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The Fourbar Linkage: Pseudographic Kinematic Analysis*

W. P. BOYLE AND K. LIU
Division of Engineering, Saint Mary's University, Halifax, Nova Scotia, Canada, B3H 3C3

Kinematics for the fourbar linkage, in both open and crossed configurations, are modelled using plane geometry rule statements written in the software TK Solver. The solution is iterative, initiating from guessed position co-ordinates for one end of the rocker, with the program generating new guesses for successive angular positions. Diagram drawing is not required, although a sketch is an aid in making the initial position co-ordinate guesses.

SUMMARY OF EDUCATIONAL ASPECTS OF THE PAPER

1. The paper discusses materials/software for a course in Dynamics of Rigid Bodies.
2. Students in all branches of engineering are taught in this course.
3. Level of the course: Sophomore—2nd year of a 5-year programme.
4. Mode of presentation is through lectures.
5. The material is presented in a regular course.
6. Class or hours required to cover the material: 3 hours of lectures per week plus 3 hours of tutorial/laboratory work for 1 semester.
7. Student homework or revision hours required for the materials: about 10 hours per semester.
8. Pseudographics offers an alternative method for the kinematic solution of planar mechanisms. The method is non-vectorial and strongly reinforces the principles of graphical methods for mechanisms.
9. The standard text recommended in the course, in addition to authors' notes: A. Bedford and W. Fowler, *Engineering Mechanics-Dynamics*, Addison-Wesley Publishing Co. Inc. (1995).

INTRODUCTION

VELOCITY and acceleration parameters for the fourbar linkage can be derived [1] by successive differentiation of the loop position vector equation.

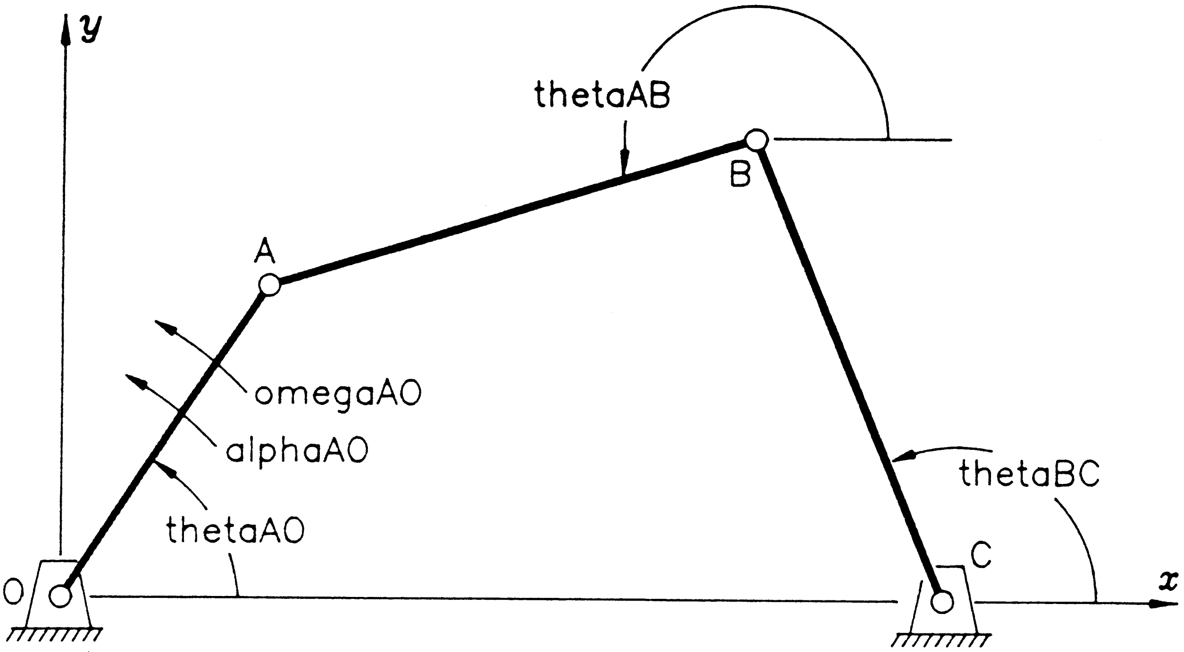

Fig. 1. Fourbar linkage, open configuration. The model is solved for nondimensional data as presented in Norton [1]. Table P7-1, row c, p 261: AO = 10, AB = 6, BC = 8, OC = 3, crossed, thetaAO = 45°, omegaAO = −15, alphaAO = −10.

