Int. J. Engng Ed. Vol. 13, No. 2, p. 98–107, 1997
Printed in Great Britain.

0949-149X/91 $3.00+0.00

A Model for the Measurement of Creativity. Part II: Creative Paths and Case Study*

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As a final step towards a methodology for the measurement of creativity, a definition for the creativity of a candidate designer is given. By introducing the concept of creative paths, it is shown how the $c_E Qe_X$ diagram is used as a nomogram for the graphic display of creative performance during the execution of a creative act. The possible appearance of a limit on the quality of a design is discussed. The design of an ore-mill for the mining industry is used as a case study.

NOTATION

Capital letters

A_o	operational availability
$A(Q)$	area associated with Q on $c_E Qe_X$ diagram
C_E	creative effort
C_{EF}	creative effort required to access facilities
C_i	i-th characteristic of product
C_{bi}	physical boundary of C_i
C_{Si}	specified requirement for C_i
$C_E(Q)$	E_X-independent part of C_E
CC	construction cost
E_d	invested education
E_{dr}	relevant invested education
E_X	expertise
IC	installation cost
K_Q	function of Q
MC	maintenance cost
Q	product quality
Q_l	limiting Q
R	constant

Lower-case letters

c_E	non-dimensional C_E wrt E_{XR}
$C_E(Q)$	e_X-independent part of c_E
c'_E	$c_E(Q)/e_X$
c_r	creativity
c_{rabs}	absolute c_r
c_{rrel}	relative c_r
e_X	non-dimensional E_X wrt E_{XR}
$g(E_X)$	function of E_X
k_Q	function of Q
m	exponent in $C_E(Q)$
n	exponent in $C_E(Q)$
p	constant
w_i	weighting value

Greek letters

Γ	constants in $C_E(Q)$
γ	constants in $c_E(Q)$
η	mechanical efficiency

Subscript

R	reference designer

* Accepted 20 January 1997.
* Part I published in Vol. 13:1.

INTRODUCTION

In the present set of two papers the author proposes a guiding model for the measurement of creativity. The model relates product quality, designer expertise and designer creative effort in such a way that creativity can be calculated as a function of time as the creative process proceeds. The model shows what demands are placed on the designer to ensure quality designs and as such hints at engineering curriculum and syllabus design. The model approaches creativity *from the outside looking in,* i.e. it evaluates creativity by looking at the quality of the product which is being formed; duly considering (a) the effort which has been spent on the process and (b) the level of appropriate education of the creator.

In Part I definitions for product design quality, designer expertise and creative effort are introduced in a quantified manner such that they are interrelated and can be graphically depicted as a set of hyperbolic curves, the $c_E Qe_X$ diagram. Product design quality Q is mathematically related to the product characteristics in such a manner that $Q = 0$ at design inception, $Q = 1$ when the requirements of the development specification are met and $Q \to \infty$ when characteristics approach physical boundaries. Designer expertise is expressed in person-years and includes all the invested tertiary education and relevant experience contained in the design team, as well as investment in design software and laboratory facilities.

The notion of a reference designer, whose performance is used for the calibration of the $c_E Qe_X$ diagram of a product type, is introduced. For a reference designer, the functional relationship between expertise (E_X) creative effort (C_E) and quality is defined such that creativity is always unity and his effort is always uniform with respect to Q.

In the present study it is shown how the $c_E Qe_X$ diagram can be used as a nomogram for the graphic portrayal of creative processes of *candidate* (i.e. not necessarily *reference*) designers and how the present model may be used for the measurement of creativity by means of a case study. It is also shown how the candidate designer could run into a cul-de-sac in the event where either creative performance was too low or if the designer had persisted with an inadequate design solution for too long—the appearance of limited quality.

THEORY

The present discussion relies heavily on the mathematical model which was introduced in Part I. For convenience the relevant equations are summarized in Appendix A, retaining the numbering system from Part I.

Creative paths and quality limit

Consider a $c_E Qe_X$ diagram which has been constructed for a *reference designer* associated with a particular product, Fig. 1. From the moment of project start, this designer will develop his product according to the uniform effort rule (see Part I) and quality Q will progress with creative effort C_E along straight line cc, the *creative path* for this case.

Consider also two other *candidate designers*, A and B (each with creative potential differing from that of the reference designer), with initial expertise levels aa and bb, respectively, as shown on the figure. At the moment of project inception no solution yet exists hence $Q = 0$ (the case of a redesign, where a prior solution might have existed, is treated by appropriately defining Q—see the case study). As work on the design commences, a number of departure conceptual designs start crystallizing on the drawing board, all most likely of low quality as measured against the development specification. As time progresses, quality Q and creative effort spent C_E grow in magnitude and, if the candidate and reference designers were equally creative, co-ordinates of Q and c'_E would describe a horizontal locus, the creative path. As, however, the creative abilities of designers could differ (even having equal expertise) the creative paths for A and B, constructed by plotting the co-ordinate pairs (Q, c'_E), will in general not be horizontal. Uncreative designers would find it impossible to maintain the required rate of increase of Q and their creative paths would drop. Creative paths for underperforming designers would typically proceed as indicated on Fig. 1.

