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A Software Tool for Mechanics of Composite Materials*

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The development and implementation of a software tool PROMAL' for teaching a course in Mechanics of Composite Materials is presented. The software complements the work on lengthy linear algebra computations for parametric studies and homework assignments. Most important, at the end of the course it allows the students to design and analyze laminated structures and appreciate their applications. PROMAL is both an instructional tool for the instructor and a guide to the student as it shows intermediate steps to the final calculations.

INTRODUCTION

TAUGHT first in 1988, as a conventional 'chalk and talk' lecture course, by the first author (hereafter referred to as 'I'), Mechanics of Composite Materials is now a model course in the College of Engineering at the University of South Florida. The course has 'integrated' modern and traditional tools such as videos, slide shows, tours of companies using composite materials, links on the World Wide Web to educational sites about composites, computerized multiple-choice question tutorials, open-ended design projects and software. This integration is based on the strong philosophy that all the tools are used only to complement the lecture course for making the course more interesting, and easier to understand and appreciate.

A typical course in mechanics of composites starts with a brief introduction of applications, types and manufacturing of composites. Then the mechanical behavior of composite materials is explained typically in four main topics.

- Macromechanical behavior of a single lamina is explained, including concepts about stress/strain relationship for a lamina, stiffness and strength of a lamina, and the stress/strain response due to temperature and moisture change.
- Mathematical models for finding mechanical and hygrothermal properties of a lamina in terms of individual properties of the constituents are developed. These properties include stiffness, strength, and coefficients of thermal and moisture expansion.
- Macromechanics of a single lamina are extended to the macromechanics of a laminate. Stress/strain equations for a laminate based on individual properties of the laminae that make the laminate are derived. Then mathematical models for stiffness and strength of a laminate, and effects of temperature and moisture on residual stresses in a laminate are developed.
- At the end of the course, procedures are made for failure analysis and designing of laminated composites structures.

All the above four topics are augmented for instruction and understanding by two software tools. The first one is a Windows-based [1] instructional software called PROMAL [2]. The second one is the use of symbolic manipulators such as MACSYMA [3] in understanding the mechanics of composite materials described in Reference [4]. The purpose of this paper is therefore only to discuss the development and implementation of the first software tool—PROMAL. For more information, please visit the WWW site at http://www.eng.usf.edu/ME/people/promal.html.

HISTORY

An introductory course in mechanics of composite materials involves simple linear algebra procedures that are otherwise lengthy [2]. So to appreciate the fundamentals, show intermediate steps in analysis, and conduct industrial strength analysis and design within a reasonable time, a suitable computer tool is essential. This has been the motivation for the development of the software tool PROMAL (acronym for PROgram for Micromechanical and Macromechanical Analysis of Lamina and Laminates).

PROMAL is an in-house product developed at the University of South Florida. In 1988, independent study student, Steven Jourdenias and I embarked on an ambitious project of developing an interactive software package for the mechanics of composite materials. So after eighteen months of development, PROMAL [5] (DOS version) was published nationally. Since then several students who took the mechanics of composites course continued working with me

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on adding other components as well as developing a graphical interface for the program. Using the Microsoft Visual Basic [6] computer language, we introduced PROMAL (Windows version) in 1996.

The combined effort of the faculty and students has generated a quality product that meets the needs of students and the instructor. The experiences of those who shared the authoring of the package with me have been rich. It has helped them not only in better understanding of the course and in learning new programming skills, but also they have left an excellent product for future students. Many of these students have found employment or pursued graduate school in the field of composites.

FEATURES OF PROMAL

PROMAL is a Windows-based interactive software package and has five main programs given below.

Matrix algebra

Throughout the course of mechanics of composite materials, the most used mathematical procedures are based on linear algebra. This feature of PROMAL allows the student to multiply matrices, invert square matrices and find the solution to a set of simultaneous linear equations. Many students now have programmable calculators and access to tools such as MATHCAD [7] to do such manipulations, and we have included this program only for convenience. This program allows the student to concentrate on the fundamentals of the course as opposed to spending time on lengthy matrix manipulations.

Lamina properties database

In this program, the properties of unidirectional laminas can be added, deleted, updated and saved as shown in Fig. 1. This is useful as these properties can then be loaded in other parts of the program without repeated inputs.

Macromechanical analysis of a lamina

Using the properties of unidirectional laminas saved in the above database, one can find the stiffness and compliance matrices, transformed stiffness and compliance matrices, engineering constants, strength ratios based on four major failure theories, and coefficients of thermal and moisture expansion of angle laminas. These results are then presented in textual, tabular and graphical forms.

The stiffness and the compliance matrices for a unidirectional ply are found by using the following equations [2], respectively.