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Vertex	x	y	Location as per TK model
0	ϕ^*	ϕ^*	Origin of coordinates placed at pin O.
A	xA	yA	Locus of A is circle of radius AO, centre O.
B	xB	yB	Locus of pin B is circle of radius BC, centre C; the locus of the disp. of B rel. to A is a circle of rad. AB, centre A.
C	xC*	ϕ^*	xC is the length of the earth link OC; yC = 0.
o, c	ϕ, ϕ^*	ϕ, ϕ^*	Vertices o, c, are at the origin of coordinates.
a	xa	ya	Locus of a is circle of radius vA, centre o, with the line oa being ±90° out of phase with the position line OA.
b	xb	yb	Vertex b is at the intersection of the lines cb (slope m5) and ab (slope m4).
o1, c1	ϕ, ϕ^*	ϕ, ϕ^*	Vertices o1, c1 are at the origin of coordinates.
a11	xa11	ya11	Locus of a11 is circle of radius aAn, centre o1, with the line o1a11 being 180° out of phase with the position line OA.
a1	xa1	ya1	Vertex a1 distanced aAt from a11, with the line a11a1 being ±90° out of phase with the position line OA.
b11	xb11	yb11	Locus of b11 is a circle of radius aBn, centre c1. The line c1b11 is 180° out of phase with the position line CB
b111	xb111	yb111	Vertex b111 distanced aBAn from a1 with the line a1b111 parallel to the position line BA.
b1	xb1	yb1	Vertex b1 located at the intersection of the line b111b1 (slope m4) and the line b11b1 (slope m5).