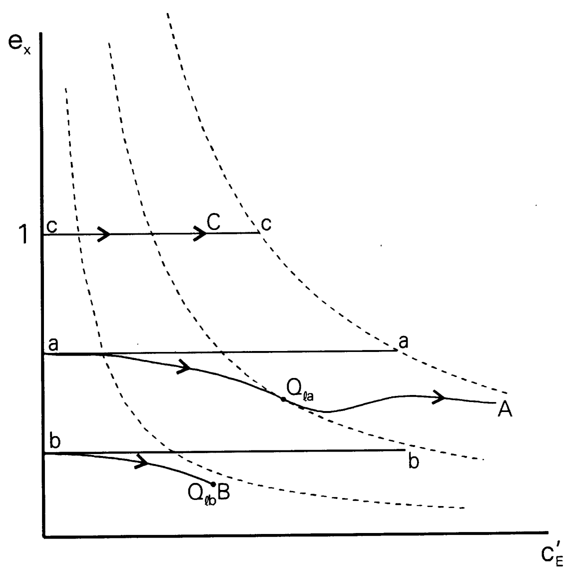


Fig. 1. Creative paths for various designers.

An interesting feature of low performance creative paths (like cases A and B) is the appearance of a limited quality solution, Q_l, Fig. 1, which is the best solution obtainable without rather spending creative effort on research or improvement of design or managerial skills than on design evolution. At these points the creative paths and the hyperbolas are tangent where after the path would typically stick to the hyperbola (which implies that the design solution has been frozen whilst all effort is being spent towards improving skills or knowledge). A formula for the estimation of Q_l is derived in Appendix B.

Measuring creativity by means of $c_E Qe_X$

The $c_E Qe_X$ approach can be used to establish the creativity of a designer. Specific features of this approach are:

- Individual or group-collective creativity can be measured.
- The method is product-type specific.
- Educational level is accounted for.

The method entails the following:

1. Identification of a reference designer followed by construction of a $c_E Qe_X$ diagram. This is done by establishing (see Appendix A, Part I) the constants Γ_0, Γ_1, Γ_2, m and n of equation (7), the function $g(E_X)$ of equation (6) and the expertise E_{XR}, (equation (A4)) whereafter the hyperbolas for chosen values of Q are given by equation (12).
2. The measurement of product quality as a function of creative effort (Q vs C_E data points) for the candidate designer as the design progresses.

3. The creative path is now constructed by noting from equations (11) and (12) that:

$$e_X = \frac{k_Q Q g(e_X) E_{XR}}{C_E} \quad (20)$$

from which e_X can implicitly be solved for every (C_E, Q) co-ordinate pair. Note that e_X as obtained by equation (20) negates that obtained by equation (A6), which is only used for reference purposes. With e_X known, c'_E is obtained from equation (11).

Two different concepts of creativity are introduced, i.e. one absolute and the other relative.

(*a*) *Absolute creativity*, c_{rabs}. In this case creativity is determined with respect to the reference level of expertise, $e_{Xref} = E_X/E_{XR}$, where E_X is given by equation (A6). Therefore, for the candidate designer, e_{Xref} is calculated and the co-ordinates (c'_E, e_X) are determined from the co-ordinates (C_E, Q) as discussed above. For the reference designer, set $C_{Eref} = C_E$. From equation (11) it follows that:

$$c'_{Eref} = \frac{c'_E g(e_X)}{g(e_{Xref})}$$

Now, redefine creativity (see equation (10)) as:

$$c_r = \frac{k_Q Q}{e_{Xref}\, c'_{Eref}}$$

With the help of equation (13) it hence follows that (see Fig. 2):

$$c_{rabs} = \frac{k_Q Q}{k_{Qref} Q_{ref}} = \frac{A(Q)}{A(Q_{ref})} \quad (21)$$

It should be clear that for the reference designer, $c_{rabs} = 1$ always.

(*b*) *Relative creativity* $= c_{rrel}$. Here creativity is determined by comparing the actual creative path followed to a reference creative path, where the latter is not necessarily horizontal. As before, for the actual creative path, (c'_E, e_X) are known from (C_E, Q). Now, the reference creative path is described by a given mathematical equation of the form $e_{Xref} = e_{Xref}(c'_{Eref})$. Hence, from equation (13), $k_{Qref} Q_{ref}$ is known as a function of e_{Xref}. Substituting $k_{Qref} Q_{ref}$ into equation (20) and again setting $C_{Eref} = C_E$ allows solution of the corresponding co-ordinates (c'_{Eref}, e_{Xref}). Hence the relative creativity is obtained from equation (21), see Fig. 3.

Determination of either c_{rabs} or c_{rrel} entails 'sampling' at a prescribed value of either C_E or Q. In education, time limits are usually enforced on tests, exams or assignments implying a prescribed C_E situation. In industry, on the other hand, work is usually not terminated before an acceptable value of product quality is obtained, implying a prescribed Q. In the latter case it is also common, however, to monitor progress and performance on a periodic basis, implying prescribed C_E values.

