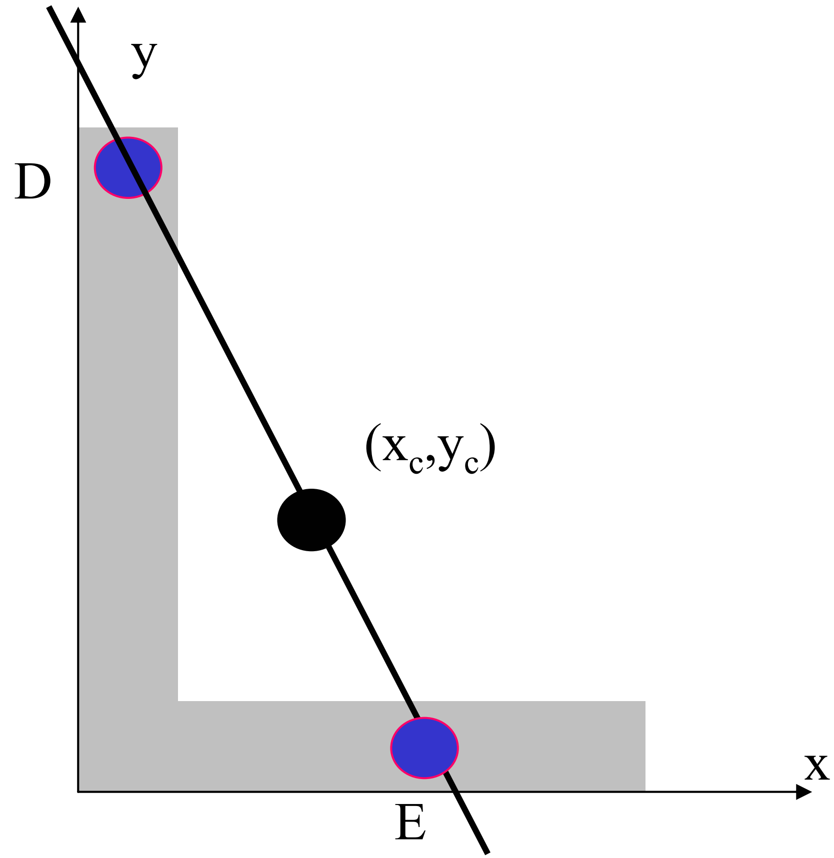
View Member from top (gravity still acts along –z)
One finger is placed at D near end of long member.
Where should support point E be?



Equilibrium in 3-D

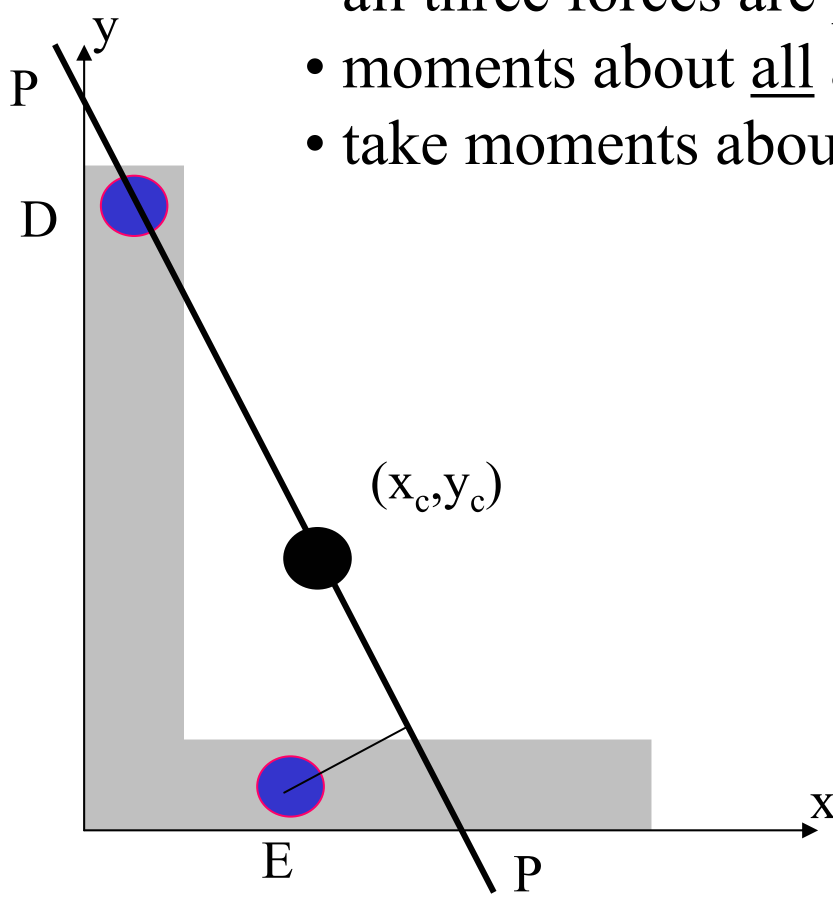
Support point E should be along line connecting D and center of gravity.