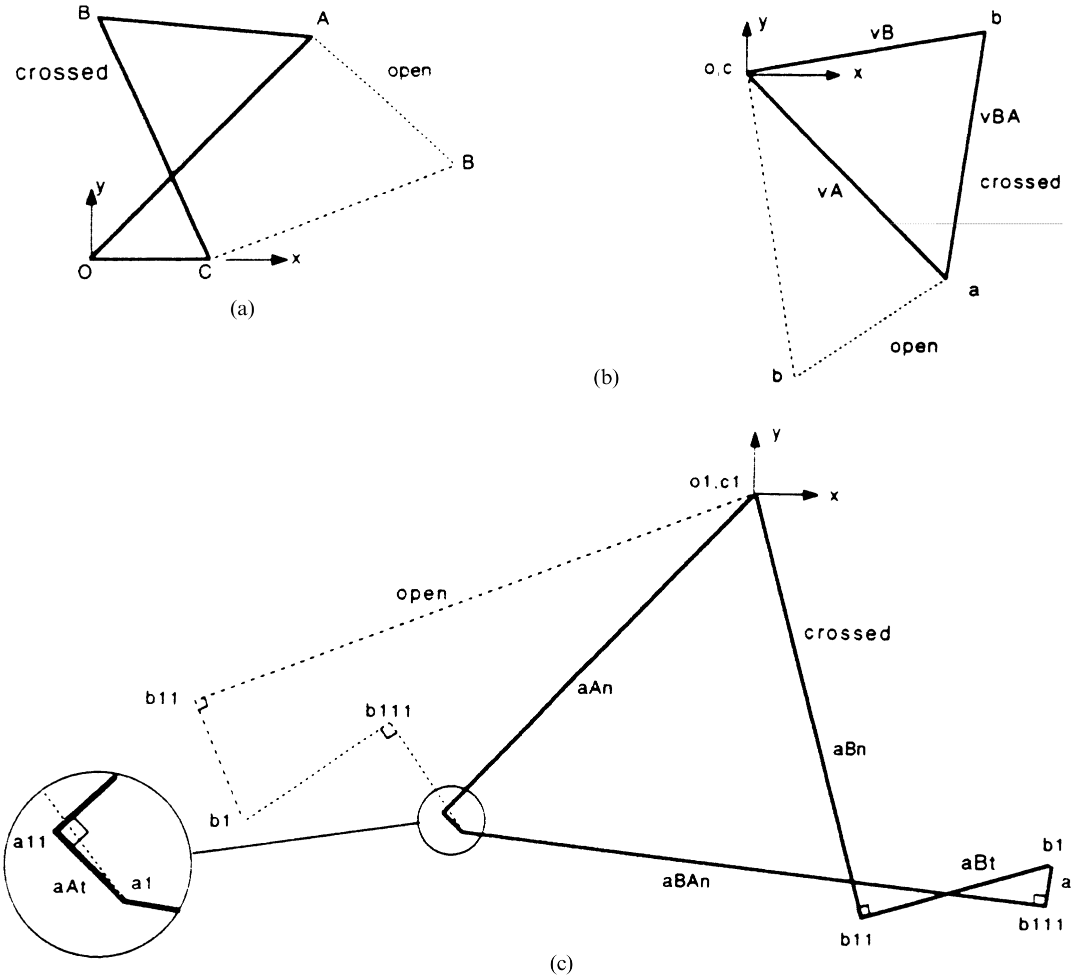

Fig. 2. Co-ordinate generation. Input co-ordinates (twelve) appear with asterisks. (a) Position diagram, crossed and open. Upper case letters denote the ends of links. (b) Velocity diagram, crossed and open. Lower case letters denote the ends of velocity vectors. (c) Acceleration diagram, crossed and open. Lower case letter with single, double, or triple numerals denote the ends of acceleration vectors.

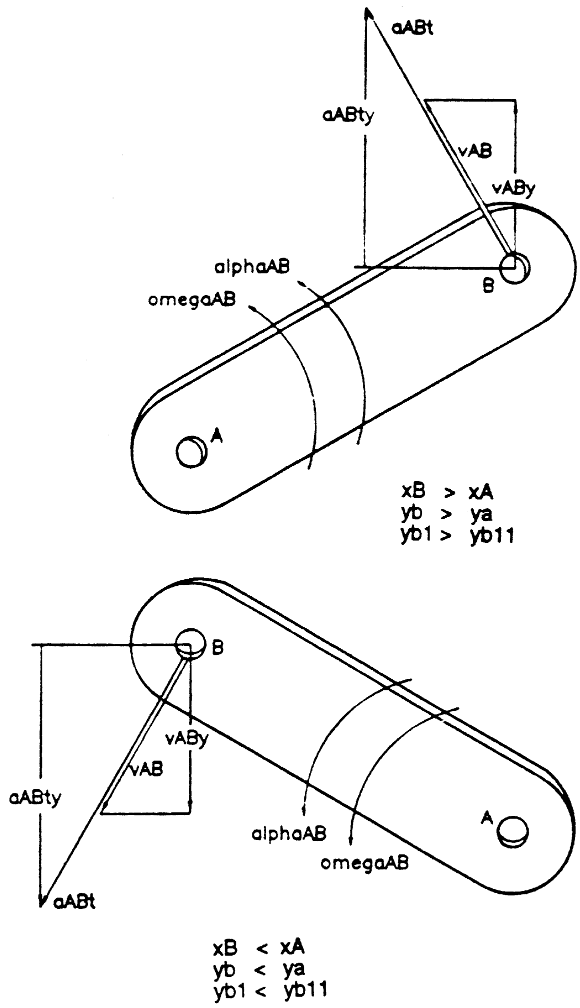

Fig. 3. The signs associated with angular velocity/acceleration of the links AB and BC are determined by comparing values of the appropriate co-ordinates of the kinematic polygons. Anti-clockwise rotation or acceleration is positive, so if one, *and only one*, of the pair of conditions below is untrue, then the rotational velocity/acceleration is negative (clockwise).

if xB > xA and yb ≥ ya then omegaAB ≥ 0
if xB > xA and yb1 ≥ yb11 then alpha AB ≥ 0

However, the mathematical steps are somewhat tortuous, and it can be difficult to relate intermediate results to physical parameters.

An alternative approach, in vogue in introductory dynamics texts [2], involves vectorial expression of the rigid body kinematic relations between pairs of points at each end of the members. Simultaneous solution of sets of scalar equations yield the unknown kinematic variables.

Graphical methods offer a more intuitive analysis, but are usually limited to a single driving link position, with determination of the complete status requiring a large number of repetitions of the diagram set. An earlier paper [3] outlined a TK Solver-based non-iterative method [4] for generation of the kinematics of the slider crank, and the present paper extends the technique to a fourbar mechanism, as shown in Fig. 1.

MECHANISM ANALYSIS

A model was used to derive numerical values for co-ordinates of the vertices of the position, velocity, and acceleration polygons, for both the open and crossed configurations of the fourbar linkage. The TK rules handle the problem in the same manner, and in the same order, as a uni-position graphical solution.

The three kinematic diagrams for a particular crossed fourbar mechanism with a driving link position of 45° above the horizontal are shown in Fig. 2. With twelve given input values, an additional eighteen co-ordinates completely define these figures.