By determination of c_{rabs} or c_{rrel} the creative performance of designers is obviously assessed. For example, consider the case where a specialised course on the design of a product type is offered and it is wished to measure the creativity of the performers as they execute an assignment. Assume a reference creative path, representing the performance of a typical first class student, say, is known. Setting $e_{Xref} = 1$, creative paths for various types of performers are shown in Fig. 4. Example creative performance figures are:

Path L: $c_{rrel} < 1$: Low.
Path F: $c_{rrel} = 1$: First-class.
Path E: $c_{rrel} > 1$: Exceptional.

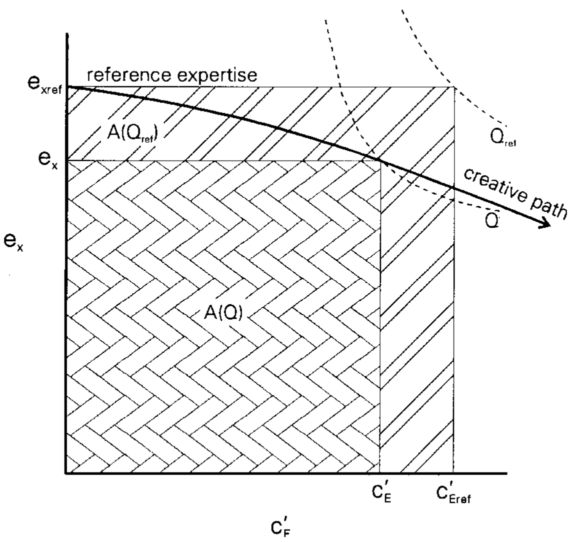


Fig. 2. Absolute creativity.

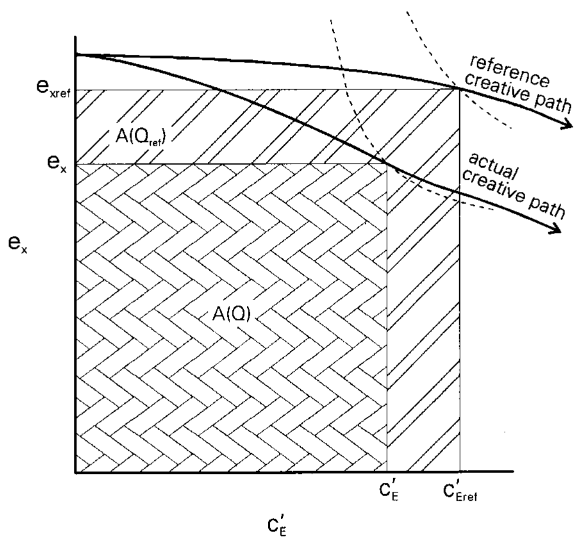


Fig. 3. Relative creativity.

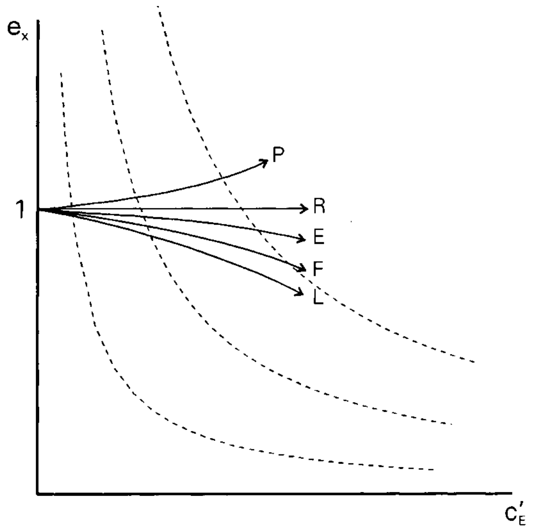


Fig. 4. Various performer types.

Path R: $c_{rabs} = 1$: Professional.
Path P: $c_{rabs} = 1$: Pioneering.

The application of the present method for the measurement of creativity relies on the ability to measure C_E and Q whilst the design effort evolves. Measurement of C_E is fairly straightforward as it is directly related to the number of person hours spent on the effort. But how do we measure Q? In general this turns out to be one of the most difficult challenges facing the designer and his management.

The definition of product quality tells us what characteristics of the design have to be measured and how they should be added to obtain Q. As was discussed earlier, for the present purposes the quality of an unfinished design is obtained by including only those characteristic values into the characteristic vector for Q which have been confirmed by means of development models. The assessment of Q should be performed by an external, objective reviewer. Hence Q will start at value zero and, if the design process is successful, gradually increase to $Q = 1$, the client's requirement.

CASE STUDY

The mining industry is constantly searching for more efficient methods of extracting minerals from rock. An energy consuming phase of the extraction process entails the mechanical grinding of the mined ore to a fine powder. Many different types of grinding mills have been developed and new designs are still appearing continually in an effort to reduce the construction, maintenance and running costs of these machines.

