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Matrix Methods, Calculators and Computers: Impact on Introductory Mechanics of Materials Courses*

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It is proposed that meaningful employment of computers in introductory mechanics courses can only be achieved if matrix methods are used to formulate the theory. An approach to do this is presented in the form of lecture material and examples for an introductory mechanics of materials course. Topics begin with a traditional introduction from which matrix formulations are constructed. These are computer-ready for solution. The sample lectures, which cover equilibrium and energy approaches to uniaxial deformation problems, demonstrate the general idea.

SUMMARY OF EDUCATIONAL ASPECTS OF THIS PAPER

1. The paper proposes the introduction of computers in introductory mechanics of materials (MOM) through employment of matrix methods and provides illustrative lecture topics and examples on how to do it.
2. Students in all engineering departments except electrical and computer engineering are taught this course.
3. The level of this course is usually second-year undergraduate in the United States.
4. The mode of presentation is typically lecture with demonstrations.
5. The course is regular for engineering students.
6. The entire course requires 45 hours to cover the material. The sample lessons in this paper require 9 hours.
7. Student homework requires 3 hours per hour of lecture.
8. The novel aspect of this paper is an approach which (1) breaks with a traditional course content in place since about 1930 and (2) introduces the use of basic computational methods and computers and/or advanced scientific calculators in a meaningful way.
9. A text which covers the material is not available, but one by Roylance [1] comes close.

INTRODUCTION

TRADITIONAL introductory mechanics of materials (MOM) modeled upon Timoshenko's classic text [2] has served the engineering community well since its introduction in 1930. But the advent of the personal computer running user-friendly software tools and the advanced scientific calculator, now normally used to teach calculus, presents a technological change so significant that modernization through radical revision of this and other introductory mechanics courses in order to exploit these technologies is essential.

The problem with the current MOM curriculum is that these technologies are not employed and the loss is significant. Computers are not actively used to solve problems and powerful scientific calculators are used to only do basic arithmetic. The reason is the underlying theory necessary to their meaningful use, namely matrix methods, is not employed. The current curriculum fails in three ways:

1. Students do not actively use computers and do not take full advantage of advanced scientific calculators which breaks the computational thread in the engineering curriculum and departs from routine practice in the profession;
2. Students are not exposed to modern conceptual thought in the form of matrices, arrays and their manifestation as physical quantities, hence their physical thinking remains locked in a scalar world whereas related mathematical and computer learning is not;
3. Students are disconnected from linear algebra by mechanicians who ignore the issue, leaving them oblivious to its usefulness to mechanics, and by mathematicians who fail to clarify the importance of theory through physical applications, in effect stifling motivation to learn the linear algebra itself.

These problems can only be solved by modernizing the MOM curriculum and revising the linear algebra curriculum. This is not to say the traditional curriculum in mechanics of materials has

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Table 1. Syllabus listing lectures for the axial deformation unit. Asterisks denote new, nontraditional material

1. Displacement, strain (linear elastic and thermal), boundary conditions and modeling (simplification, conventional symbols and free-body diagrams)
2. Force, concept of stress (uniaxial, bearing and simple shear) and equilibrium
3. Properties of linear elastic materials (mechanical and thermal behavior)
4. Fundamental applications (solution for forces, stress, strain and displacement)
5. Stiffness matrix formulation (physical approach)*
6. Applications: statically determinant and indeterminate problems*
7. Applications: statically indeterminate problems with thermal strain*
8. Elastic strain energy, work and potential energy (equilibrium wells)*
9. Energy minimization formulations (advanced topic) and future directions*

not served the engineering fraternity well. It has, but neither can the community rest upon past laurels.

At issue here is how to modernize MOM without losing concepts long viewed fundamental to understanding mechanics, for instance, construction of free-body diagrams, imposition of boundary and compatibility conditions and conceptual insight to problem solution. The solution proposed in the present paper is to blend traditional approaches with the new matrix methods by developing the latter from the former and to integrate modeling and analysis so that the focus seldom becomes the mathematics. It is presented in the form of a series of lecture topics which illustrate the following approach:

1. Initiate the formulation from first principles using traditional scalar methods and equilibrium laws and apply these to basic problems.
2. Construct the matrix methods from the scalar methods and apply these to more advanced problems.
3. Introduce potential energy formulations consistently as an advanced topic in order to complete a well-rounded treatment.
4. Point to future directions.

The lecture topics chosen here cover axial deformation. Student exercises to complement these topics are not discussed, but one may start from examples given here and expand upon those given in traditional texts.