Position, Fig. 2(a)

The locus of pin A is circular, with the co-ordinates (xA, yA) defined by the angle thetaAO and the link length AO. The point B is constrained to move in a circular path, centred at pivot C and with radius equal to the rocker length BC; xB is

Rule

```
call position(thetaAO;xA,yA,xB,yB,xP,yP,thetaAB,thetaBC,m4,m5)
call velocity(thetaAO,xB,xA,m4,m5;vB,omegaAB,omegaBC)
call acceleration(thetaAO,thetaAB,thetaBC,xA,xB,m4,m5,omegaAB,omegaBC;alphaAB,alphaBC)
if and (solved(),elt()<length('thetaAO)) then place ('xB,elt()+1)=xB
if and (solved(),elt()<length('thetaAO)) then place ('yB,elt()+1)=yB
```

Fig. 4. TK Solver Rule Sheet. The 'if and' conditional statements serve to (a) delay the insertion of generated guesses until each pass is complete, (b) prevent the length of the guess list from becoming longer each time the model is solved.

Comment: position
Parameter Variables: AO,AB,BC,xC,ratio
Argument Variables: thetaAO
Result Variables: xA,yA,xB,yB,xP,yP,thetaAB,thetaBC,m4,m5

S Rule
```
(xA,yA)=(AO*cosd(thetaAO),AO*sind(thetaAO)) ;xA,yA
AB^2=((xB-xA)^2+(yB-yA)^2) ;xB,yB
BC^2=((xB-xC)^2+yB^2) ;xB,yB
thetaAB=atan2d(yA-yB,xA-xB) ;thetaAB
thetaBC=atan2d(yB,xB-xC) ;thetaBC
m2=tand(thetaAB) ;m2
m3=tand(thetaBC) ;m3
m2*m4=-1 ;m4
m3*m5=-1 ;m5
(xP,yP)=(xA+AB*ratio*cosd(thetaAB+180),yA+AB*ratio*sind(thetaAB+180)) ;xP,yP
```

(a)

Comment: velocity
Parameter Variables: AO,AB,BC,xC,omegaAO
Argument Variables: thetaAO,xB,xA,m4,m5
Result Variables: vB,omegaAB,omegaBC

S Rule
```
vA=omegaAO*AO ;vA
(xa,ya)=(vA*cosd(thetaAO+90),vA*sind(thetaAO+90)) ;xa,ya
xb*(m5-m4)=ya-m4*xa ;xb
yb=m5*xb ;yb
vB=sqrt(xb^2+yb^2) ;vB
vBA=sqrt((xb-xa)^2+(yb-ya)^2) ;vBA
if and (xB>=xA,yb>=ya) then omegaAB*AB=vBA ;omegaAB
if and (xB<=xA,yb<=ya) then omegaAB*AB=vBA ;omegaAB
if and (xB>=xA,yb<=ya) then omegaAB*AB=-vBA ;omegaAB
if and (xB<=xA,yb>=ya) then omegaAB*AB=-vBA ;omegaAB

if and (xB>=xC,yb>=0) then omegaBC*BC=vB ;omegaBC
if and (xB<=xC,yb<=0) then omegaBC*BC=vB ;omegaBC
if and (xB>=xC,yb<=0) then omegaBC*BC=-vB ;omegaBC
if and (xB<=xC,yb>=0) then omegaBC*BC=-vB ;omegaBC
```

(b)