The example chosen for the application of the $c_E Q e_X$ technique is concerned with the designs of novel tumbling grinding mills which have recently been put forward in South Africa. The author, a colleague and a number of postgraduate students have been actively participating with industry in some of this work. The designs and innovations as will be referred to below closely resemble actual solutions which were arrived at. The case histories of the designs (including financial and progress data) are not factual as the required information was not recorded or is, from the developers' point of view, confidential at this stage.

Two engineering companies were separately contracted by the mining fraternity to develop prototypes of a new class of semi-autogenous tumbling mill. The new designs were required to have reduced construction, installation and maintenance costs at a specified delivery rate of 250 tons per hour. The companies will henceforth be individually referred to as designer A and designer B, respectively.

Product characteristics and required quality

For the client of the companies, five design characteristics were deemed most important. These were:

- construction cost
- installation cost
- maintenance cost
- operational availability
- mechanical efficiency.

Although some of these characteristics are interdependent from an engineering point of view (e.g.

Table 1. Grinding mill quality characteristics

Characteristic	Transformed Characteristic*, C_i	Threshold value ($)	Boundary value, C_{bi}	Specified value, C_{Si}	Weight w_i
1. Construction cost, CC	$\frac{CC_t}{CC} - 1$	500 000	50	4	0.27
2. Installation cost, IC	$\frac{IC_t}{IC} - 1$	50 000	50	4	0.27
3. Maintenance cost (monthly), MC	$\frac{MC_t}{MC} - 1$	2500	50	4	0.27
4. Availability, A_o	A_o	—	1.00	0.90	0.095
5. Efficiency, η	η	—	1.00	0.94	0.095

* CC_t, IC_t and MC_t are threshold values

Table 2. Relevant expertise of designers

	Designer	
Design team	A	B
Managers Engineers	} 8	} 5
Technologists	11	7
Design draughtspeople	5	5
Expertise (equation (A6))	34.2	18.3
Facilities		
Computers	0.5	1/6
Software	1.5	0.5
Workshop Laboratories	To be contracted	To be contracted

Note: Values are in person-year. For conversion of facilities to expertise, 1 person-year = $27 000

maintenance cost and availability), this dependence was of no concern to the client who defined the various product characteristics as shown in Table 1. Note that the characteristics associated with cost are mathematically transformed such that the transformed characteristic increases when the basic characteristic decreases. Also shown are threshold, boundary and specified values. If the value of a characteristic is more than its threshold value, the transformed characteristic is equated to zero.

The boundary values for the cost characteristics were set equal to a number (50) which appears to be far out of reach of contemporary designers. Quality was to be defined according to the 'stubborn' model.

Designer expertise

Company A was a natural selection for the design due to its vast experience in the field of heavy mechanical engineering, both in the mining and in the shipbuilding industries. They had been directly involved with the design of mill systems before. From their ranks they assembled a design team consisting of two engineers, two technologists and a design draughtsperson. At their disposal were three top-of-the-line desktop computers, standard software packages as well as powerful numerical finite-element design packages. Although A possessed extensive workshop and laboratory facilities, none of these were allocated to the mill project. The design team's needs in these areas had to be addressed via contracting.

Company B was considerably smaller than company A. The design team of B, which consisted of the leader, two technologists and a design draughtsperson, was lead by an experienced metallurgist. Their prior design experience had mostly been associated with the design of smallish supporting mechanical gear (e.g. trunnion bearing supports) for the mining industry. They had at their disposal two high performance desktop computers and some standard software packages and a small mechanical workshop. They were given the opportunity to attempt the mill design due to their reputation of being creative.

Table 2 summarizes the *relevant* education and facilities of the two design teams. Notice that for both designers the 'workshop expertise' is excluded as a component of the designer's expertise, as the required workshop contribution to the design effort will be expressed as an equivalent amount of creative effort, via contracting.

The progress of the two designers was monitored by an independent systems reviewing authority on a regular basis. Table 3 contains the *case histories* of the various product characteristics as they were projected at the completion of design reviews which were held at regular increments of spent creative effort.

