Int. J. Engng Ed. Vol. 9, No. 6, pp. 453–457, 1993
Printed in Great Britain.

0742–0269/91 \$3.00+0.00

A Technical Note on Optimum Inter-Stage Pressure and Specific Work Input for Multi-Stage Reciprocating Air Compressors*

K. T. OOI
School of Mechanical and Production Engineering, Nanyang Technological University, Nanyang Avenue, Singapore 2263, Republic of Singapore

This note comments on the way in which most of the currently available applied thermodynamics textbooks introduce the term 'optimum inter-stage pressure' of a multi-stage compressor system. It attempts to modify and suggest a more appropriate introduction of the optimum inter-stage pressure based on the minimum specific work input rather than minimum work input as currently stated in most textbooks. Attempts have also been made to highlight the importance of the specific work in the study of the air compressor.

NOMENCLATURE

$\dot{m}$ air mass flow rate (kg/s)
n polytropic index
P_i inter-stage pressure (N/m^2)
P_1 suction pressure (N/m^2)
P_2 discharge pressure (N/m^2)
R_c clearance ratio
V_s swept volume (m^3/s)
W (total polytropic power) (W)
w specific work (kJ/kg)

INTRODUCTION

RECIPROCATING machines are generally a topic of study in undergraduate mechanical engineering curricula. Reciprocating air compressors are usually introduced to students as an example of a work-absorbing device. Multi-stage compressor systems are always presented as a means to overcome some of the limitations of the single-stage compressor. In a multi-stage reciprocating air compressor system, the optimum inter-stage pressure is introduced to show the practically possible ideal operating conditions under which the compressor system operates most efficiently. In this respect, textbooks [1–4] usually begin with some of the commonly applied assumptions that eventually arrives at the statement where the optimum inter-stage pressure for a two-stage compressor system is equal to $P_i = \sqrt{P_1 P_2}$, where P_i and P_2 represent suction and discharge pressures respectively. In these textbooks [1–4], this optimum inter-stage pressure is quoted as the inter-stage pressure under which the polytropic power input is minimum. Reference [5] obtained the expression based on minimum cycle work. In the author's opinion this optimum inter-stage pressure should be referred to the condition where the specific work input is minimum, as is shown in [6]. So far, none of the textbooks cited [1–6] mentions the significance of the specific work input in this field of study. The importance of the 'specific work input' in the study of work-absorbing devices like air compressors is comparable to the 'specific fuel consumption' in the study of work-absorbing devices like internal combustion engines.

* Accepted 10 April 1993.

OPTIMUM INTER-STAGE PRESSURE AS ILLUSTRATED IN MOST TEXTBOOKS

The textbooks at the author's disposal [1–5] all give the same derivation of the optimum inter-stage pressure, which is briefly illustrated below. (In [5], the optimum P_i is obtained based on work per cycle.)

For a two-stage compressor system (Fig. 1) the total polytropic power can be expressed as:

$$W = \dot{m}\,\frac{n}{1-n}\,RT_1\left(\left(\frac{P_i}{P_1}\right)^{\frac{n-1}{n}} + \left(\frac{P_2}{P_i}\right)^{\frac{n-1}{n}} - 2\right) \tag{1}$$

The above equation represents the total polytropic power input (termed 'total work input' in these textbooks, which is technically inappropriate) for a two-stage compressor system. It is derived under the conditions that the polytropic index n is assumed to be the same for both stages and the intercooling is assumed complete such that

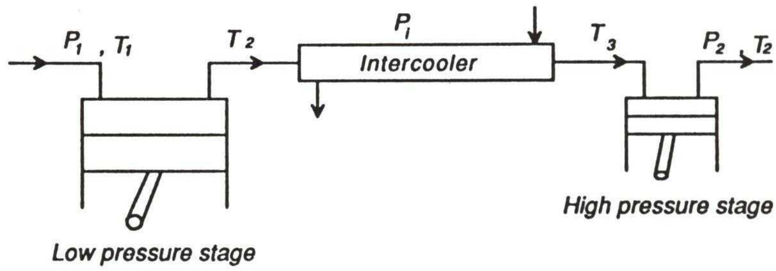

Fig. 1. Schematic of a two-stage compressor system

$T_3 = T_1$. If P_1, T_1 and P_2 are fixed, then the optimum P_i that resulted in the minimum work input can be obtained by equating $\mathrm{d}W/\mathrm{d}P_i$ to zero, i.e.

$$\frac{\mathrm{d}}{\mathrm{d}P_i}\left(\left(\frac{P_i}{P_1}\right)^{\frac{n-1}{n}}+\left(\frac{P_2}{P_i}\right)^{\frac{n-1}{n}}-2\right)=0 \qquad (2)$$