Why?



Equilibrium in 3-D

- all three forces are perpendicular to plane.
- moments about <u>all</u> axes must sum to zero.
- take moments about axis P-P:

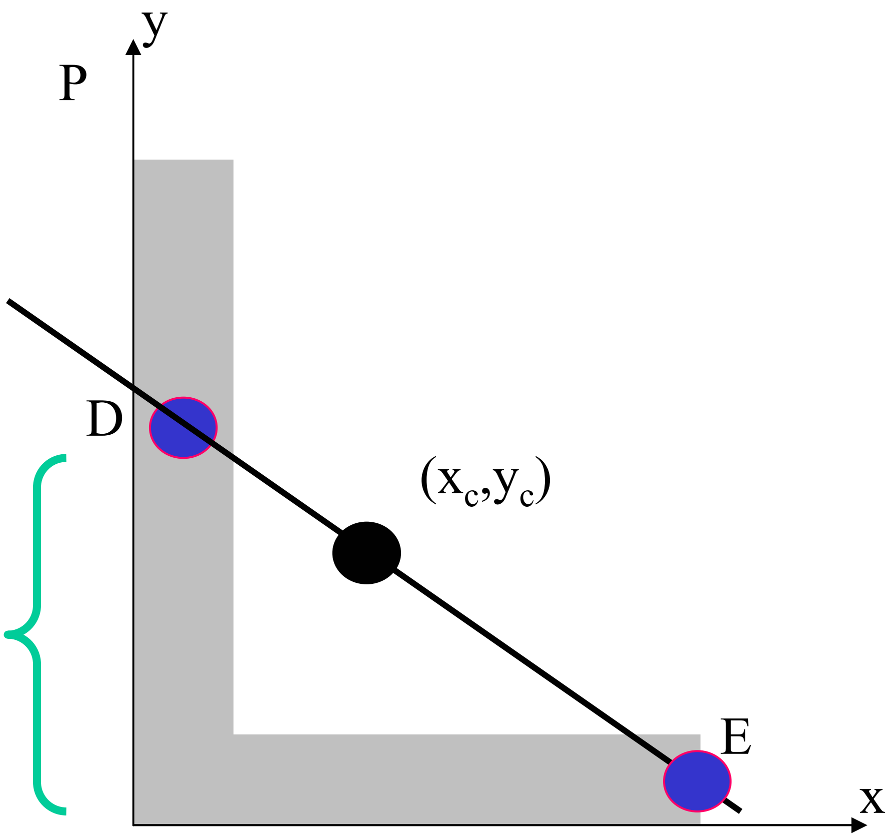


If E is not on line P-P, it produces an unbalanced moment

Equilibrium in 3-D

- let E be at end of short member.
- again D must be on the line joining E and centroid.
- if D is any lower, E would have to be off the end of member