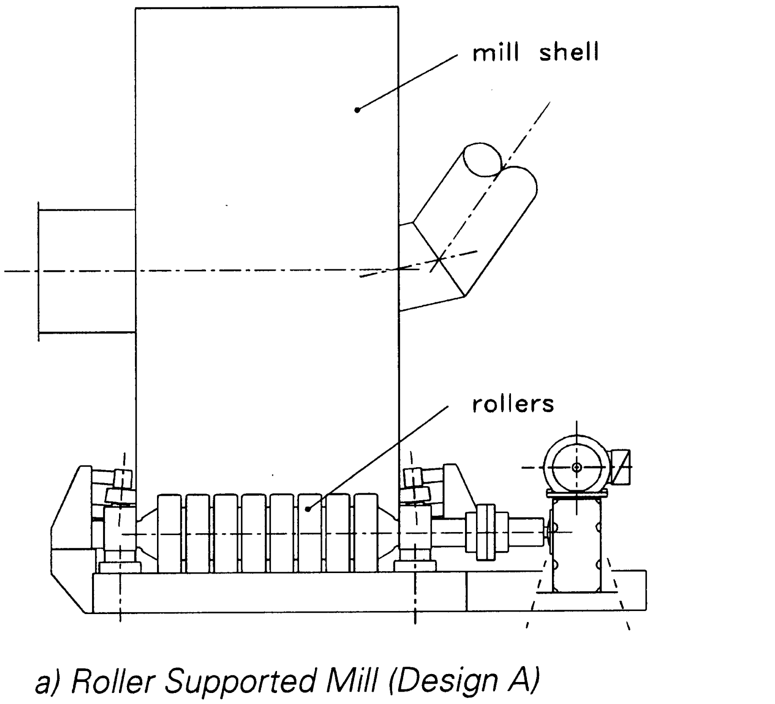
After one person-year of creative effort designer A had selected a favourite solution concept which was based on supporting and driving the mill shell on sets of rollers, thus eliminating the conventional ring gear and pinion drive and the normal trunnion bearing supports. Estimates of maintenance costs, availability and efficiency were considered to be unreliable at this stage. The next review revealed

Table 3. History of C_i and Q for designers

		Creative effort (person-year)							
Characteristic	Team	0	1	2	3	6	9	12	15
CC	A	0	6	6	5	5	5	5	5
	B	0	8	7	6	6	4	4	—
IC	A	0	6	5	5	5	5	5	5
	B	0	7	6	6	5	4	4	—
MC	A	0	0	0	0	0	3	3	6
	B	0	0	0	4	0	4	4	—
A_o	A	0	0	0	0	0	0.85	0.85	0.95
	B	0	0	0	0.90	0	0.90	0.93	—
η	A	0	0	0.95	0.95	0.95	0.95	0.95	0.95
	B	0	0	0	0.95	0.90	0.90	0.95	—
	Reference	0	0.600	0.767	0.840	0.953	1.019	1.067	1.104
Quality	A	0	0.517	0.613	0.613	0.613	0.921	0.921	1.000
	B	0	0.517	0.517	1.000	0.609	0.996	1.000	—

that installation costs had previously been overestimated and sounder projections for mechanical efficiency had been obtained via analysis and tests. At the three person-year review stage a set-back in the design process, due to unacceptably high roller wear rates measured on test rigs, was reported. It was decided to seek a solution to the problem by exploring the usage of a wide variety of roller materials. These experiments subsequently consumed considerable effort, with limited success, until a spectacular breakthrough was reported at the 15 person-year review. A new polyurethane material had been found to perform excellently, thus ending the design process on a high note.

At the first review of designer B's efforts a very novel solution concept was reported. As was done by A, the conventional mill shell drive and support system was replaced by something different, in



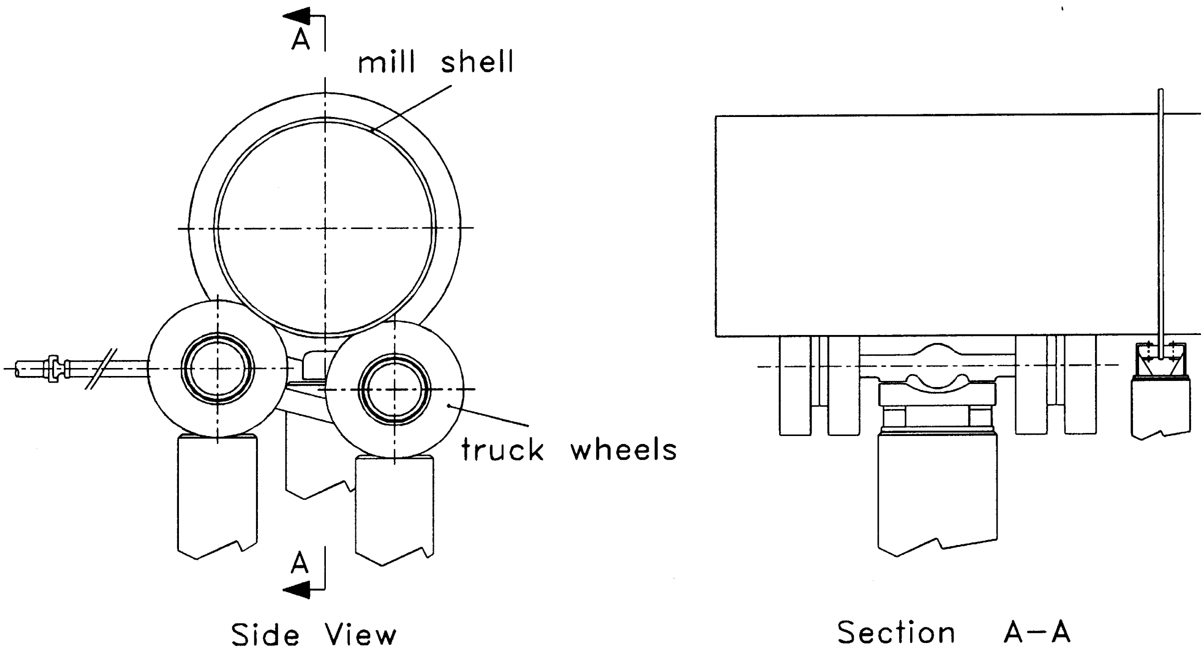


Fig. 5. Ore mill designs.

