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An Experimental Analysis and Demonstration of the Non-Steady Flow in a Shock Tube*

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It is well known that the nonlinear features of non-steady compressible flows exhibiting finite amplitude wave patterns can be conveniently demonstrated by the hydraulic analogy. A comparative study between air and hydraulic analogy experiments, along with numerical solutions was investigated for one-dimensional shock tube flow. An educational tool in the form of a user-friendly program was written for personal computers which generated numerical solutions for homentropic flows, in compressible ideal gas and free surface water flow. A variety of initial and boundary conditions were possible. A useful tool included in the program was storage of numerical results in spreadsheets, to be subsequently presented in graphical form. Also developed were idealized, but mathematically as well as physically rigorous, numerical solutions for the shock tube. These dealt with the initial phases of shock motion up to the return of the first characteristic of the expansion fan from the driver section. This treatment provided benchmarks for the accuracy of the numerical and experimental studies.

INTRODUCTION

TRANSIENT flow phenomena are germane to many important engineering problems and it is instructive to consider experimental, analytical, and computational methods for their study and demonstration. A special case, considered here for its potential illustrative and educational values is one-dimensional non-steady motion.

Within well defined restrictions, the well known hydraulic analogy is an attractive tool to study one and two-dimensional non-steady flows. The approach is based on the comparison of the nonlinear partial differential potential equations for isentropic gas motion and the free surface flow of an incompressible liquid on a level plane [1]. The wave propagation velocity for 'shallow' water is much smaller than the acoustic velocity in a gas (usually of the order 1/1000) so that the non-steady phenomena can be observed and recorded with ease [2]. While the water table analogy introduces the limitation to long wavelengths compared to the water depth, only one property for the water determines the state [3]: it is the surface height, h. Comparatively, for an ideal gas, two independent properties are used for determining the state.

Both gas flow and the analogous water flow are complicated by the very nature of the nonlinearity of their governing potential equations. Compression waves in the gas develop towards shocks while the free surface water flow encounters the formation of hydraulic jumps. The irreversibilities cause stagnation pressure (p_0) degradation in the gas and stagnation height (h_0) loss in the water. However, while the shock front in the gas is essentially a discontinuity, the hydraulic jump in the water extends over a finite distance.

These one-dimensional non-steady flows were experimentally investigated at the University of Illinois and Texas A&M, with an air shock tube and hydraulic shock tube device, respectively. A user-friendly computer program was also developed to generate numerical solutions for both sets of experiments. For accuracy, a theoretical analysis was completed.

The theoretical analysis, restricted to the initial phase of wave travel, was carried out in considerable detail, including the effects of irreversibility due to shock formation. It also dealt with different driver-and-driven gas combinations. This analysis provided a benchmark for the accuracy of both air and hydraulic experimental data.

If it is desired to follow through with the theoretical analysis beyond the establishment of the initial wave pattern, one may select the hodograph concept based on the Method of Characteristics [4] and pursue a combined graphical and numerical procedure. While this approach can account for shock formation, changes in entropy, cross section, and even wall friction, it is extremely cumbersome to pursue beyond a few

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wave interactions, let alone towards asymptotically reached steady flow conditions. Numerical codes, developed for digital computer utilizations are now available in highly sophisticated packages.

For educational purposes it is well justified to restrict ourselves to the less ambitious task of presenting tools which give engineering students both insight into the governing mechanisms of non-steady flows and also allow a quantitative assessment of the usefulness of results which require only access to microcomputers.

NUMERICAL SOLUTIONS

The idealized shock tube performance was treated for its initial stages by tracing pressure and expansion waves in rigor (for ideal gases) by the concept of traveling shock waves and expansion waves (characteristics), respectively. The program SHTUBE.BAS (EXE) [5] generated an idealized solution. As wave patterns for one-dimensional non-steady flow increased in complexity due to multiple interactions and diverse initial and boundary conditions, a more general approach by numerical computational procedures was required than the original graphical computational methods [6, 7] became extremely tedious.

For the present objective, to provide an educational tool not exceeding the requirements for microcomputers (PCs), one must be content to remain within the restrictions imposed by homentropic flow conditions for a perfect gas. This was achieved by carrying out numerical integrations based on the finite difference formulations for the method of characteristics for one-dimensional non-steady flow [8–10].