D cannot be in here

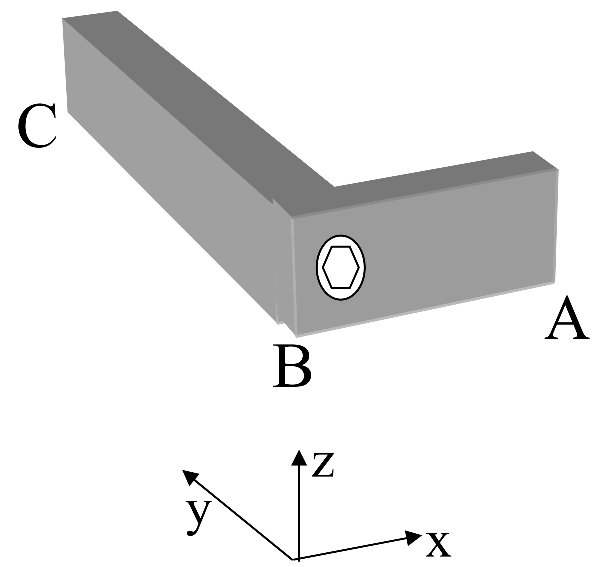


Equilibrium in 3-D

Consider supporting the member in the orientation shown by:

- **two fingers applying upward forces**
- **a nut driver applying <u>only a couple</u> to the nut located near B**

The member can be balanced by a couple of the right magnitude and direction acting at B and forces applied to the following pairs of points:



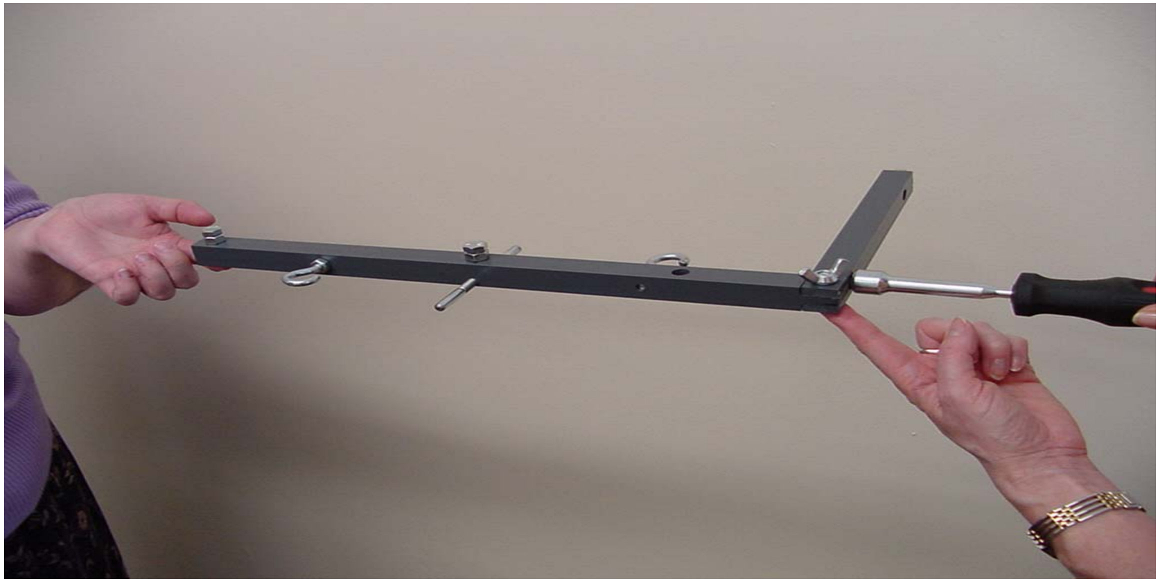
A and B: **Yes** **Gr** **No** **Pi**

A and C: **Yes** **Gr** **No** **Pi**

B and C: **Yes** **Gr** **No** **Pi**

Equilibrium in 3-D

Consider supporting the member in the orientation shown by applying:
a couple to the nut located near B + two forces with the fingers at B and C



Equilibrium in 3-D

Consider supporting the member in the orientation shown by applying:
a couple to the nut located near B + two forces with the fingers at B and C

The couple applied to the nut to maintain equilibrium is described by:

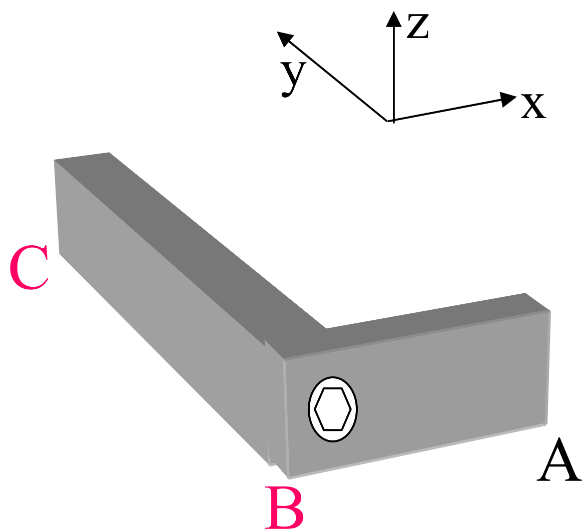
$M_x > 0$ **Gr**

$M_x < 0$ **Pi**

$M_y > 0$ **Bl**

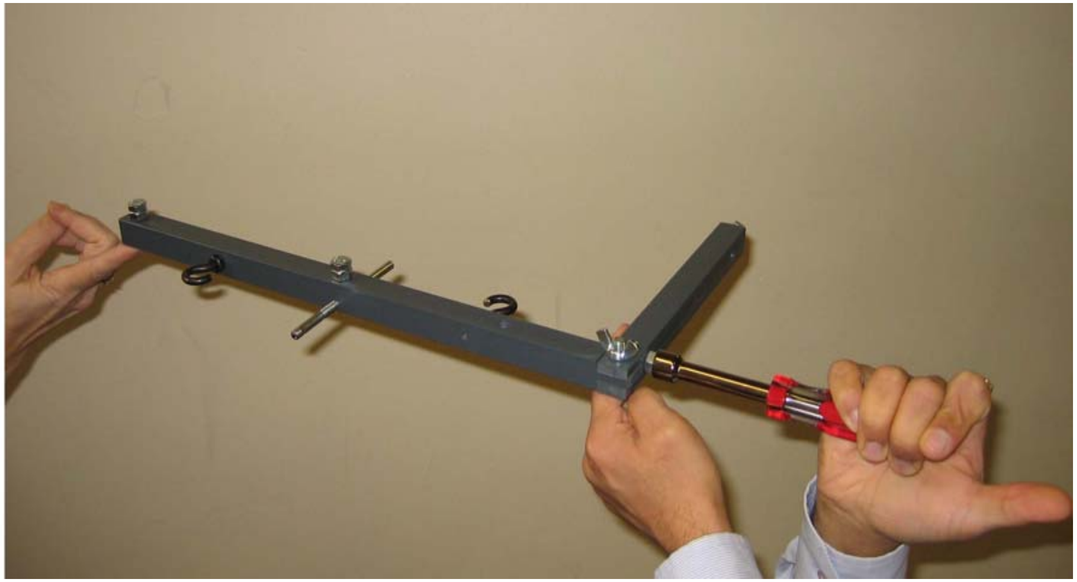
$M_y < 0$ **Ye**

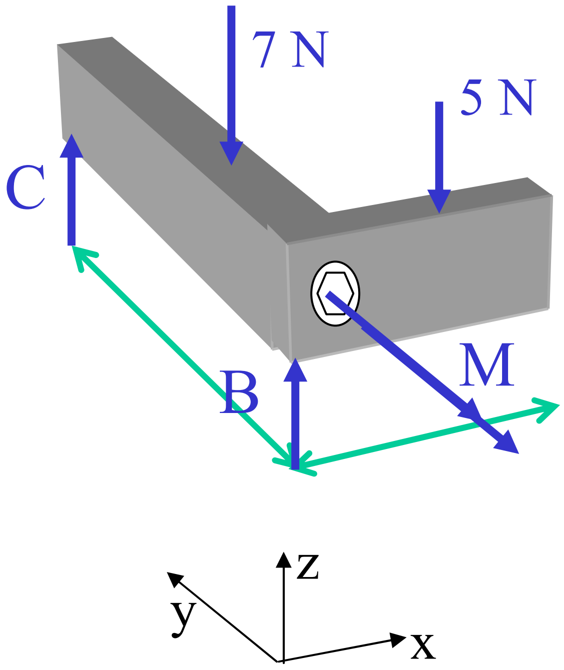
$M_z > 0$ **Wh**



Equilibrium in 3-D

Consider supporting the member in the orientation shown by applying:
a couple to the nut located near B + two forces with the fingers at B and C





$M_y < 0$ **Ye**

Equilibrium in 3-D

Consider supporting the member in the orientation shown by applying: a couple to the nut located near B + two forces with the fingers at B and C