It should be understood that for this proposal to be successful, independent prerequisite courses in linear algebra and computer science are envisioned and they must be co-ordinated with the MOM course. These supporting courses are not treated here.

EQUILIBRIUM: DEFORMATION OF A BAR

Lectures in this section present first principles of uniaxial deformation and introduce concepts of stress and strain in one dimension together with associated material properties and an explanation of the tests necessary to obtain them. These concepts are integrated with simple exercises and applied to basic problems over lectures 1 through 4 in Table 1.

The approach is traditional, hence details are omitted, but one important result of these lectures is development of the axial displacement u of a bar (Fig. 1) given by:

$$u = \frac{PL}{AE} \tag{1}$$

where E is the modulus of elasticity, L and A are length and section area, respectively, and P denotes the internal force in the bar. The distinction between the internal and applied load is an important one to make.

Other discussion points include: (1) Displacement u is relative and an internal function of all variables on the right-hand side. Hence the bar can move with constant velocity under a constant stress and this can be clarified by writing:

$$u \to u_{1/2} = u_2 - u_1 \tag{2}$$

where 1 and 2 denote stations or nodes at each end of the bar, respectively. Clearly u is the same as long as the difference between u_1 and u_2 is constant. (2) So what happens if a boundary condition is applied at 1 or 2? (3) What happens if a numerical solution is attempted for u_2 if u_1 is not set (or vice-versa), that is, a boundary condition is not applied?

The importance of boundary conditions can again be addressed in the solution of simultaneous equations. Equations (1) and (2) and notions they engender serve as a transition into the matrix

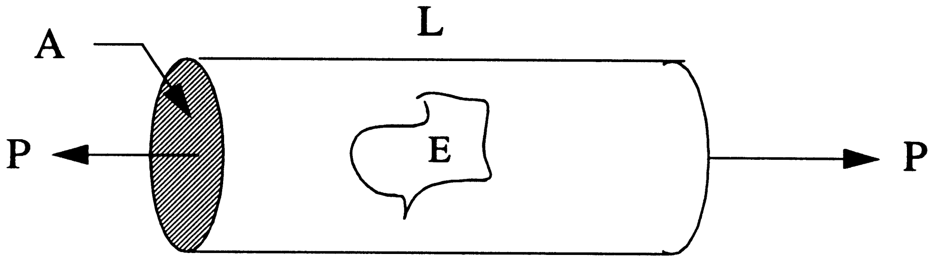

Fig. 1. Bar with elastic modulus E, length L and section area A under tensile force P.

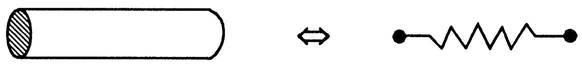

Fig. 2. Bar/spring analogy.

method covered next. If equation (1) is rearranged as follows:

$$\left(\frac{AE}{L}\right)u = P \iff ku = P \tag{3}$$

one has the connection

$$k = \frac{AE}{L} \tag{4}$$

where k is the stiffness of the bar and indeed provides the important and simplifying spring analogy, Fig. 2, given by the second equation in (3). It should be emphasized that the bar and its spring model are now synonymous.

THE MATRIX METHOD: A PHYSICAL APPROACH

The matrix method for a uniaxial bar, listed as lecture 5 in Table 1, introduces the concept of a properly posed problem as well as derivation of the matrix method. Well-posedness, so necessary to matrix inversion, provides a setting for discussion of boundary conditions and their physical importance. An example is presented to illustrate application of the method and, in particular, engineering modeling. The principal result of this section is a problem formulation which emphasizes engineering constructs and is in the form of simultaneous algebraic equations ready for solution by computer or advanced scientific calculator.

The matrix method is derived using a physical approach which permits application of the above scalar methods. To begin, the displacement influence function is defined as:

$$\sum_j k_{ij}u_j = P_i \iff \begin{bmatrix} k_{11} & k_{12} \\ k_{21} & k_{22} \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} P_1 \\ P_2 \end{bmatrix} \tag{5}$$

where the matrix form is emphasized and the nodal quantities are illustrated in Fig. 3; the matrix equation is called the bar stiffness equation and the entries of the 2×2 matrix are, for descriptive purposes, called influence factors which must yet be determined. The question is posed: How does each nodal displacement of an elastically resisting bar influence the forces at the nodes?

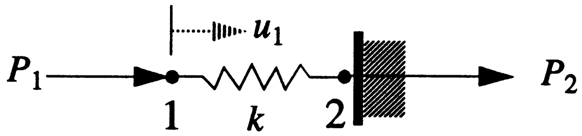

Fig. 4. Bar element under the influence of u_1 with u_2 fixed.