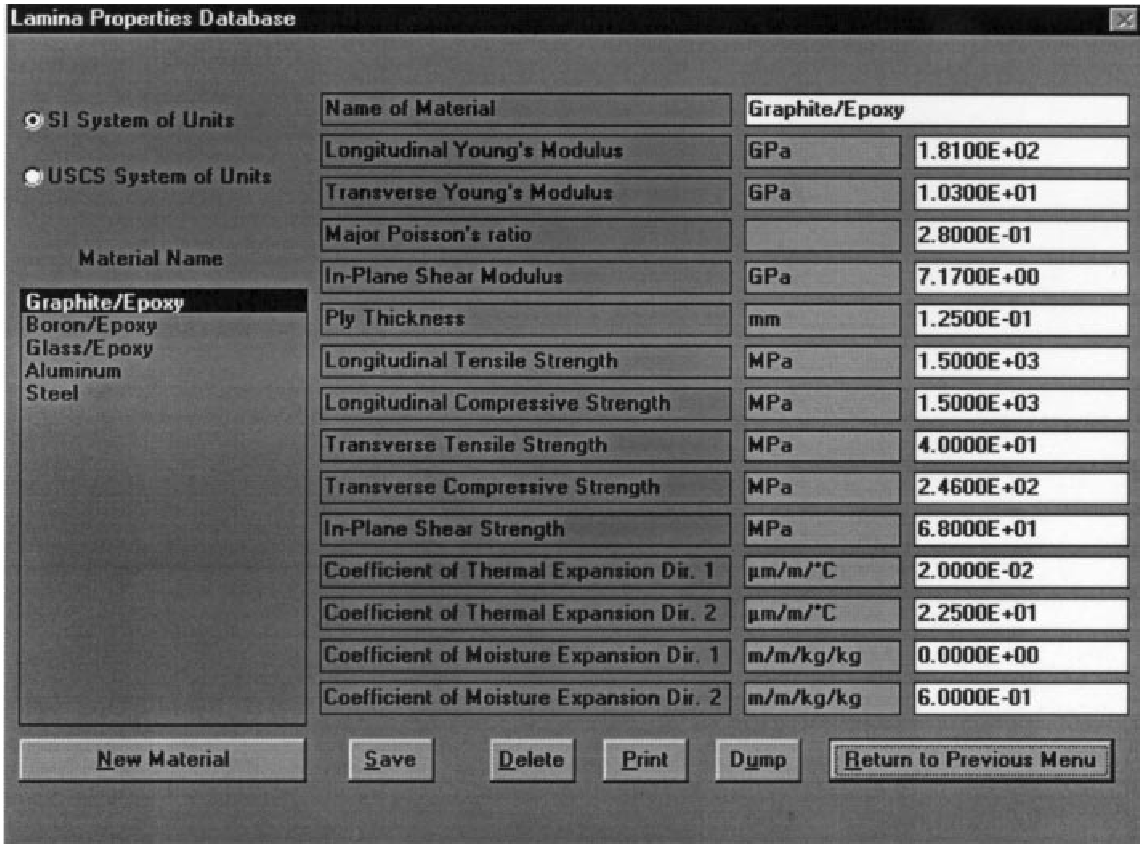

Fig. 1. Lamina properties database form.

$$[Q] = \begin{bmatrix} Q_{11} & Q_{12} & 0 \\ Q_{12} & Q_{22} & 0 \\ 0 & 0 & Q_{66} \end{bmatrix}, \quad [S] = \begin{bmatrix} S_{11} & S_{12} & 0 \\ S_{12} & S_{22} & 0 \\ 0 & 0 & S_{66} \end{bmatrix}, \tag{1a, b}$$

where,

$$S_{11} = 1/E_1, \quad S_{22} = 1/E_2, \quad S_{12} = -\nu_{12}/E_1, \quad S_{66} = 1/G_{12}, \tag{2a, b}$$

and

$$Q_{11} = E_1/(1 - \nu_{12}\nu_{21}), \quad Q_{22} = E_2/(1 - \nu_{12}\nu_{21}), \tag{3a, b}$$

$$Q_{12} = E_1\nu_{21}/(1 - \nu_{12}\nu_{21}), \quad Q_{66} = G_{12} \tag{3c, d}$$

$$[\bar{S}] = [\bar{Q}]^{-1} \tag{4}$$

The elastic moduli for an angle lamina (Fig. 2) are given as [2]:

$$1/E_x = \cos^4\theta/E_1 + (1/G_{12} - 2\nu_{12}/E_1)\sin^2\theta\cos^2\theta + \sin^4\theta/E_2 \tag{5a}$$