Couple is $M_y < 0$.

What is the magnitude M of the couple applied at the nut?

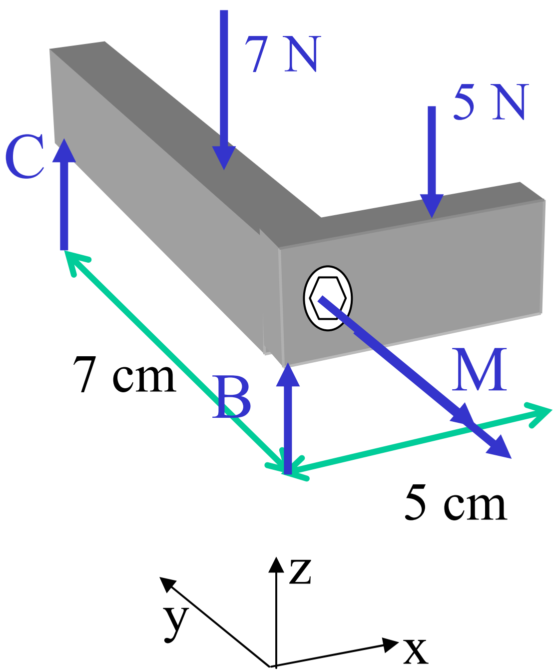
$M = (7.0)(3.5)$ N-cm **Gr**

$M = (2.5)(2.5)$ N-cm **Pi**

$M = (7.0)(2.5)$ N-cm **Bl**

$M = (12)\ (2.5)$ N-cm **Ye**

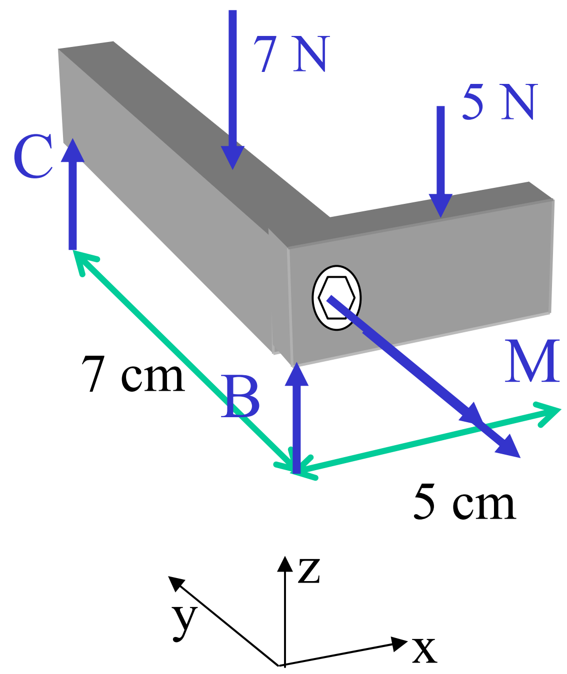
$M = (5.0)\ (2.5)$ N-cm **Wh**



Equilibrium in 3-D

Consider supporting the member in the orientation shown by applying:
a couple to the nut located near B + two forces with the fingers at B and C

With fingers under B and C, what is the magnitude of the couple applied at the nut?