Program ONDINSTT.BAS (EXE)

Specifically, the method of solution is an inverse marching (implicit) Euler predictor-corrector procedure. Developed in IBM-Basic, the program is adapted for use on PCs and can be run in Basic, Quickbasic, and Turbobasic. Turbobasic is required for dealing with cases demanding higher resolution in spacing and time. The program ONDINSTT.BAS (EXE) runs on DOS.

The program can deal with a wide variety of initial conditions (pressure and velocity distributions) at time = 0 and boundary conditions. These include diaphragm openings; open, gradually opening or closed ends; subcritical inflow and outflow, critical discharge; and reed valves at the end.

Of special convenience is the use of the program in connection with spreadsheets. It gives the choice of detailed printouts or data file storage for later processing in graphical form. Graphical presentations of velocity and pressure records for selected stations are generated as well as hodographs.

Illustrative cases

With a theoretical method based on homentropic concepts one must expect that rarefaction

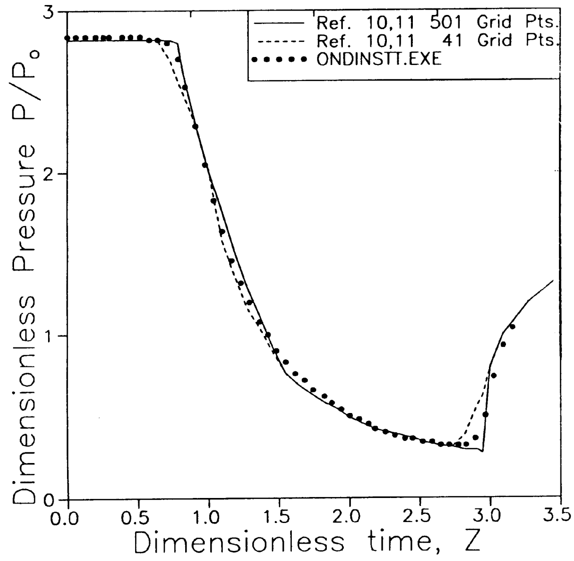

Fig. 1. Outflow from an initially pressurized tube.

waves are more accurately treated than compression (shock) waves. In addition, the calculation procedure leads to numerical wave dispersion; hence shock waves are not only deficient in resolution (steepness) but also tend to be modified in strength (reversible pressure rise) and shape (reduction in duration of pressure plateaus).

Outflow from an initially pressured tube

An example is outflow from an initially pressurized tube. The case selected corresponds to earlier studies [10, 11] and illustrates the accuracy achieved by the numerical procedure. The initial pressure level in the tube is reduced by the sudden opening at the high pressure end to the atmospheric pressure ratio. When the pressure ratio is equal to 2.83, the pressure at the closed end is calculated as a function of time. The pressure in dimensionless form, is shown in Fig. 1.

Outflow from a partially pressurized tube

For outflow from a partially pressurized tube where the left end opens instantaneously while the right end is equipped with a reed valve, admitting

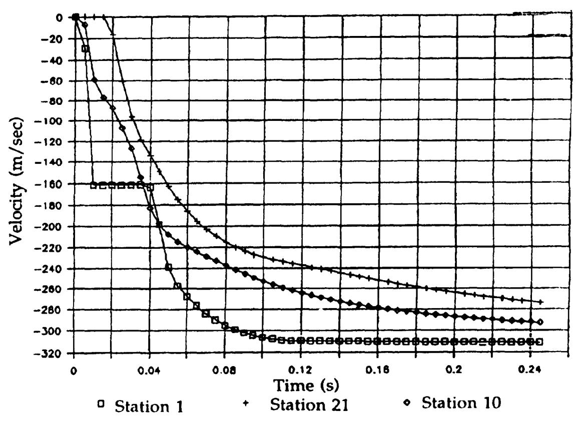

Fig. 2. Outflow from a partially pressurized tube.

the air from a pressurized reservoir leads to the eventual establishment of steady flow conditions. In Fig. 2, the velocity versus time plot for three stations within the tube is shown. The initial pressure levels are as follows: 200 kPa in the pressurized section of the tube, 110 kPa atmospheric pressure, and 200 kPa reservoir pressure (reed valve controlled right tube end). The velocity values are negative, indicating flow from right to left.