$$\nu_{xy} = E_x[\nu_{12}(\sin^4\theta + \cos^4\theta)/E_1 - (1/E_1 + 1/E_2 - 1/G_{12})\sin^2\theta\cos^2\theta] \tag{5b}$$

$$1/E_y = \sin^4\theta/E_1 + (1/G_{12} - 2\nu_{12}/E_1)\sin^2\theta\cos^2\theta + \cos^4\theta/E_2 \tag{5c}$$

$$1/G_{xy} = 2(2/E_1 + 2/E_2 + 4\nu_{12}/E_1 - 1/G_{12}) \times \sin^2\theta\cos^2\theta + (\cos^4\theta + \sin^4\theta)/G_{12} \tag{5d}$$

$$m_x = \sin 2\theta(K - M\cos^2\theta) \tag{5e}$$

$$m_y = \sin 2\theta(K - M\sin^2\theta) \tag{5f}$$

where

$$K = E_1/E_2 + \nu_{12} - E_1/(2G_{12}), \quad M = 1 + E_1/E_2 + 2\nu_{12} - E_1/G_{12} \tag{6}$$

The stress-strain relations in the global x-y axes are found by using [2]:

$$\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} = \begin{bmatrix} 1/E_x & -\nu_{yx}/E_y & -m_x/E_1 \\ -\nu_{xy}/E_x & 1/E_y & -m_y/E_1 \\ -m_x/E_1 & -m_y/E_1 & 1/G_{xy} \end{bmatrix} \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} \tag{7}$$

The stresses and strains in the local 1–2 axes, and the stiffness and compliance matrices for an angle ply are found by using the transformation matrices [2].

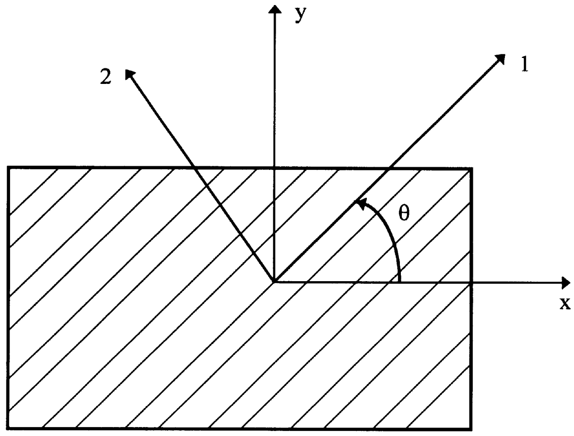

Fig. 2. Local and global axes of an angle lamina.

Micromechanics analysis of lamina

Using elastic moduli, coefficients of thermal and moisture expansion, and specific gravity of fiber and matrix, one can find the elastic moduli, and coefficients of thermal and moisture expansion of a unidirectional lamina. Again, the results are available in textual, tabular and graphical forms.

The above properties of a unidirectional lamina are found by using the following equations [2].

$$E_1 = E_f V_f + E_m V_m \tag{8a}$$

$$E_2 = E_f E_m/(E_f V_m + E_m V_f) \tag{8b}$$

$$G_{12} = G_f G_m/(G_f V_m + G_m V_f) \tag{8c}$$

$$\nu_{12} = \nu_f V_f + \nu_m V_m \tag{8d}$$

$$\nu_{21} = \nu_{12} E_2/E_1 \tag{8e}$$

$$P_f/P_c = E_f V_f/(E_f V_f + E_m(1 - V_f)) \tag{8f}$$

$$\alpha_1 = (\alpha_f E_f V_f + \alpha_m E_m[1 - V_f]) \div (E_f V_f + E_m[1 - V_f]) \tag{8g}$$

$$\alpha_2 = (1 + \nu_f)\alpha_f V_f + (1 + \nu_m)\alpha_m(1 - V_f) - \alpha_1 v_{12} \tag{8h}$$

$$\beta_1 = (\beta_m E_m \rho_c)/(E_1 \rho_m) \tag{8i}$$

$$\beta_2 = (1 + \nu_m)\beta_m \rho_c/\rho_m - \beta_1\nu_{12} \tag{8j}$$

Macromechanics of a laminate

Using the properties of the lamina properties from the database, one can analyze laminated structures. These laminates (Fig. 3) may be hybrid and unsymmetric. The output includes finding stiffness and compliance matrices, global and local strains, and strength ratios in response to mechanical, thermal and moisture loads. This program is used for design of laminated structures such as plates and thin pressure vessels at the end of the course.