Table 4. Characteristics of reference designer

Characteristic	Value	Unit
Education, E_{dR}	16	Person-year
Facilities, C_{EFR}	2	Person-year
Creative effort to $Q = 1$, C_{ER}	8	Person-year
Expertise (equation (A4)), E_{XR}	20	Person-year
Γ_0	0	Person-year^2
Γ_1	20.79	Person-year^2
Γ_2	139.2	Person-year^2
m	0.5	—
n	7	—

this case a set of truck axle, differential and wheel assemblies. The relatively low cost of these components implied huge potential savings in construction and installation costs and there appeared to be maintenance benefits as well. Due to the novelty of the solution it was also realized, however, that the risks were higher and the mechanical efficiency inherent to the solution was considered suspect. A thorough study of the drive system efficiency was hence conducted and initial results were favourable. As development progressed, a serious structural problem was discovered. The high stresses in the mill shell in the vicinity of the truck wheel contact areas resulted in fatigue crack appearance in the steel. This setback, which would imply huge maintenance costs, was reported at the 6 person-year review. The problem was subsequently alleviated by a structural modification to the mill shell, which increased construction costs. To add insult to injury, advanced experimentation revealed that the early estimates of mechanical efficiency had been optimistic for certain modes of mill operation. It was finally decided by the client to accept the design, but for future usage in limited modes only, thus moving from a 'stubborn' towards a more 'accommodating' attitude.

Sketches of the two designs are shown in Fig. 5.

Creativity analysis

To perform a creativity analysis on the design cases as were discussed above, a reference designer is to be found and his performance as a designer is to be quantified. As such data are not available, the following assumptions were made:

- The expertise of the reference designer is determined by the data in Table 4.
- The constants Γ_0, Γ_1. Γ_2, m and n of equation (7) have values as shown in Table 4.
- The parameter $g(E_X)$ of equation (6) equals unity.

As a comparison between the creative performances of the two designers is the main concern here, inaccuracies in these assumptions are of little consequence.

The equations as discussed before were programmed in FORTRAN to enable computerized application of the $c_E Qe_X$ technique. The two mill designs were subsequently analyzed and the results are summarized below. Calculated histories for Q are shown in Table 3.

The calculated values for absolute creativity for the two designers are shown as a function of creative effort in Fig. 6. The fluctuations of these curves suggest that the creativity of a designer (as defined here) would sporadically rise or slump during the creative process. Such oscillations reflect a changing comparative creative rate between the candidate and the reference designer and could be due to one or more of the following reasons:

- Dropping of the curve due to creative effort in progress, with little visible improvement of the design quality, resulting from:
  (a) the finite time interval required for development work to show positive results (e.g. segments ab and cd);
  (b) a technical obstacle which might threaten the viability of the concept (e.g. segment ef).

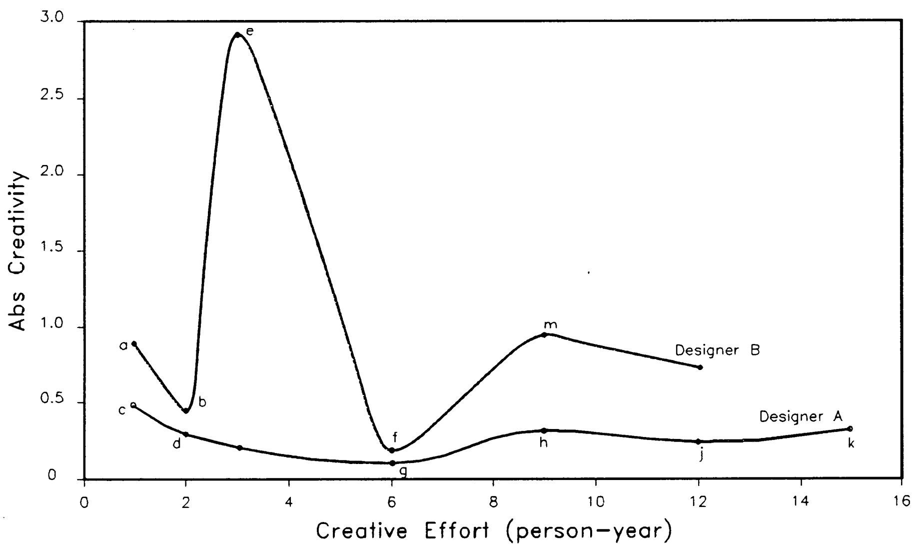


Fig. 6. Creativity of designers.