It can be easily shown that equation (2) yields

$$P_i = \sqrt{P_1 P_2} \qquad (3)$$

Notice that in the differentiation above, the term $\dot{m}$ is assumed constant and is taken to be independent of P_i. This is not true since the mass flow rate $\dot{m}$ can be expressed as:

$$\dot{m}=\left[1-R_c\left(\left(\frac{P_i}{P_1}\right)^{\frac{1}{n}}-1\right)\right]\frac{P_1\dot{V}_s}{RT_1} \qquad (4)$$

Equation (4) shows that the mass flow rate of a two-stage compressor system is a function of inter-stage pressure P_i only, since other parameters in equation (4) are practically fixed. If equation (4) is substituted into equation (1) the expression for total polytropic power input can be expressed as:

$$W=\left[1-R_c\left(\left(\frac{P_i}{P_1}\right)^{\frac{1}{n}}-1\right)\right]\frac{nP_1\dot{V}_s}{1-n}\left(\left(\frac{P_i}{P_1}\right)^{\frac{n-1}{n}}+\left(\frac{P_2}{P_i}\right)^{\frac{n-1}{n}}-2\right) \qquad (5)$$

Differentiating equation (5) with respect to P_i and equating it to zero yields,

$$R_c\left(\frac{P_i}{P_1}\right)^{\left(\frac{1}{n}\right)}\left[\left(\frac{P_i}{P_1}\right)^{\left(\frac{n-1}{n}\right)}+\left(\frac{P_2}{P_i}\right)^{\left(\frac{n-1}{n}\right)}-2\right]\left[\frac{P_1\dot{V}_s}{P_2(1-n)}\right]+\left[\frac{P_1\dot{V}_s}{P_2}\right]\left[1-R_c\left(\left(\frac{P_i}{P_1}\right)^{\left(\frac{1}{n}\right)}-1\right)\right]\left[\left(\frac{P_i}{P_1}\right)^{\left(\frac{n-1}{n}\right)}-\left(\frac{P_2}{P_i}\right)^{\left(\frac{n-1}{n}\right)}\right]=0 \qquad (6)$$

It may be noticed that equation (6) is not as simple an expression as that shown in equation (3). Apart from depending on P_i and P_2, it is also dependent on the polytropic index n, swept volume rate $\dot{V}_s$ and clearance ratio R_c. Figure 2 shows the dependency of the inter-stage pressure on the polytropic index n for minimum polytropic power input. It also shows that the value of the optimum inter-stage pressure increases as P_2 increases. As P_2 increases there is a pressure from which there exists no turning point for the power input. The comparison is also presented in the figure for the case where the inter-stage pressure is obtained from equation (3).

SPECIFIC WORK INPUT

In the study of reciprocating air compressors the overall throughput of the machine is measured by the specific work input w. The specific work input measures the amount of work input required to produce a kilogram of air. In any compressor system, it is this parameter that should be minimized. The specific work input can be expressed as:

$$w=\frac{W}{\dot{m}}=\frac{n}{1-n}RT_1\left(\left(\frac{P_i}{P_1}\right)^{\frac{n-1}{n}}+\left(\frac{P_2}{P_i}\right)^{\frac{n-1}{n}}-2\right) \qquad (7)$$

For minimum specific work input, differentiate equation (7) and equate it to zero. This yields

$$\frac{\mathrm{d}}{\mathrm{d}P_i}\left(\left(\frac{P_i}{P_1}\right)^{\frac{n-1}{n}}+\left(\frac{P_2}{P_i}\right)^{\frac{n-1}{n}}-2\right)=0 \qquad (8)$$

which leads to equation (9):

$$P_i = \sqrt{P_1 P_2} \qquad (9)$$

Notice that equations (8) and (9) are similar to equatons (2) and (3). It is obvious that euations (2) and (3) are obtained based on the condition for minimum specific work output and not the total polytropic power input as stated. It is clear that equation (3) refers to the value of the inter-stage pressure P_i when the specific work input is minimum, and not when the total polytropic power