Case 1. Fix node 2, determine the influence of u_1

Using modeling concepts already covered, construct the free-body diagram with the boundary condition shown in Fig. 4. Then application of the first row of the stiffness equation (5) yields:

$$k_{11}u_1 + 0 = P_1 \tag{6}$$

Again referring to the diagram in Fig. 4, traditional scalar methods for static equilibrium of the spring yields:

$$P_1 = ku_1 \tag{7}$$

Since equations (6) and (7) must be equal, one obtains $k_{11} = k$ where k is the intrinsic stiffness property of the bar. Referring to Fig. 4, application of the second row of the stiffness equation (5) yields:

$$k_{21}u_1 + 0 = P_2 \tag{8}$$

Again referring to Fig. 4, static equilibrium and equation (7) yields:

$$P_2 = -P_1 = -ku_1 \tag{9}$$

By comparison of equations (8) and (9), one finds $k_{21} = -k$.

Case 2. Fix node 1, determine the influence of u_2

This case is very similar and will not be presented. It is recommended as a student exercise. The results are $k_{12} = -k$ and $k_{22} = k$. All influence factors are now known. Substituting these results into the stiffness equation (5) yields:

$$k\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} P_1 \\ P_2 \end{bmatrix} \tag{10}$$

which is the **bar element stiffness equation**. It contains in order, respectively, the bar element stiffness matrix, the element displacement vector and the element nodal applied force vector (which include reactions). Class discussion should include:

1. Correct forces are obtained for any given set of displacements u_1, u_2.
2. The inverse is not true; one cannot arbitrarily assign forces P_1, P_2 because the problem in Fig. 3 is not well posed which returns the discussion to equilibrium and boundary conditions.

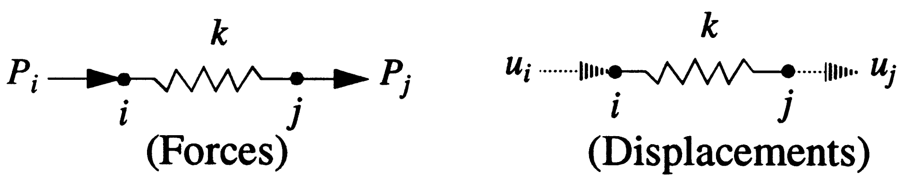

Fig. 3. Bar element showing nodal forces and nodal displacements.

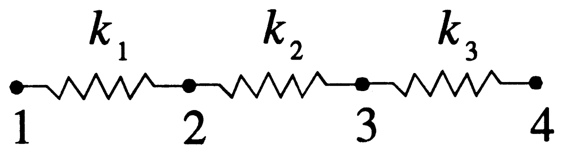

Fig. 5. Four-node, three-element chain-like structure.

3. Reanalysis of Fig. 4 using equation (10) with actual numbers and with emphasis on the importance and effects of boundary conditions.

Example. This example demonstrates application of (10) to a chain-like structure. Here only the matrix equation assembly is illustrated, but a more complete example should be provided to students.

The structure in Fig. 5 is composed of an assembly of elements each of which can be represented by equation (10). It is appealing to refer to (10) as a template. Here detailed explanation is bypassed, but the process of overlaying templates will be obvious by inspection and students should prove as a homework exercise that any row of the resulting matrix equation represents a scalar equilibrium equation written about a node of the structure. Assembling the structure results in the matrix equation:

$$\begin{bmatrix} k_1 & -k_1 & 0 & 0 \\ -k_1 & (k_1+k_2) & -k_2 & 0 \\ 0 & -k_2 & (k_2+k_3) & -k_3 \\ 0 & 0 & -k_3 & k_{33} \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{bmatrix} = \begin{bmatrix} P_1 \\ P_2 \\ P_3 \\ P_4 \end{bmatrix} \quad (11)$$

which is called the **structural stiffness equation**. It contains in order, respectively, the structural stiffness matrix, the structural displacement vector and the structural nodal applied force vector (which includes reactions). Clearly, for each element i, $k_i = (AE/L)_i$ and each k_i can be different. Class discussion should include:

1. The nature of applied forces at nodes. Can they be zero? Yes.
2. At each node, nature dictates that either the applied force or the displacement must be known, but not both. Having one, the other is obtained from solution of equation (11). Once all displacements are known, the unknown reactions can be found by substitution.
3. What happens in the solution of equation (11) if boundary conditions are not specified?