- Rising of the curve due to creative effort in progress, with significant perceived improvement of design quality, resulting from:
  - (c) consolidation of prior development work into the design (e.g. segment gh);
  - (d) an assessment error by the reviewing group (e.g. segment be), something which often happens in practice;
  - (e) a technical breakthrough such as shown by segments fm and jk.

Figure 6 also shows that the creativity of designer B exceeds that of designer A despite the fact that B's final product is slightly inferior technically. This is due to the present definition of creativity, which gives increased creativity for reduced values of expertise or creative effort spent (equation (10)). As B managed to create a competitive product with less initial expertise and at a lower cost than A, the present definition gives B credit for his superior ability to 'form out of nothing', which is in line with the Oxford definition of creativity.

The computed $c_E Qe_X$ diagram is shown in Fig. 7. Superimposed on the diagram are the creative paths of three designers, i.e. the reference designer and designers A and B, respectively. The oscillations in these paths are due to the reasons discussed above. To recapitulate the meaning of these paths, a typical point on designer B's path, such as point R with approximate co-ordinates (0.25; 0.4), would imply:

1. A total amount of $E_{XR} g(E_X) c'_E = 20 \times 1 \times 0.25 = 5.0$ person-years of creative effort has been spent on the project (see equation (11)).
2. The quality of the design at this stage is about 0.77 (interpolate between hyperbolas or solve for Q from equations (13) and (14)).
3. The expertise which the reference designer would have required to reach a $Q = 0.77$ design after 5.0 person-years of creative effort would have been $0.4 \times 20 = 8$ person-years. As this value is much lower than the expertise of designer B (18.3 person-years), it clearly shows creative underperformance at this point of the process.

These considerations would be valuable to an assessor of a design effort. Poor management or assessment of the design team could lead not only to a low creative path, but also to the path oscillating with low damping. A sustained non-horizontal creative path could also indicate an inappropriately defined reference designer.

CONCLUSIONS

In this paper the concept of creative paths was introduced and it was shown how these loci on the $c_E Qe_X$ diagram indicate the history of a candidate designer's creative performance. Application of the theory to an ore mill design was presented and it was indicated how the shape of a particular creative path could illuminate the creative performance of a design team.

Research is needed to establish whether the present model for the measurement of creativity is in harmony with established knowledge regarding favourable creative characteristics (such as is discussed in [1]) of the creator.

Further work on the $c_E Qe_X$ technique is needed,

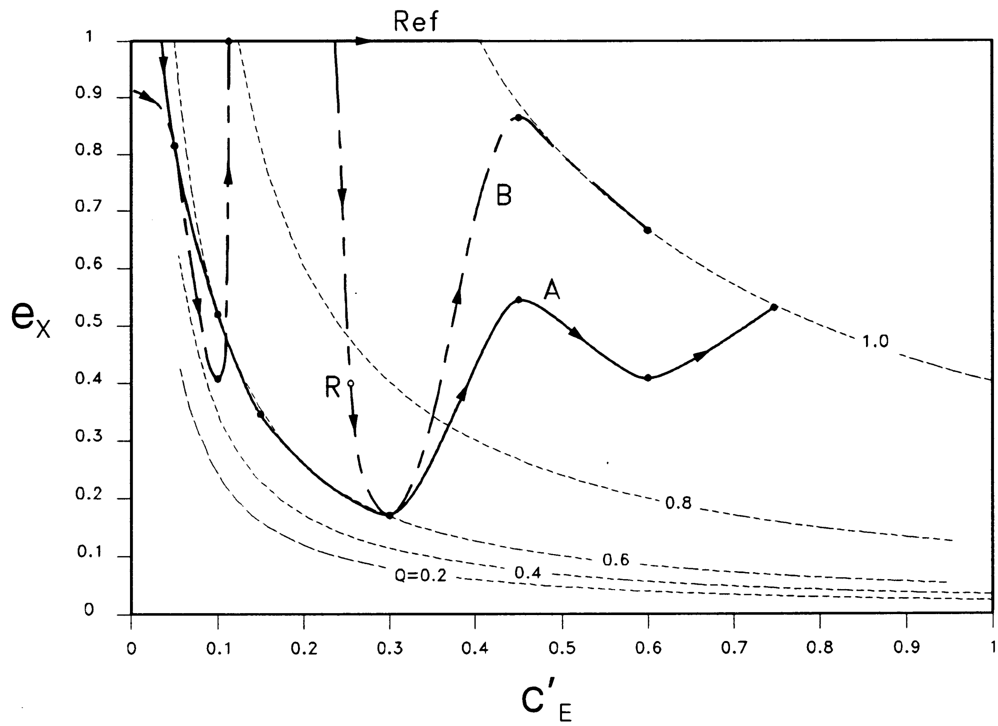


Fig. 7. $c_E Qe_X$ diagram for mill designs.

particularly with regard to the construction of diagrams based on the performance of actual companies. Such comparative data would be of value to the companies themselves, to economic analysts and to educational institutions. Research projects for postgraduate students could be designed to include the collection and analysis of appropriate data. For undergraduate design courses, suitable benchmark assignments should be designed which are of such a nature that the construction of their creative paths during execution can be done. This will allow comparative creativity measurements of individuals and groups to be made.