The (extensional) [A], (coupling) [B], and

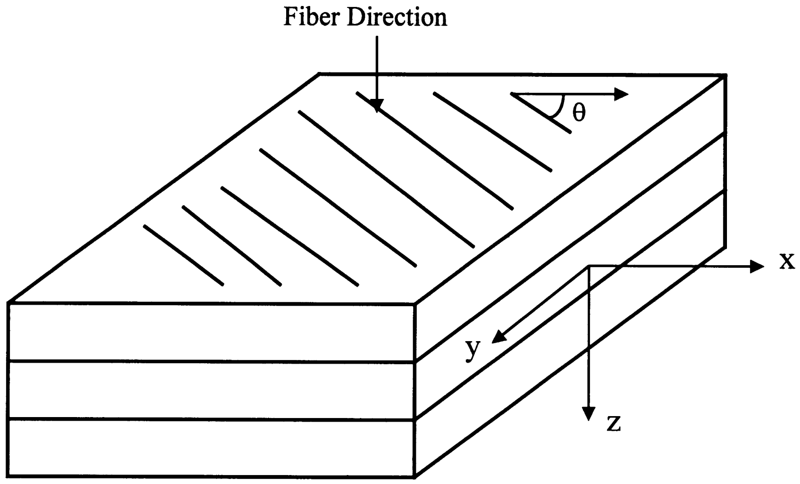

Fig. 3. Schematic of a laminate.

(bending) [D] stiffness matrices and the normalized [A], [B], and [D] stiffness matrices are calculated using the following equations[1].

$$A_{ij} = \sum_{k=1}^{n} [(\overline{Q_{ij}})_k (z_k - z_{k-1})] \tag{9a}$$

$$B_{ij} = \left(\frac{1}{2}\right) \sum_{k=1}^{n} [(\overline{Q_{ij}})_k (z_k^2 - z_{k-1}^2)] \tag{9b}$$

$$D_{ij} = \left(\frac{1}{3}\right) \sum_{k=1}^{n} [(\overline{Q_{ij}})_k (z_k^3 - z_{k-1}^3)] \tag{9c}$$

The normalized stiffness matrices are given by [2]:

$$[A^*] = (1/t)[A],$$
$$[B^*] = (2/t^2)[B], \tag{10a–c}$$
$$[D^*] = (12/t^3)[D],$$

Note that the above summations, and all other summations in this text, are from $k = 1$ to n where n is the number of plies in the laminate, and k is the ply number from top to bottom. Also, z_k is the location of the kth ply relative to the midplane of the laminate. The total thickness of the laminate is t.

The laminate stiffness and compliance matrices are defined as follows.

$$\text{Laminate stiffness matrix} = \begin{bmatrix} A & B \\ B & D \end{bmatrix} \tag{11}$$

$$\text{Laminate compliance matrix} = \begin{bmatrix} A & B \\ B & D \end{bmatrix}^{-1} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \tag{12}$$

The compliance matrices normalized with respect to the laminate thickness are given by [2]:

$$[a^*] = (t)[a],$$
$$[b^*] = (t^2/2)[b], \tag{13a–c}$$
$$[d^*] = (t^3/12)[d],$$

The equivalent plate properties are valid only if there is no coupling between the forces and moments. This is the case only for symmetric

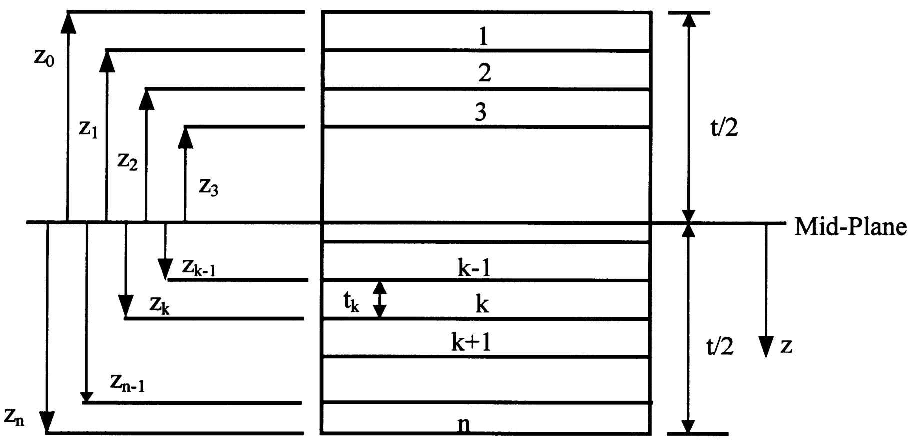

Fig. 4. Co-ordinate locations of laminae in a laminate.

laminates. These quantities are found by using the following equations [2].