Fig. 5. TK Solver Rule Function Subsheets for (a) position, (b) velocity, (c) acceleration.

```
Comment:                 acceleration
Parameter Variables:     AO,AB,BC,xC,omegaAO,alphaAO
Argument Variables:      thetaAO,thetaAB,thetaBC,xA,xB,m4,m5,omegaAB,omegaBC
Result Variables:        alphaAB,alphaBC
S  Rule
   aAn=omegaAO^2*AO  ;aAn
   aAt=alphaAO*AO  ;aAt
   (xa11,ya11)=(aAn*cosd(thetaAO+180),aAn*sind(thetaAO+180))  ;xa11,ya11
   (xa1,ya1)=(xa11+aAt*cosd(thetaAO+90),ya11+aAt*sind(thetaAO+90))  ;xa1,ya1
   aBn=omegaBC^2*BC  ;aBn
   (xb11,yb11)=(aBn*cosd(thetaBC+180),aBn*sind(thetaBC+180))  ;xb11,yb11
   aBAn=omegaAB^2*AB  ;aBAn
   (xb111,yb111)=(xa1+aBAn*cosd(thetaAB),ya1+aBAn*sind(thetaAB))  ;xb111,yb111
   xb1*(m4-m5)=m4*xb111+yb11-yb111-m5*xb11  ;xb1
   yb11-yb1=m5*(xb11-xb1)  ;yb1
   aBAt=sqrt((xb111-xb1)^2+(yb111-yb1)^2)  ;aBAt
   aBt=sqrt((xb11-xb1)^2+(yb11-yb1)^2)  ;aBt
   if and (xB>=xA,yb1>=yb111) then alphaAB*AB=aBAt  ;alphaAB
   if and (xB<=xA,yb1<=yb111) then alphaAB*AB=aBAt  ;alphaAB
   if and (xB>=xA,yb1<=yb111) then alphaAB*AB=-aBAt  ;alphaAB
   if and (xB<=xA,yb1>=yb111) then alphaAB*AB=-aBAt  ;alphaAB

   if and (xB>=xC,yb1>=yb11) then alphaBC*BC=aBt   ;alphaBC
   if and (xB<=xC,yb1<=yb11) then alphaBC*BC=aBt   ;alphaBC
   if and (xB>=xC,yb1<=yb11) then alphaBC*BC=-aBt  ;alphaBC
   if and (xB<=xC,yb1>=yb11) then alphaBC*BC=-aBt  ;alphaBC
```

(c)

Fig. 5. Continued.

found by the simultaneous solution of the equation for this path, and the equation for the circle centered at A, with a radius equal to the length of the coupler AB. TK Solver performs an iterative solution for this pair of equations, and initial guessed values are required for xB, yB. TK rules are shown in Fig. 4, the position Function Subsheet in Fig. 5(b), and the Variable Sheet in Fig. 6.

A sketch of the mechanism in both open and closed configurations provides these starting values. There is no need to specify the branch—the initial guessed co-ordinates for pin B determine which version is implemented. TK generates the corrected values, and these values are used as the guessed co-ordinates at the next angular position of the crank AO. The success of this iteration depends on the guessed values being reasonably close to the true solutions. Incrementing link AO by 5° keeps the guesses acceptably close, but for mechanisms with dramatic positional changes a smaller interval may be required. TK is very accommodating *vis-à-vis* the guesses used for the solution of linear equations, but as the system becomes more nonlinear, then the guesses must be more judicious.

St	Input	Name	Output	Unit	Comment
	+10	AO			length of link AO
	+6	AB			length of link AB
	+8	BC			length of link BC
	+3	xC			horizontal location of pin C
	+4	ratio			length AP/length AP
	-15	omegaAO			angular velocity of link AO
	-10	alphaAO			angular acceleration of link AO
L	+45	thetaAO			angle of link AO
L		xB	+1.11		coordinate of pin B
L		yB	+7.77		coordinate of pin B
		thetaAB	-6.73		angle of link AB
		thetaBC	+103.65		angle of link BC
L		xP	-16.76		coordinate of point P
L		yP	+9.88		coordinate of point P
L		omegaAB	-22.77		angular velocity of link AB
L		omegaBC	-15.7		angular velocity of link BC
L		alphaAB	-65.25		angular acceleration of link AB
L		alphaBC	-148.03		angular acceleration of link BC

Fig. 6. TK Solver Variable Sheet. Values of intermediate variables employed in the Rule Function Subsheets are not displayed.

Velocity, Fig. 2(b)

The locus of vertex a is circular, with radius equal to the speed of pin A, vA. The co-ordinates (xa, ya) are defined by the fact that the velocity vector for point A is ±90° out of phase with the spatial line OA. The velocity of point B is normal to the link BC; the velocity of point B relative to point A is represented by the line ab perpendicular to the link AB. Thus the slope, m4, of line ab is known, and the co-ordinates (xb, yb) are found by the simultaneous solution of the equations for the lines ab and cb. The velocity Function Subsheet is shown in Fig. 5(b).

Links AB and BC have angular velocities omegaAB and omegaBC, with the associated signs determined by the two nests of four conditional (if and) statements appearing at the end of the velocity Function Subsheet, explained in Fig. 3.

Acceleration, Fig. 2(c)

The locus of vertex a11 is circular, with radius equal to the normal acceleration of pin A, aAn, with the co-ordinates of a11 being (xa11, ya11). This normal acceleration is 180° out of phase with the spatial line OA.

The distance of point a1 from point a11 corresponds to the tangential acceleration of pin A, aAt, with the line a11a1 being ±90° out of phase with the position line OA. The appropriate sign is determined by the sense of the angular acceleration.