Finally, it is hoped that the introduction of the $c_E Q e_X$ technique will revitalize debates on the measurement of creativity. It is anticipated that the technique could be used in fields other than engineering such as economy, physics, defence, music and sport.

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APPENDIX A: SUMMARY OF FORMULAE FROM PART I

The following formulae were developed in Part I and are used in Part II. For ease of reference, the equation numbering system from Part I is retained.

$$C_E(Q, E_X) = \frac{C_E(Q)}{E_X} g(E_X) \tag{6}$$

$$C_E(Q) = \Gamma_0 + \Gamma_1 Q^m + \Gamma_2 Q^n \tag{7}$$

$$\begin{aligned} c_r &= \frac{K_Q Q g(E_X)}{E_X C_E} \\ &= \frac{k_Q Q}{e_X c'_E} \end{aligned} \tag{10}$$

$$\begin{aligned} c'_E &= \frac{C_E}{g(E_X) E_{XR}} \\ &= \frac{c_E(Q)}{e_X} \end{aligned} \tag{11}$$

$$e_X = \frac{k_Q Q}{c'_E} \tag{12}$$

$$\begin{aligned} A(Q) &= k_Q Q \\ &= c'_E e_X \end{aligned} \tag{13}$$

$$A(Q) = \gamma_0 + \gamma_1 Q^m + \gamma_2 Q^n \tag{14}$$

$$E_{XR} = E_{dR} \left\{ \frac{C_{EFR}}{C_{ER}} + 1 \right\} \frac{g(E_{XR})}{g(E_{dR})} \tag{A4}$$

$$E_X \frac{g(E_d)}{g(E_X)} = \frac{E_d}{1 - \dfrac{C_{EF}}{C_{ER}} \dfrac{g(E_{XR})}{g(E_d)} \dfrac{E_d}{E_{XR}}} \tag{A6}$$

APPENDIX B: LIMITING QUALITY Q_L

Suppose that, due to a lack of appropriate expertise, the absolute creativity of a designer deteriorates with creative effort, as follows:

$$c_{rabs} = 1 - R \left[\frac{C_E}{g(E_{Xref}) E_{XR}} \right]^p \tag{B1}$$

where p and R are constants and E_{Xref} equals the E_X value obtained from equation (A6). But, from equations (21), (13) and (14) it follows that:

$$c_{rabs} = \frac{A(Q)}{A(Q_{ref})} = \frac{\gamma_0 + \gamma_1 Q^m + \gamma_2 Q^n}{c'_{Eref}\, e_{Xref}} \tag{B2}$$

where e_{Xref} is a constant. It was shown earlier that:

$$c'_{Eref} = \frac{c'_E g(e_X)}{g(e_{Xref})} \tag{B3}$$

As, from equation (11):

$$C_E = c'_E g(E_X) E_{XR}$$

equation (B1) becomes, with the help of equation (B3):

$$c_{rabs} = 1 - R[c'_{Eref}]^p \tag{B4}$$

Equating equations (B2) and (B4) gives:

$$[1 - R(c'_{ref})^p]c'_{Eref}\, e_{Xref} = \gamma_0 + \gamma_1 Q^m + \gamma_2 Q^n \tag{B5}$$

The co-ordinates for the creative path can be obtained by solving (numerically, in general), equation (B5) for c'_{Eref} as a function of Q. The corresponding co-ordinates (c'_E, e_X) are obtained from the simultaneous solution of equations (13) and (B3). Taking the derivative of equation (B5):

$$\therefore \quad [1 - (1+p)R(c'_{Eref})^p]\, \mathrm{d}c'_{Eref}\, e_{Xref} = [m\gamma_1 Q^{m-1} + n\gamma_2 Q^{n-1}]\, \mathrm{d}Q \tag{B6}$$

Now, limiting quality is obtained when:

$$\frac{\mathrm{d}Q}{\mathrm{d}c'_{Eref}} = 0$$

Hence, from equation (B6):

$$1 - (1+p)R(c'_{Erefl})^p = 0$$

or

$$c'_{Erefl} = [(1+p)R]^{-1/p} \tag{B7}$$

Q_l can hence be obtained by substituting equation (B7) into equation (B5):

$$\gamma_0 + \gamma_1 Q_l^m + \gamma_2 Q_l^n = \left\{1 - \frac{R}{(1+p)R}\right\}\left\{\frac{1}{(1+p)R}\right\}^{1/p} e_{Xref} = \frac{p e_{Xref}}{(1+p)\{(1+p)R\}^{1/p}} \tag{B8}$$

Equation (B8) has to be solved numerically for Q_l by means of a technique such as Newton-Raphson, in general. It is also useful to note that, by substituting equation (B7) into equation (B4):

$$c_{rabsl} = \frac{p}{1+p} \tag{B9}$$

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