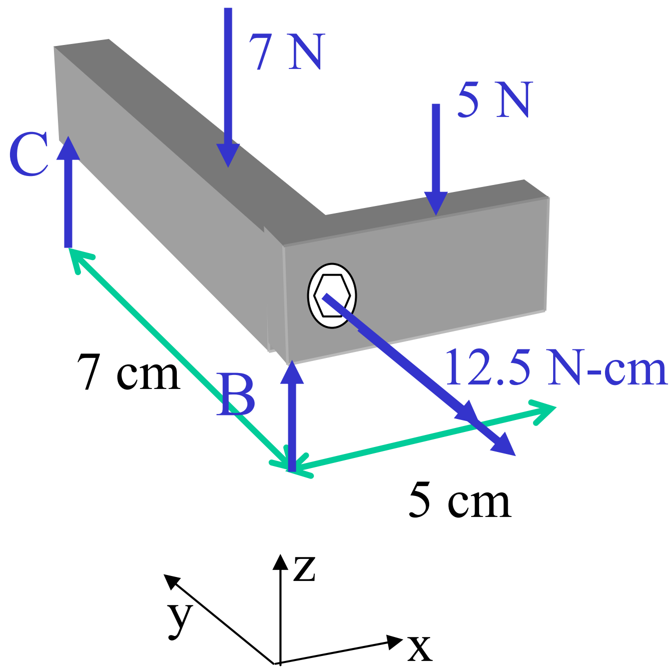
Set $\Sigma M|_{By} = 5(5/2) - M = 0$

$M = (5)(2.5)$ N-cm $\boxed{\textbf{Wh}}$

Equilibrium in 3-D

Consider supporting the member in the orientation shown by applying:
a couple to the nut located near B + two forces with the fingers at B and C

Given the whole member weighs 12 N, what are the forces exerted by the fingers?



B = 5.0 N, C = 7.0 N **Gr**

B = 8.5 N, C = 3.5 N **Pi**

B = 7.5 N, C = 4.5 N **Bl**

B = 3.5 N, C = 8.5 N **Ye**

B = 9.5 N, C = 3.5 N **Wh**

Equilibrium in 3-D

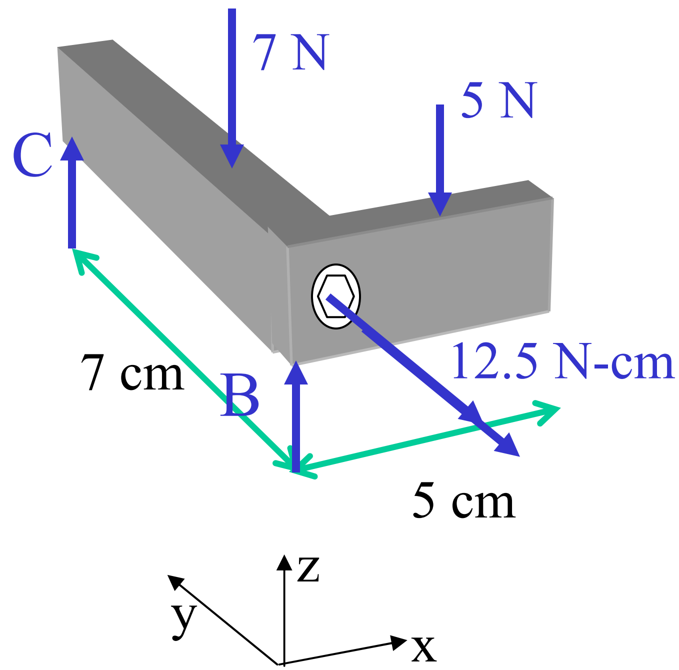
Consider supporting the member in the orientation shown by applying:
a couple to the nut located near B + two forces with the fingers at B and C

Given the whole member weighs 12 N, and there are fingers under B and C, what are the forces exerted by the fingers?

$$\Sigma M|_{Bx} = 7(7/5)\text{-}C(7) = 0$$

$$\Sigma F_z = B + C - 12 = 0$$

B = 8.5 N, C = 3.5 N **Pi**



Equilibrium in 3-D

Consider supporting the member in the orientation shown by applying:
a couple to the nut located near B + two forces with the fingers at C and A