Inplane longitudinal Young's modulus $E_x^0 = 1/a_{11}^*$ (14a)

Inplane transverse Young's modulus $E_y^0 = 1/a_{22}^*$ (14b)

Inplane shear modulus $G_{xy}^0 = 1/a_{66}^*$ (14c)

Inplane Poisson's ratio $\nu_{xy}^0 = -a_{12}^*/a_{11}^*$ (14d)

Flexural longitudinal Young's modulus $E_x^f = 1/d_{11}^*$ (14e)

Flexural transverse Young's modulus $E_y^f = 1/d_{22}^*$ (14f)

Flexural shear modulus $G_{xy}^f = 1/d_{66}^*$ (14g)

Flexural Poisson's ratio $\nu_{xy}^f = -d_{12}^*/d_{11}^*$ (14h)

The coefficients of thermal expansion for a laminate (α_x, α_y) are found by calculating the midplane strains resulting from a unit change in temperature. Likewise, the swelling coefficients for a laminate (β_x, β_y) are found by calculating the midplane strains caused by absorption of a unit moisture content.

The effective forces and moments, and the midplane strains and curvatures are found from the stress/strain relations [2] given below.

$$\begin{Bmatrix} N_x \\ N_y \\ N_{xy} \\ M_x \\ M_y \\ M_{xy} \end{Bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{16} & B_{11} & B_{12} & B_{16} \\ A_{12} & A_{22} & A_{26} & B_{12} & B_{22} & B_{26} \\ A_{16} & A_{26} & A_{66} & B_{16} & B_{26} & B_{66} \\ B_{11} & B_{12} & B_{16} & D_{11} & D_{12} & D_{16} \\ B_{12} & B_{22} & B_{26} & D_{12} & D_{22} & D_{26} \\ B_{16} & B_{26} & B_{66} & D_{16} & D_{26} & D_{66} \end{bmatrix} \times \begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \\ \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{Bmatrix} - \begin{Bmatrix} N_x^N \\ N_y^N \\ N_{xy}^N \\ M_x^N \\ M_y^N \\ M_{xy}^N \end{Bmatrix} \tag{15}$$

where,

$$\begin{Bmatrix} N_x \\ N_y \\ N_{xy} \end{Bmatrix} = \sum_{k=1}^{n} [\overline{Q_{ij}}]_k \left\{ \int_{z_{k-1}}^{z_k} \begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{Bmatrix} \mathrm{d}z + \int_{z_{k-1}}^{z_k} \begin{Bmatrix} \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{Bmatrix} z \, \mathrm{d}z \right\} \tag{16}$$

$$\begin{Bmatrix} M_x \\ M_y \\ M_{xy} \end{Bmatrix} = \sum_{k=1}^{n} [\overline{Q_{ij}}]_k \left\{ \int_{z_{k-1}}^{z_k} \begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{Bmatrix} z \, \mathrm{d}z + \int_{z_{k-1}}^{z_k} \begin{Bmatrix} \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{Bmatrix} z^2 \, \mathrm{d}z \right\} \tag{17}$$

$$\begin{Bmatrix} N_x^N \\ N_y^N \\ N_{xy}^N \end{Bmatrix} = \sum_{k=1}^{n} \left[\int_{k-1}^{z_k} [\overline{Q_{ij}}]_k \begin{Bmatrix} \alpha_x \\ \alpha_y \\ \alpha_{xy} \end{Bmatrix}^k \Delta T \, \mathrm{d}z + \int_{z_{k-1}}^{z_k} [\overline{Q}_{ij}]_k \begin{Bmatrix} \beta_x \\ \beta_y \\ \beta_{xy} \end{Bmatrix} C \, \mathrm{d}z \right] \tag{18}$$

$$\begin{Bmatrix} M_x^N \\ M_y^N \\ M_{xy}^N \end{Bmatrix} = \sum_{k=1}^{n} \left[\int_{z_{k-1}}^{z_k} [\overline{Q_{ij}}]_k \begin{Bmatrix} \alpha_x \\ \alpha_y \\ \alpha_{xy} \end{Bmatrix}_k \Delta T z \, \mathrm{d}z + \int_{z_{k-1}}^{z_k} [\overline{Q}_{ij}]_k \begin{Bmatrix} \beta_x \\ \beta_y \\ \beta_{xy} \end{Bmatrix}_k C z \, \mathrm{d}z \right] \tag{19}$$

N_x, N_y, and N_{xy} are stress resultants over the thickness t of the laminate; M_x, M_y, and M_{xy} are the resultant moments. ε_x^0, ε_y^0, γ_{xy}^0, κ_x, κ_y, and κ_{xy} are the midplane strains and curvatures; α and β are the transformed coefficients of thermal expansion and swelling, respectively for each ply, ΔT is the temperature difference, C is the moisture content, and N_x^N, N_y^N, N_{xy}^N, M_x^N, M_y^N, and M_{xy}^N are the fictitious forces and moments induced by temperature difference and moisture.