APPLICATIONS: STATICALLY INDETERMINANT PROBLEMS

Applications to problems, statically indeterminant and otherwise, are covered in lectures 6 and 7 in Table 1. They clarify application of the matrix method, but more importantly explain engineering modeling, prescription of boundary conditions and compatibility conditions. First a mechanical example is presented. Subsequently, the formulation for thermal problems, rather subtle to grasp, but easy to employ, is done and followed by an example. Overall this process is a radical departure from scalar procedures.

Example: a composite cylinder

The composite cylinder shown in Fig. 6 illustrates Problem 233 in Pytel and Singer [3] who state:

'Problem 233. A steel bar 50 mm in diameter and 2 m long is surrounded by a shell of cast iron 5 mm thick. Compute the load that will compress the combined bar a total of 0.8 mm in the length of 2 m. For steel, $E = 200$ GPa, and for cast iron, $E = 100$ GPa. Answer: P = 192 kN.'

The solution is found as follows. (1) Construct the model also shown in Fig. 6. To do this, it must be understood that both cylinders move compatibly together at node 1, hence their displacements are equal. Note: The two concentric cylinders are modeled as parallel springs. Although shown offset from each other for visual reasons, both connect between nodes 1 and 2; the bold bar denoting node 1 is not permitted to rotate. (2) Prescribe the obvious boundary conditions at node 2. (3) Using values given in the problem statement, compute for steel, $k_s = 196$ MN/m; for cast iron, $k_c = 43.2$ MN/m. (4) Assemble the structural stiffness equation:

$$\begin{bmatrix} (k_s+k_c) & -(k_s+k_c) \\ -(k_s+k_c) & (k_s+k_c) \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} P_1 \\ P_2 \end{bmatrix} \quad (12)$$

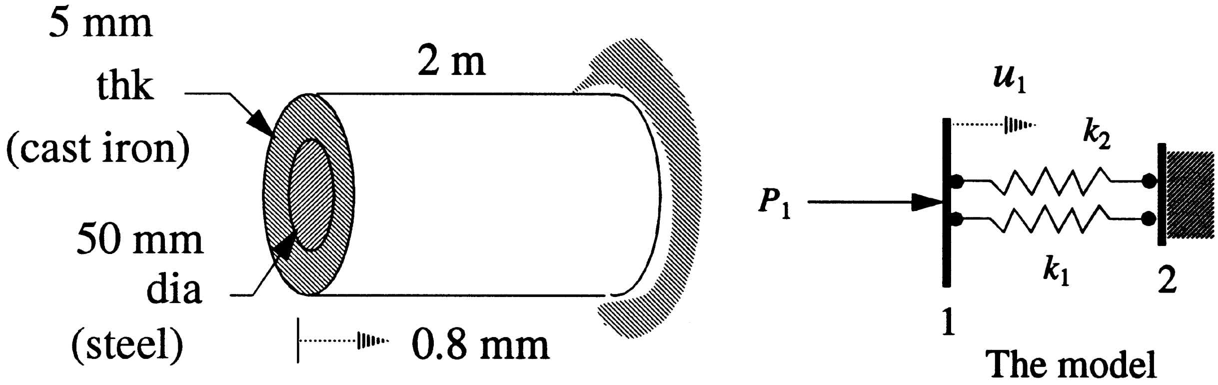

Fig. 6. Example problem, Problem 233, from Ref. 3.

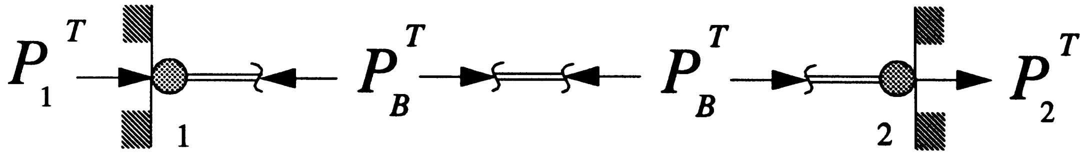

Fig. 7. Free-body diagram of bar between rigid walls in compression under increase in temperature. Nodes shown shaded.

(5) One can clearly see the assembly; however it may need more explanation for students. But note: the system matrix is singular! So the boundary condition $u_2 = 0$ must be applied. Doing this and substituting the given displacement $u_1 = 0.8\,\text{mm}$ yields directly $P_1 = (k_1 + k_2)u_1 = 192\,\text{kN}$, in agreement with the textbook answer. Discussion points include prescription of boundary conditions (repetition perhaps ad nauseam) and the notion of compatibility of displacements. (6) The reaction is recovered by substituting the given u_1 and boundary condition $u_2 = 0$ into the second row of the matrix equation and solving as follows:

$$P_2 = -(k_1 + k_2)u_1 + (k_1 + k_2)u_2$$
$$= -(k_1 + k_2)u_1 = -192\,\text{kN}.$$

(7) Other quantities such as strain and stress can be obtained using classical equations. With the stresses in hand, a discussion point is the distinction between internal and external (applied) forces (which include reactions).