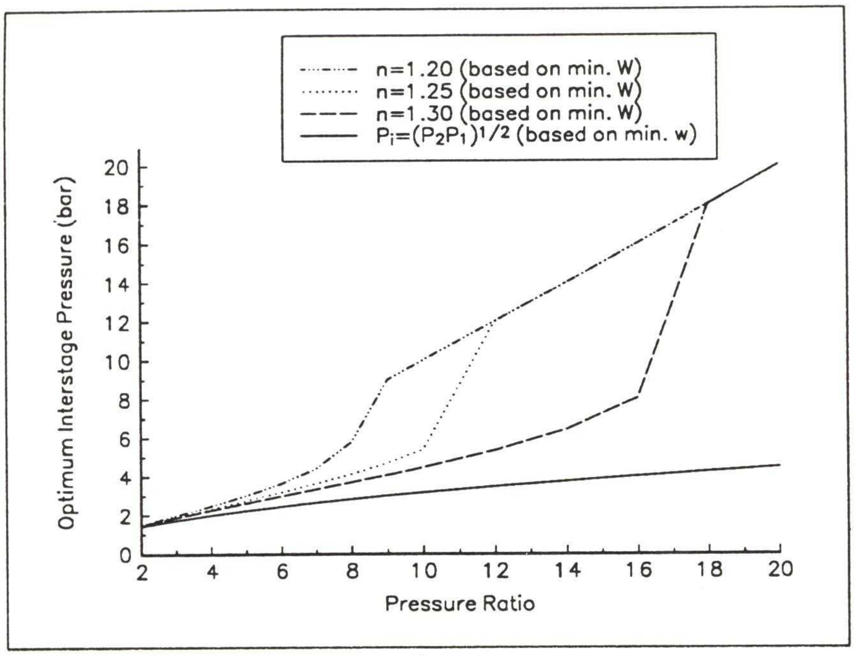

Fig. 2. Variation of optimum inter-stage pressure for a two-stage compressor.

input is minimum, as stated in the textbooks under discussion. In most work-absorbing devices, the lower power input condition is always associated with the lower mass flow rate; hence if the total power input is to be minimized, the minimum power input condition is always the one that results in a minimum mass flow rate. This condition is undesirable in this field of study. However, if the specific work is considered for minimization, it provides the condition where the machine operates most efficiently. That is, it minimizes the power input and at the same time maximizes the mass flow rate of air output. Unfortunately the importance of the specific work input as an overall throughput is never mentioned in these textbooks [1–6].

Figure 3 shows a comparison of the total power input for the case where the system operates at the inter-stage pressure when the total power input is minimized, with that when the specific power is minimized. The former is dependent on the polytropic index n, and is represented by broken lines; the solid line represents the case when the specific work is minimized. The minimum power input shown in Fig. 3 corresponds to the lower mass flow

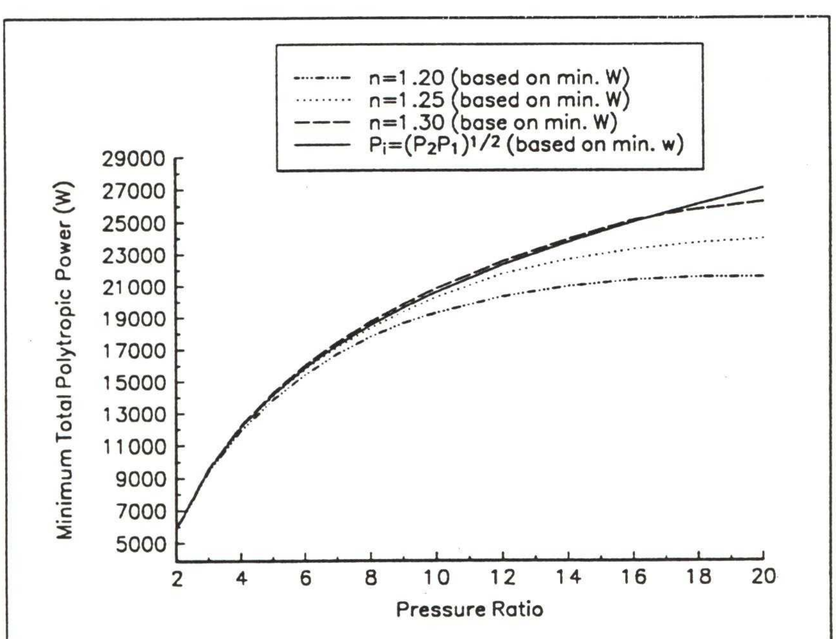

Fig. 3. Variation of minimum polytropic power with pressure ratio.

rate produced, as shown in Fig. 4. This figure also shows that for the case when specific work input is minimized, the mass flow rate of the air is maximized. Figure 5 shows the specific work plotted against the pressure ratio under all conditions mentioned above. The solid line represents the case where the specific work is minimized and this effect is clearly shown in the figure.

CONCLUSION

The author believes that it is essential to introduce and highlight the importance of the specific work input as an overall throughput in the study of air compressors. In any studies on improving industrial compressors it is this overall throughput that should be minimized. The ways in which most textbooks [1–5] introduce conditions for optimum operating condition should be modified, and more appropriately should be based on the condition for minimum specific work input, as this is more appropriate and suitable in the context of air compressor study.