Pressure equalization in shock tube when both ends are closed

Another illustration is pressure equalization in the shock tube with both ends closed. Computed here is the development towards pressure equilibrium for a shock tube run (for the geometry see next Section), with the initial conditions of 350 kPa in the driver section and 99 kPa in the downstream section. Absolute temperature is initially 290 K in both sections.

Within the restriction of limited memory in a microcomputer one has to trade resolution in space for extended time coverage. Hence only 11 points along the tube have been selected.

Figure 3 shows the pressure history for stations 2 and 5, corresponding closely to the locations of pressure transducers in the experimental shock tube. As can be seen, the overall pattern for the experiment shown in Fig. 4, and the simulated run show reasonable agreement. It is noteworthy, albeit surprising, that the trend towards the equilibrium pressure for the homentropic computational solution approaches closely the level of 161.4 kPa, which would be predicted on the basis of a thermodynamic systems analysis using conservation of internal energy and mass.

Hodograph mapping

The quasi-linear nature of the potential equation governing one-dimensional non-steady flow allows the establishment of an intermediate general hodograph solution, independent of the particular solutions sought for specified initial and boundary conditions. While the hodograph had a decisive role in the earlier graphical solution methods, it also can serve to gain additional insight into the

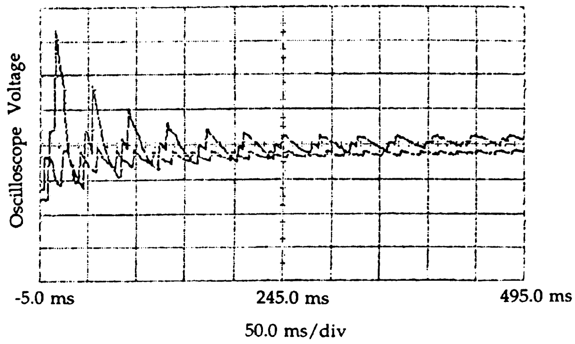

Fig. 4. Experimental air shock tube long duration, both ends closed.

nature and structure of the flow process. Data storage in spreadsheet form facilitates the calculation of hodograph curves and their graphical presentation.

The hodograph shown in Fig. 5 represents the case of outflow from a partially pressurized tube, as discussed above. It is clearly seen that the outflow condition at the left end is both on the Bernoulli Ellipse through the stagnation state in the reservoir and reaches critical velocities as time progresses towards establishment as the asymptotic steady-state solution.

Program MELTUBE.BAS (EXE)

The program MELTUBE.BAS (EXE) is used conveniently when the initial conditions are such that only pressure differences, but no velocity distributions are present; such is the case for shock tube operation. It has, therefore, been used to generate solutions corresponding to experimental conditions produced with the shock tube located in the gas dynamics laboratory of the Mechanical and Industrial Engineering Department at the University of Illinois at Urbana-Champaign.

Program ONDINSTW.BAS (EXE)

The program ONDINSTW.BAS (EXE) is needed for hydraulic shock tube investigations.

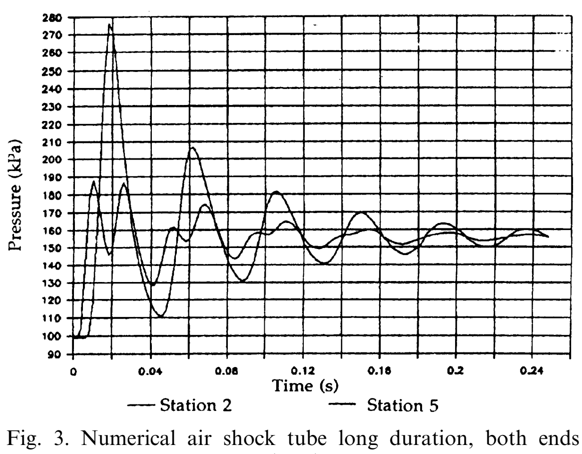

Fig. 3. Numerical air shock tube long duration, both ends closed.