The global strains and stresses are found by using the following equations [2].

$$\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_{xy} \end{Bmatrix} = \begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{Bmatrix} + z \begin{Bmatrix} \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{Bmatrix} \tag{20}$$

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \sigma_{xy} \end{Bmatrix}_k = \begin{bmatrix} \overline{Q}_{11} & \overline{Q}_{12} & \overline{Q}_{16} \\ \overline{Q}_{12} & \overline{Q}_{22} & \overline{Q}_{26} \\ \overline{Q}_{16} & \overline{Q}_{26} & \overline{Q}_{66} \end{bmatrix}_k \times \left[\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix}_k - \begin{Bmatrix} \alpha_x \\ \alpha_y \\ \alpha_{xy} \end{Bmatrix}_k \Delta T - \begin{Bmatrix} \beta_x \\ \beta_y \\ \beta_{xy} \end{Bmatrix}_k C \right] \quad (21)$$

IMPLEMENTATION

Each student is given a copy of the PROMAL software in the beginning of the course. Also, it is set up in all the engineering and departmental laboratories at the university. The following are the key features of the implementation of the software.

1. Using a laptop computer and a projection system, PROMAL is set up for each lecture. It is used only if a student asks a question which I can answer using PROMAL. Typical questions relate to parametric studies or showing several other examples of the work shown in class. Out of three class meetings per week, I hold one meeting in a computer laboratory. The laboratory has one computer for every student and each desk has a video monitor connected to the instructor's computer. This allows me to conduct numerical investigations which the student can directly see. Then the student conducts similar investigation with different inputs to reinforce the fundamentals. During this class, I conduct parametric studies and also teach how to use the computer program. Although I could teach this class on all days in the computer laboratory, I have not done so because of the nature and need for efficient use of computer laboratories. Because of the monitor heights being at eye level, students have a harder time concentrating on the material unrelated to the computer implementation. Also, the demand of computer laboratories by students for other courses and general use does not make it an efficient use of such facilities.
2. With a few exceptions, I do not allow students to use PROMAL to submit their homework assignments. They can, however, use the Matrix Algebra program of PROMAL or use mathematical tools such as MATHCAD [7] to submit their assignments. They can also verify their results as it not only shows the results but also intermediate steps.
3. Only nonprogrammable (but scientific) calculators are allowed in the examinations. This is mainly done to reinforce the fact that they have to understand the fundamentals of the course as opposed to taking a 'black box' approach to the course.
4. At the end of the course, I require that students design structures made of composite materials. The final design project carries 20% of the overall grade. Students use PROMAL and they come up with different designs as most problems are open ended. Then students discuss the various alternatives with me and see how different people approach the same problem differently.
5. Some key features of the program which make PROMAL different from many other similar programs are that one can switch between the system of units with a single click, choose printing the outputs to a printer or a text file which they can merge with other reports, and show each input and output clearly with used system of units. The emphasis on international system of units is critical [8] with ever increasing global markets where our students will not only be competing but also forming partnerships.

ILLUSTRATIVE EXAMPLES

The following examples illustrate how PROMAL can be used in teaching the mechanics of composite materials course. These examples show how PROMAL illustrates the fundamentals of the course and also shows the application of PROMAL to practical industrial designs.

Example for macromechanics of a lamina

Structures subjected to torsion, such as a driving shaft, use angle laminas. This is because the angle plies provide more shear stiffness and strength than unidirectional lamina. For a graphite/epoxy unidirectional lamina with the following properties:

Longitudinal Young's modulus = 181 GPa
Transverse Young's modulus = 10.3 GPa
Major Poisson's ratio = 0.28
In-plane shear modulus = 7.17 GPa
Longitudinal tensile strength = 1500 MPa
Longitudinal compressive strength = 1500 MPa
Transverse tensile strength = 40 MPa
Transverse compressive strength = 246 MPa
In-plane shear strength = 68 MPa

Find the angle of the lamina for which the shear modulus is a maximum. Also, find this value of the maximum shear modulus and compare with the shear modulus of the unidirectional lamina.

Solution.