Equilibrium in 3-D

Consider supporting the member in the orientation shown by applying:
a couple to the nut located near B + two forces with the fingers at C and A

The couple applied to the nut to maintain equilibrium is described by:

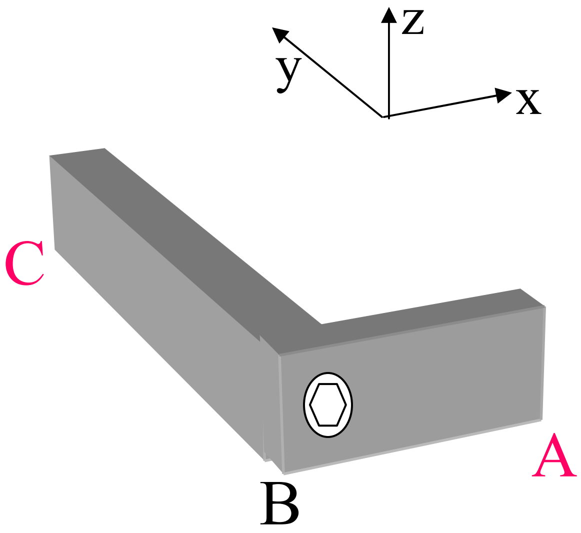
$M_x > 0$ **Gr**

$M_x < 0$ **Pi**

$M_y > 0$ **Bl**

$M_y < 0$ **Ye**

$M_z > 0$ **Wh**



Equilibrium in 3-D

Consider supporting the member in the orientation shown by applying:
a couple to the nut located near B + two forces with the fingers at C and A

Given the whole member weighs 12 N, what are the forces exerted by the fingers?