Thermal problems

A very similar, but more subtle procedure applies to thermal problems. First it is necessary to derive an effective force vector due to thermal strain. A physically appealing approach is to assume that a positive temperature change ΔT acts upon an initially stress-free element fixed between rigid walls as shown in Fig. 7.

Intuition guides one to the conclusion that the constraint against thermal expansion generates internal compression P_B^T in the bar. By making a sharp distinction between thermal wall reactions at the nodes P_1^T, P_2^T (like applied forces, both shown positive) and element thermal forces P_B^T which act internally in the bar, the following steps become palatable. (1) Using traditional methods and the free-body diagram of the central portion of the bar in Fig. 7, one finds for a completely constrained bar, $P_B^T = \alpha EA \cdot \Delta T$ where α is the coefficient of thermal expansion and the sense of these forces are as shown in Fig. 7. (2) Knowing that $u_1 = u_2 = 0$ at the

```
k=100; p1=10; p2=-100;
pie[u1_,u2_] := (1/2)*k*(u2-u1)^2 - p1*u1 - p2*u2
Plot3D[pie[u,v], {u,-.2,.5},{v,-.2,.5}]
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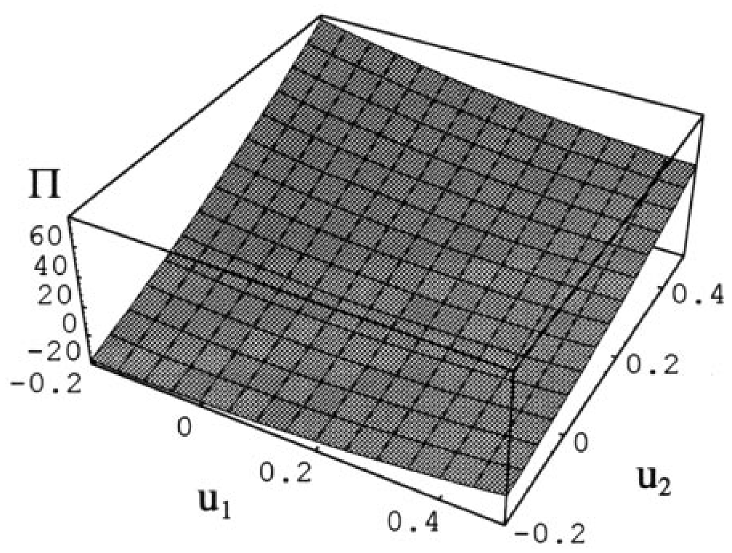


Fig. 8. The P functional for non-equilibrium with Mathematica code.

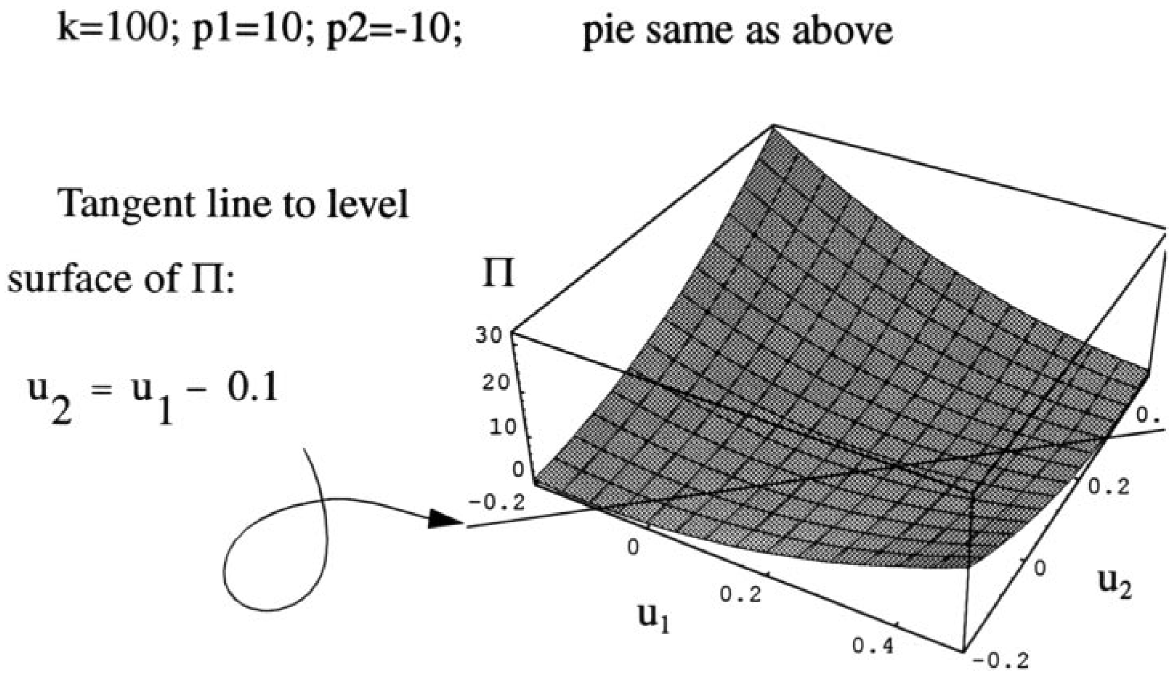


Fig. 9. The P functional for neutral equilibrium with Mathematica code.

walls, the matrix assembly procedure on the left gives a vanishing load vector:

$$k\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} = \begin{bmatrix} \text{Applied forces} \\ \text{at nodes} \end{bmatrix}$$
$$= \begin{bmatrix} P_1^T \\ P_2^T \end{bmatrix} + \begin{bmatrix} -P_B^T \\ +P_B^T \end{bmatrix} \qquad (13)$$

On the right, equilibrium is employed together with strict adherence to direction of forces at the nodes to obtain proper signs. (3) In general, a problem may not be completely constrained (u_1, u_2 may be nonzero) and may involve both mechanical and thermal loads, hence the applied forces (or reactions) are not self-equilibrating (do not sum to zero) as in equation (13). Since the nodal forces are now general, superscripts T can be dropped and the matrix method formulation for an element with both applied nodal forces (or equivalently reactions) and element thermal forces is:

$$k\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} P_1 \\ P_2 \end{bmatrix} + \alpha EA\Delta T\begin{bmatrix} -1 \\ +1 \end{bmatrix} \qquad (14)$$

where step 1 is recalled and it is important to view the trailing vector as an internal element thermal load.

Example. A rod composed of two components of equal length in series, but different properties, is attached on the left to a rigid wall and separated initially from a rigid wall on the right by a gap g. Solve for the mechanical state of the rod if the temperature rises ΔT. Using two bar elements to model the rod, the structural matrix equation is:

$$\begin{bmatrix} k_1 & -k_1 & 0 \\ -k_1 & (k_1+k_2) & -k_2 \\ 0 & -k_2 & k_2 \end{bmatrix}\begin{bmatrix} 0 \\ u_2 \\ g \end{bmatrix} = \begin{bmatrix} P_1 \\ 0 \\ P_3 \end{bmatrix} + \begin{bmatrix} -P_{B1}^T \\ P_{B1}^T - P_{B2}^T \\ P_{B2}^T \end{bmatrix} \qquad (15)$$

where $P_{Bi}^T = (\alpha EA \cdot \Delta T)_i$ is known for each part i of the rod as designated.

Many discussion points exist for this problem, among them the trade-off between known and unknown variables. If the rod is uniform and node 2 is in the middle so that all properties are equal, for instance $k_i = k = 2AE/L$ where L is the entire length of the rod, then row 2 yields:

$$k(2u_2 - g) = 0 \quad \Rightarrow \quad u_2 = g/2$$

Row 3 yields:

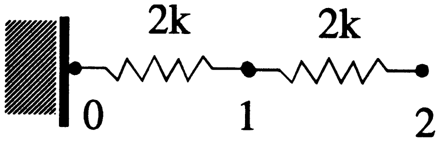
$$P_3 = -kg/2 + kg - \frac{\alpha EA}{L/2}\Delta T\frac{L}{2} = \frac{k}{2}(g - \alpha L\Delta T)$$

Hence, if $\alpha L \cdot \Delta T > g$, the rod is in compression. Otherwise P_3 either produces a tensile stress and must be applied to the end of the rod or $P_3 = 0$ and the right end just touches the wall generating no mechanical stress since the constraint is not activated.

POTENTIAL ENERGY AND WORK

Potential energy and work, covered in lectures 8 and 9, round out the unit. Strain energy would first

The Model:




```
k=100; p2=-10;
pie[u1_,u2_] := k*u1^2 + k*(u2-u1)^2 - p2*u2
plt=Plot3D[pie[u,v], {u,-10,10},{v,-10,10}]
stripes[f_] :=
   If[Mod[f, 1] > 0.5, GrayLevel[1], GrayLevel[0]]
Show[plt, ColorFunction :> (stripes[10 #]&)]
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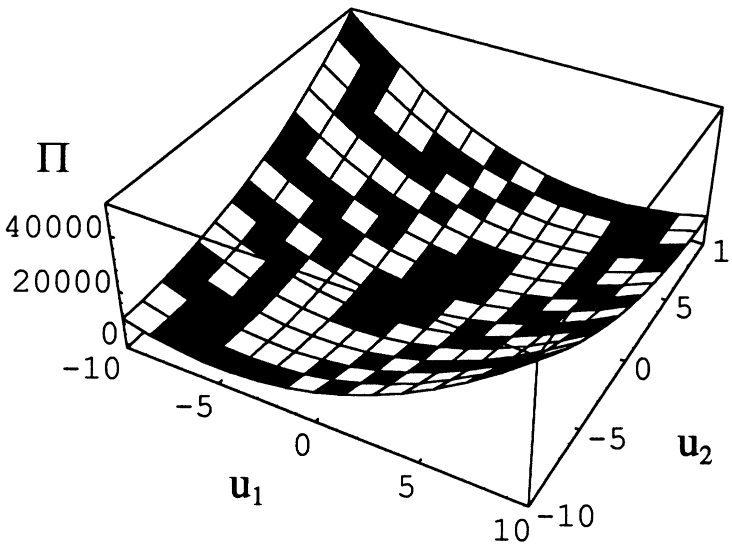


Fig. 10. The Π functional for equilibrium displayed in 3-D with Mathematica code.

be introduced in lecture 3 and here energy concepts are expanded to include the net potential of a mechanical system. Work and the minimization of the potential energy functional are introduced and used to re-derive the bar stiffness equation (5). Concepts on the meaning of well-posedness, definiteness and the importance of boundary conditions in numerics is elaborated upon through graphics. An important result is the concept of the potential well which ties energy and equilibrium together.

These two lectures are viewed as an advanced topic with the aim of conveying concepts rather than methods to solve problems. It also serves to launch future directions.

The matrix method: an energy approach

To derive the matrix method from an energy approach, the strain energy density (lecture 3) is integrated over the volume of the bar (Figs 1–3) to obtain the potential energy PE:

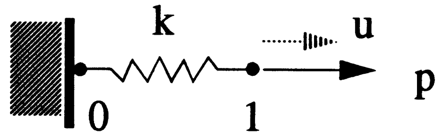
$$PE = \int_{\text{Vol}} U\,\mathrm{d}V = \frac{EA}{2}\int_0^L \left(\frac{\mathrm{d}u}{\mathrm{d}x}\right)^2 \mathrm{d}x$$
$$= \frac{EA}{2L}(u_2 - u_1)^2 = \frac{k}{2}(u_2 - u_1)^2 \qquad (16)$$

where U is strain energy density, x is the axial coordinate along the length of the bar and $\mathrm{d}u/\mathrm{d}x = (u_2 - u_1)/L$. Define work done by the applied loads to be $W = P_1u_1 + P_2u_2$, then the net potential energy Π is:

$$\Pi = PE - W$$
$$= k(u_2 - u_1)^2/2 - P_1u_1 - P_2u_2 \qquad (17)$$

It is easily accepted by students that the u's are

The Model:




```
k=100; p=-10;
pie[u_] := (1/2)k*u^2 - p*u
Plot[pie[u], {u,-.5,.5}]
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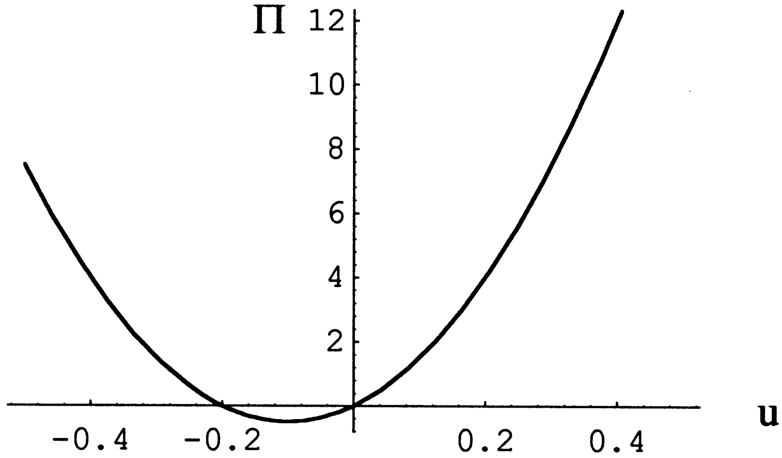

Fig. 11. The Π functional for equilibrium displayed in 2-D with Mathematica code.

the independent variables and that their equilibrium values occur at a minimum. It then remains to say that Π is what needs to be minimized. Hence:

$$
\begin{aligned}
0 &= \frac{\partial \Pi}{\partial u_1} = k(u_2 - u_1)(-1) - P_1 \\
0 &= \frac{\partial \Pi}{\partial u_2} = k(u_2 - u_1)(1) - P_2
\end{aligned} \qquad (18)
$$

By solving equation (18) and casting it into matrix form, the bar stiffness equation (10) is recovered.

Meaning of energy and equilibrium

The meaning of energy, non-equilibrium and equilibrium can be explored using graphics through the following series of cases based upon Fig. 3 for the most part. With no loss in generality, the spring constant $k = 100$ units. *Mathematica* [4] is used to produce the graphics and the code to do it is reproduced below where Π is denoted by pie.

Case: Non-Equilibrium. A plot of the net potential energy Π for non-equilibrium is shown in Fig. 8. It is observed that Π has no level surface. Students may be asked why this case is termed indefinite. Of course the applied forces are not equal and the boundary conditions have not been set.

Case: Neutral Equilibrium. Π is plotted in Fig. 9. It has a level surface along the tangent line shown and is said to be positive semi-definite which serves as a discussion point. In this case the applied forces are equal, but the spring can drift yet be in equilibrium as long as the displacements satisfy the tangent line equation shown in Fig. 9; the spring position is not unique. Why? Because the boundary conditions are not set. Discussion point: How does this effect numerical solution?

Case: Equilibrium. First Π is plotted in three dimensions in Fig. 10. In order to achieve this, the single spring in Fig. 3 must be modeled (inefficiently) as two elements in Fig. 10 in order to obtain two nodal displacements. (Students might show that if k is the stiffness for the full length of spring, then $2k$ is the stiffness for a half-length.) The potential well, observed with the help of contours superposed onto the plot, locates the equilibrium solution; Π is said to be positive definite and convex.

But Π plotted in Fig. 10 is not very accurate.

Improved resolution is obtained by again solving the problem, but in two dimensions and the results are shown in Fig. 11. It is reasonably clear that the minimum occurs at $u = u_1 = -0.1$ units.

FUTURE DIRECTIONS

Future directions (lecture 9) motivate students by reinforcing the utility of what they have learned and introducing them to other courses and professional areas open to study. For one example, the nodes in a discrete model can be endowed with mass m. Then application of D'Alembert's principle to nodal mass point i yields inertial forces $m_{ii}\ddot{u}_i$ which can be treated statically (oppositely directed to the acceleration). By including the spring and applied forces, one easily obtains for one node:

$$-m_{ii}\ddot{u}_i - k_1(u_i - u_{i-1}) + k_2(u_{i+1} - u_i) + P(t) = 0 \tag{19}$$

and, upon assembly of the matrix equation for a chain-like structure,

$$[m_{ii}]\{\ddot{u}\} + [k]\{u\} = \{P(t)\} \tag{20}$$

where the applied force is now a function of time.

Discussion may address solution approaches, simulation packages, natural frequencies, etc., and it is certain to generate interest.

CONCLUSION

This paper is based upon the proposition that an effective way to meaningfully integrate computers into introductory mechanics courses is to formulate the theory in terms of matrices and solve problems by matrix methods. This approach permits full utilization of advanced scientific calculators and even more powerful mathematical software tools. To demonstrate feasibility of the proposition, sample lectures and example problems covering uniaxial deformation for introductory mechanics of materials (MOM) is developed. The advantages of this radical revision over traditional course content are (1) a consistent, comprehensive coverage that provides several viewpoints of the topic, (2) a formulation that is computer- (or calculator-) ready for solution and usefully integrates supporting courses, namely linear algebra and computer methods and (3) a directly related spring-board to advanced courses and professional practice. Nonetheless, traditional material is also included, however the principal disadvantage is less time for traditional exercises, the repetition of which 'burn in fundamental concepts'. This new course may move along faster than traditional courses, but topics are laid out to reinforce each other and some traditional approaches can be dropped.

Each topic in this paper has been taught in part, but the complete sequence has not been taught as a whole. Hence it is a model curriculum. Future work is to co-ordinate supporting courses and teach it as a pilot class.

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