Inputting the above properties in the lamina database, one can load the properties in the macromechanics of a lamina program. Using tabular or graphical output choice, one finds that the maximum shear modulus is for a 45° lamina (Fig. 5). The value of shear modulus at this angle

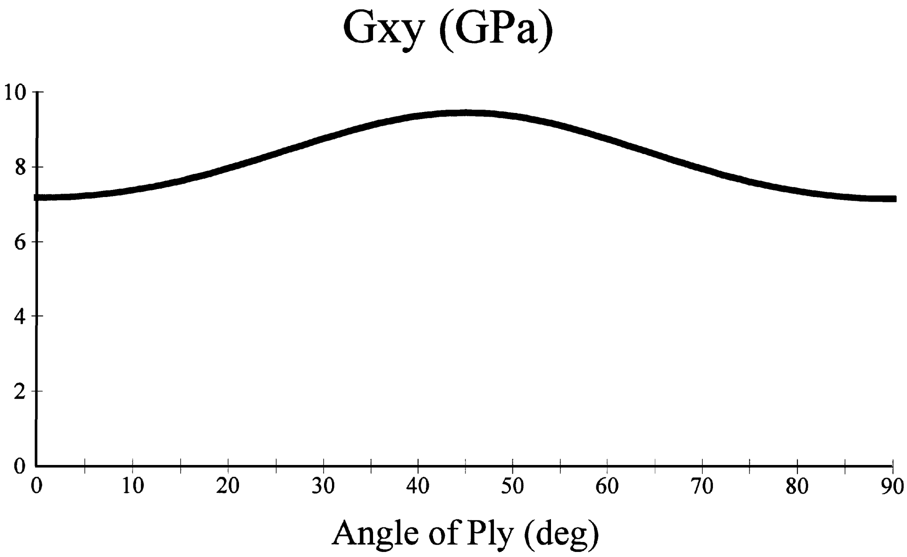

Fig. 5. Shear modulus as a function of the angle of lamina for a typical graphite/epoxy lamina.

is found as 9.46 GPa as compared to 7.17 GPa for a unidirectional lamina. An extension of this example would be to study the influence of the ratio of the longitudinal to transverse modulus of a unidirectional lamina on the shear modulus of the angle lamina.

Example for micromechanics of a lamina

In space applications such as antennas, dimensional stability in certain directions under thermal loading is a key factor in its performance. Some types of graphite fibers have a negative thermal expansion coefficient in the axial direction. By combining it with a matrix such as epoxy, the longitudinal coefficient of thermal expansion can be made zero.

Find the fiber volume fraction for which the longitudinal coefficient of thermal expansion of a graphite/epoxy is zero if the properties of graphite and epoxy are given below. *For sake of simplicity and since we seek only the longitudinal coefficient of thermal expansion, we assume that the graphite fiber is isotropic and longitudinal properties of an otherwise transversely isotropic fiber are used.*

Table 1. Longitudinal coefficient of thermal expansion of a lamina as a function of fiber volume fraction

Fiber volume fraction	Longitudinal CTE μm/m/°C
0.0	60.0
0.1	5.385
0.2	2.341
0.3	1.249
0.4	0.6880
0.5	0.3460
0.6	0.1158
0.7	0.04975
0.8	0.1745
0.9	0.2719
1.0	0.3500

Young's modulus of graphite = 300 GPa
Young's modulus of epoxy = 3.5 GPa
Poisson's ratio of graphite = 0.20
Poisson's ratio of epoxy = 0.35
Coefficient of thermal expansion of graphite = -0.35×10^{-6} m/m/°C
Coefficient of thermal expansion of epoxy = 60×10^{-6} m/m/°C

Solution.

From the micromechanics of lamina program, after inputting the above properties, one can tabulate the coefficients of thermal expansion as given in Table 1. From Table 1, for zero longitudinal coefficient of thermal expansion, the fiber volume fraction is between 0.6 and 0.7. Using different values between 0.6 and 0.7, one finds that the required fiber volume fraction is 0.6667.

Example for macromechanics of laminate

Pressure vessels made of composite materials are generally manufactured by filament winding. The winding angle determines the two parameters—resistance to pressure and resistance to expansion in diameter. In some cases of cylindrical pressure vessels, such as casings used for burning solid propellants, both design parameters are important. Why [9]? The solid fuel burns and creates high pressure. This causes large hoop strains and expansion in the diameter of the casing. If this expansion is allowed, the propellant would crack and the rate of burning would increase and build increasing pressure. Casings made of isotropic materials can reduce the expansion by using very thick casings which is not an acceptable solution. However, filament-wound casings made of composite materials can reduce the expansion due to the orthotropic nature of composites without resorting to large thicknesses.