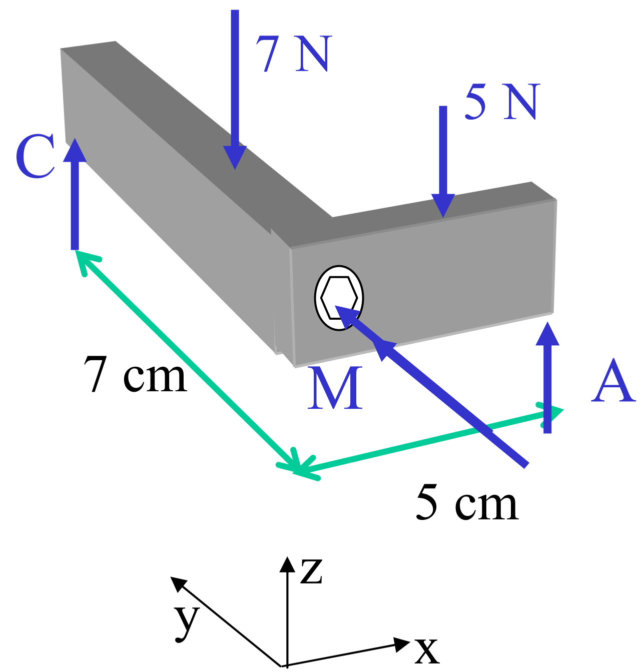
A = 5.0 N, C = 7 N **Gr**

A = 8.5 N, C = 3.5 N **Pi**

A = 7.5 N, C = 4.5 N **Bl**

A = 3.5 N, C = 8.5 N **Ye**

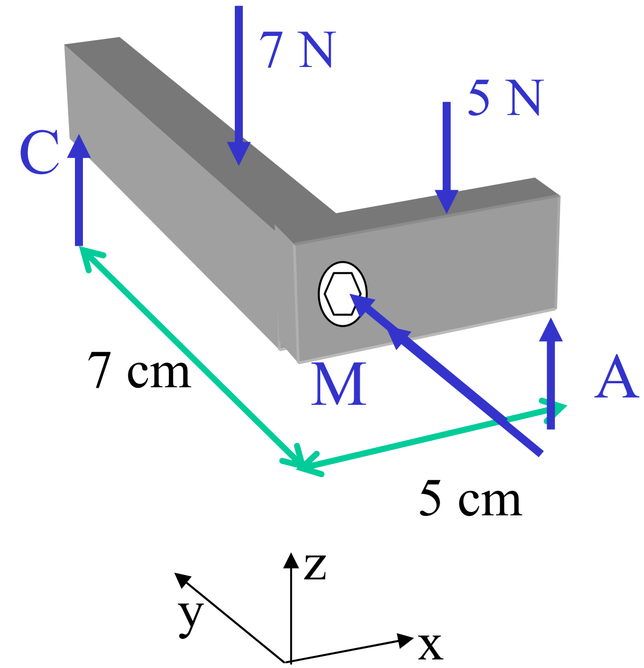
A = 9.5 N, C = 3.5 N **Wh**



Equilibrium in 3-D

Consider supporting the member in the orientation shown by applying: a couple to the nut located near B + two forces with the fingers at C and A

Given the whole member weighs 12 N, and there are fingers under A and C, what are the forces exerted by the fingers?



$$\Sigma M|_{Ax} = 7(7/5)\text{-}C(7) = 0$$

$$\Sigma F_z = A + C - 12 = 0$$

A = 8.5 N, C = 3.5 N **Pi**

Equilibrium in 3-D

Consider supporting the member in the orientation shown by applying: a couple to the nut located near B + two forces with the fingers at C and A

The couple applied to the nut to maintain equilibrium is described by $M_y > 0$:

What is the magnitude of the couple applied at the screw?

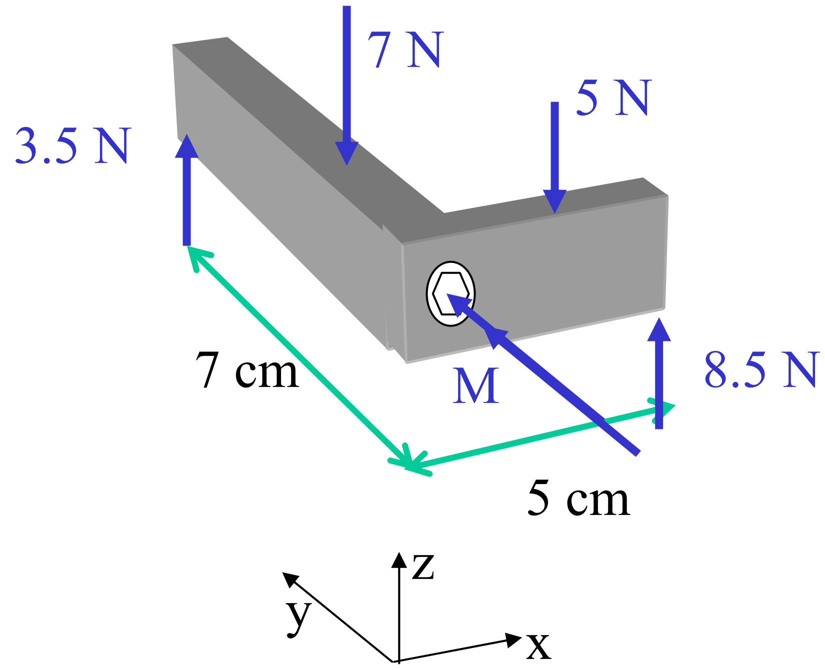
$M = 8.75$ N-cm **Gr**

$M = 12.5$ N-cm **Pi**

$M = 30$ N-cm **Bl**

$M = 42.5$ N-cm **Ye**

$M = 55$ N-cm **Wh**



Equilibrium in 3-D

Consider supporting the member in the orientation shown by applying: a couple to the nut located near B + two forces with the fingers at C and A

The couple applied to the nut to maintain equilibrium is described is $M_y > 0$:

$$\Sigma M|_{By} = +5(5/2) - 8.5(5) + M = 0$$

$$M = 30 \text{ N-cm}$$ **Bl**