The line a1b111 of the acceleration diagram represents the normal acceleration of pin B relative to pin A, aBAn, with an inclination parallel to the spatial tine BA of slope m2, i.e., a1B111 = aBAn = $(\text{omegaAB})^2$AB.

With the angular speed of the coupler AB determined in the velocity Function Subsheet, the vertex b111 is thus located.

The point B has two components of acceleration; the normal component, aBn, is directed from B to C, and is of magnitude, aBn = $(\text{omegaBC})^2$BC.

With the angular speed of the rocker link BC determined in the velocity Function Subsheet, the vertex b11 is thus located. The tangential component of acceleration of pin B, aB_t, is perpendicular to the link BC of slope m3, and is represented by the acceleration polygon line b11b1.

The tangential acceleration of pin B relative to pin A is represented by the line b111b1, perpendicular to the link AB. Thus the slope, m4, of line

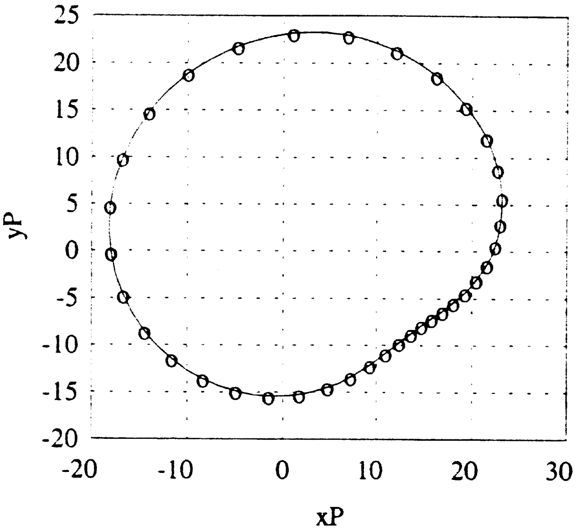

Fig. 7. Pathline for point P on the extended copler rod, ABP; AP = AB $*$ 4.

b111b1 is known, as are two of four co-ordinates, allowing xb1 and yb1 to be found by the simultaneous solution of the equations representing the lines b11b1 and b111b1. The acceleration Function Subsheet is shown in Fig. 5(c).

Links AB and BC have angular accelerations alphaAB and alphaBC, with the associated signs determined by the two nests of four conditional (if and) statements appearing at the end of the acceleration Function Subsheet, and explained in Fig. 3.

CONCLUSION

The computer model presented here offers a system, with a simple mathematical content, for the analysis of both branches of the fourbar linkage. Student feedback on their experiences with the technique has been positive. Vector algebra is not required to grasp the features of pseudographics, allowing the method to be introduced at an early stage of education.

The iterative position analysis is convenient to execute in TK Solver, with the requirement of a 'reasonable' estimate of some initial co-ordinates imposing a demand only on the common sense of the user.

Easy to display results give students good insight of less obvious kinematic features. So, for example, the pathline of an arbitrarily selected point, P, on the extended coupler rod ABP, Fig. 7, allows observation of the approximately linear motion in the crank range $145^\circ \leq$ thetaAO $\leq 265^\circ$, and also provides a qualitative perspective on the velocity of point P. Design data can be conveniently generated using pseudographics, in combination with TK Solver, while employing only elementary Cartesian geometry.

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W. Peter Boyle holds B.Sc. and Ph.D. degrees in Mechanical Engineering from The Queen's University of Belfast, is Professor of Engineering at Saint Mary's University, Halifax, and was previously a lecturer in the Department of Mechanical Engineering, University of Cape Town. He is the author of a text book *Introductory Fluid Mechanics*, and about twenty papers in a variety of topics in mechanical engineering. He has a current interest in the application of emergent software packages to engineering pedagogy, particularly in the area of mechanism analysis.

Kefu Liu, holds a Bachelor's degree in Mechanical Engineering and a Master's degree in Applied Science from The Central South University of Technology, China. He was a lecturer at that university until commencing his doctoral work in 1988 at the Technical University of Nova Scotia, receiving his Ph.D. in Mechanical Engineering in 1992. For the past two years he has been an Assistant Professor at Saint Mary's University, Halifax, and at present is an Adjunct Assistant Professor in the Department of Mechanical Engineering at TUNS. His research interests are in the areas of vibrations, machine condition monitoring, robotics, dynamics of large space structures, having a number of publications in these topics.