Determine the optimum angle for resisting maximum pressure and minimizing expansion of

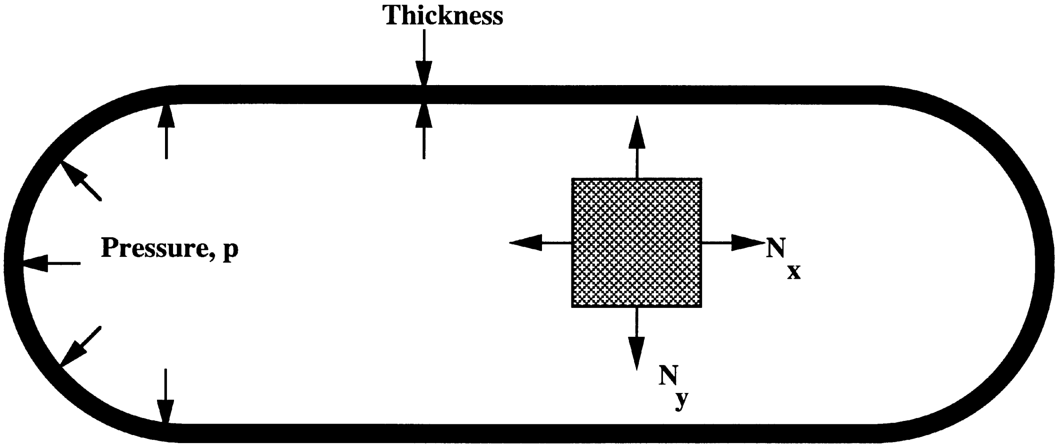

Fig. 6. A filament-wound cylindrical pressure vessel subjected to a uniform pressure.

the diameter for a filament-wound cylindrical pressure vessel. Assume the vessel is a four-ply symmetric laminate $[+\theta/-\theta]_S$ made of graphite/epoxy. The properties of a unidirectional graphite/epoxy are the same as the first example in this section. Use the maximum strain failure theory as your failure criterion.

Solution.

For a thin cylindrical pressure vessel under uniform internal pressure loading (Fig. 6), the *resultant* hoop stress (y-direction) is twice the *resultant* longitudinal stress (x-direction). Maintaining this ratio between the hoop stress and the longitudinal stress resultants, assume that the normal forces per unit width $N_x = 1000$ N/m and $N_y = 2000$ N/m. Note, only the ratio of the two values of the normal forces will affect the solution to this problem. You can now find the optimum angle for maximum pressure and minimum expansion of the diameter.

First enter the properties of the graphite/epoxy lamina in the database as shown in Fig. 1. Then enter the laminate stacking sequence $[+\theta/-\theta]_S$ and changing the angle, θ, one can calculate the minimum strength ratio of the laminate as a function of the angle, θ. As shown in Table 2, the optimum angle is 52°. Strength ratio is the ratio between the failure load and the applied load.

For minimizing the expansion of the diameter, one calculates the hoop strain, that is the normal strain in the y-direction as a function of the angle, θ. From Table 3, it shows that the optimum angle is 70°.

Table 2. Strength ratio as a function of winding angle

Angle (degrees)	Strength ratio
45	68
60	80
54	171
53	191
52	192
51	153

So the two optimum angles are different. Depending on requirements such as weight, one can find a family of filament winding angles and pressure vessel thicknesses which meet the requirements of maximum pressure and minimum expansion of the diameter.

CONCLUSIONS

An interactive software tool is used in teaching the mechanics of composites to reinforce the fundamentals of the course, verifying homework assignments, conducting parametric studies and designing laminated composite structures. PROMAL increases the capabilities of the student to solve pragmatic problems and helps the instructor in using it as an instructional tool.

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NOMENCLATURE

$[A]$ extensional stiffness matrix
$[A^*]$ normalized extensional stiffness matrix
$[a]$ extensional compliance matrix
$[a^*]$ normalized extensional compliance matrix
$[B]$ coupling stiffness matrix

Table 3. Minimum hoop strain as a function of winding angle

Angle (degrees)	Minimum hoop strain (μm/m)
0	385
90	19
73	12
70	11
64	13

$[B^*]$	normalized coupling stiffness matrix
$[b]$	coupling compliance matrix
$[b^*]$	normalized coupling compliance matrix
C	moisture content
$[D]$	bending stiffness matrix
$[D^*]$	normalized bending stiffness matrix
$[d]$	bending compliance matrix
$[d^*]$	normalized bending compliance matrix
E	Young's modulus
G	shear modulus
m	coefficient of mutual influence
M	bending moment per unit length
M^N	fictitious bending moment per unit length
N	normal force per unit length
N^N	fictitious normal force per unit length
$[Q]$	reduced stiffness matrix
$[\overline{Q}]$	transformed reduced stiffness matrix
$[S]$	reduced compliance matrix
$[\bar{S}]$	transformed reduced compliance matrix
t	laminate thickness
V	volume fraction
z	location on z-axis from laminate midplane
α	coefficient of thermal expansion
β	swelling coefficient
γ	shear strain
ΔT	change in temperature
ε	normal strain
θ	fiber angle
κ	curvature
ν	Poisson's ratio
σ	axial stress
τ	shear stress

Subscripts

1	along the direction of the fiber
2	perpendicular to the direction of the fiber
f	fiber property
m	matrix property
x	along the x-axis
y	along the y-axis

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