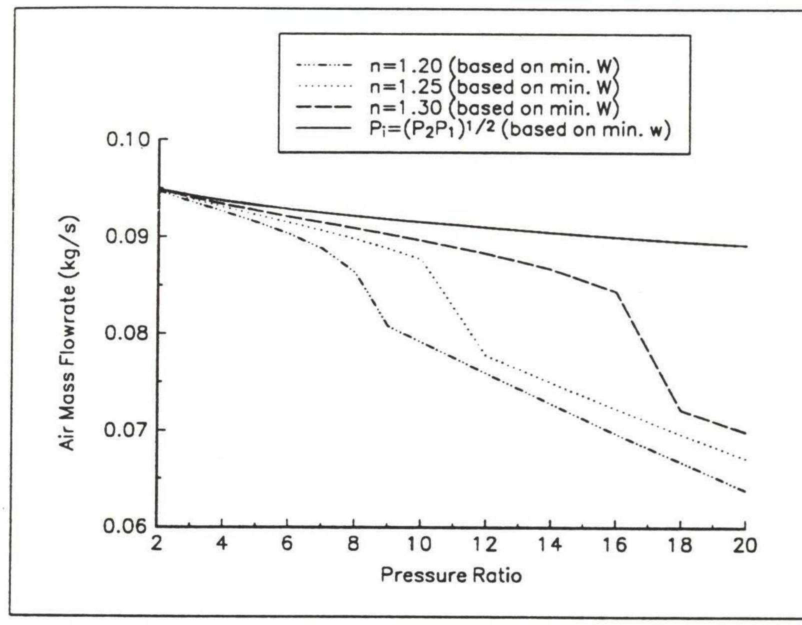

Fig. 4. Variation of air mass flow rate under 'optimum inter-stage pressure'.

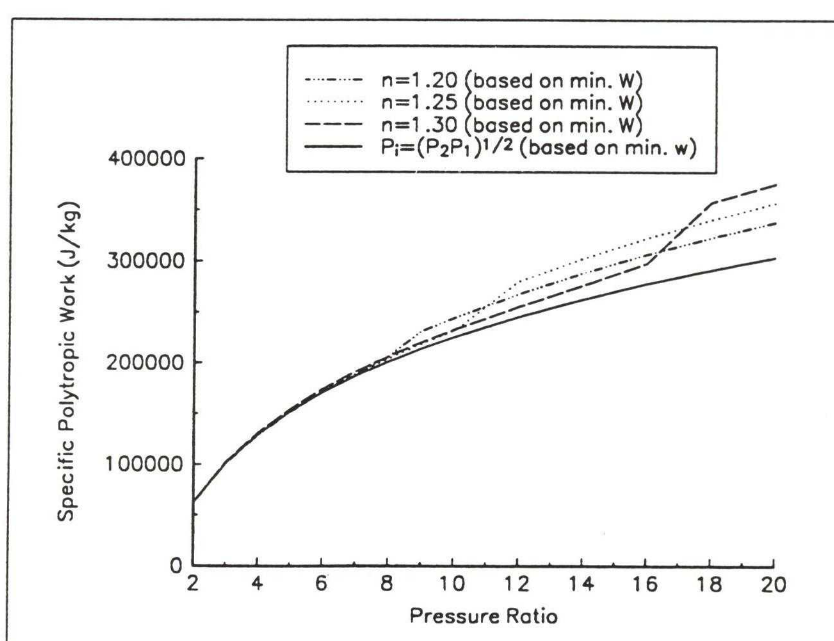

Fig. 5. Variation of specific work at 'optimum inter-stage pressure'.

REFERENCES

1. G. F. C. Rogers and Y. R. Mayhew, *Engineering Thermodynamics, Work and Heat Transfer*, 3rd edn. ELBS Longman, pp. 356–357 (1989).
2. M. D. Burghardt, *Engineering Thermodynamics with Applications*, 2nd edn. Harper & Row, New York, pp. 354–356 (1982).
3. T. D. Eastop and A. McConkey, *Applied Thermodynamics for Engineering Technologists*, 2nd edn. Longman, London, pp. 262–264 (1976).
4. R. K. Will, *Thermodynamics for Engineering Technologists*. Marcel Dekker, New York, pp. 170–172 (1979).
5. R. C. Fellinger and W. J. Cook, *Introduction to Engineering Thermodynamics*. William C. Brown, Dubuque, IA, pp. 510–511 (1985).
6. R. Joel, *Basic Engineering Thermodynamics*, 4th edn, Longman Scientific and Technical, London, pp. 399–401 (1987).