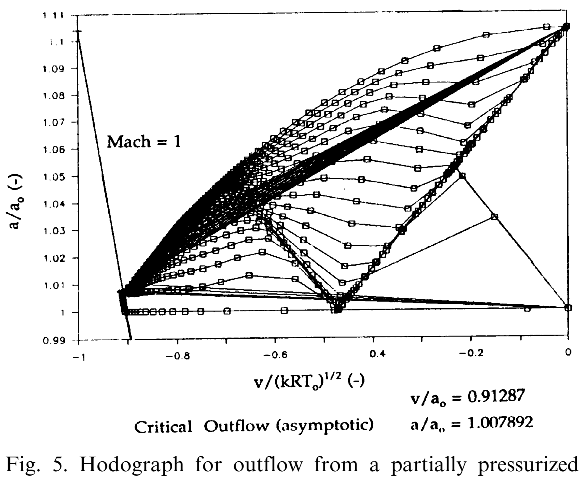

Fig. 5. Hodograph for outflow from a partially pressurized tube.

Instead of initial pressure differences in the simulated shock tube, differences in water height are present. The program ONDINSTW.BAS (EXE) has been used to generate numerical solutions corresponding to experimental hydraulic shock tube operations conducted at Texas A&M University.

UNIVERSITY OF ILLINOIS SHOCK TUBE FACILITY

Air shock tube experiments were conducted at the University of Illinois. This work was then compared to numerical solutions, idealized solutions, and experimental and numerical hydraulic work completed at Texas A&M University.

The shock tube

The compressible air shock tube at the University of Illinois had an overall interior length of 6.759 m, a driver length of 1.721 m, and a driven length of 5.038 m. The inside diameter of the circular driven tube was 0.058 m. The driver section contained transducer ports for monitoring pressure at 1.65 m and 3.6875 m, measured from the diaphragm. The single diaphragm was located at the driver end of the diaphragm holder.

Instrumentation

Initially, the driver pressure was calibrated over a range of 0 to 689.5 kPa using a pressure gage in the control console of the test unit. The transient pressures which occurred as the shock and expansion fan moved back and forth in the tube were measured using pressure transducers and associated charge amplifiers. The output from the transducers was recorded and stored using a digitizing oscilloscope. After an initial preview of the screen, where form and time frame were analyzed, the output was sent to an attached printer.

Results

Even though no direct measurements of the initial temperature in the driver section were available, two limiting conditions were surveyed. For the purpose of the analysis, the room temperature at 290 K and the theoretical temperature rise due to compression by loading from a compressed air line at 364 K, were considered. Results from the program SHTUBE.BAS (EXE) were used.

Two experimental examples are given with comparisons made to theoretical and computational results. The first involves short duration records covering a running time of approximately 0.05 s, as shown in Figs 6 and 7. These show experimental results, theoretical pressure records, and computational data from the program MELTUBE.BAS (EXE) for two transducer locations.

An experimental long duration run containing raw data output is shown in Fig. 4. When compared to a long duration run of numerical data, shown in Fig. 3, the two illustrate the usefulnses of

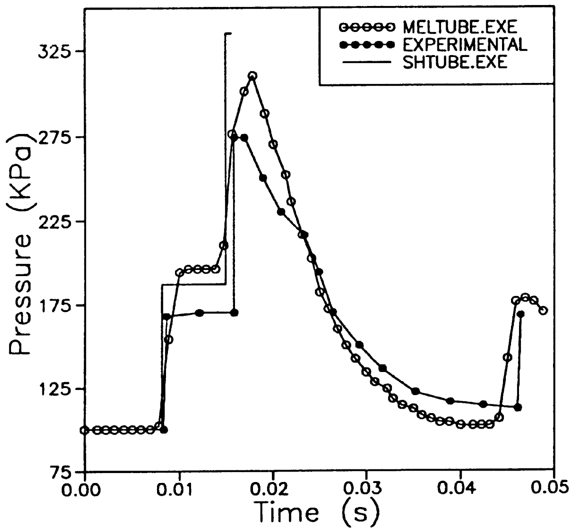

Fig. 6. Air shock tube short duration, Station 8.

the program MELTUBE.BAS (EXE). It is noteworthy that the final, equalized, pressure closely agrees with its theoretical value of 164 kPa, obtained by systems analysis based on fully mixed final conditions.

TEXAS A&M HYDRAULIC SHOCK TUBE DEVICE

Hydraulic experiments which simulated the University of Illinois air shock tube experiments were completed at Texas A&M. Numerical solutions generated from the program ONDINSTW.BAS (EXE) checked the accuracy of the experiments. Finally, hydraulic and air results were compraed by using the gas density/water height ratio relationship.

Listing of corresponding physical quantities

Because of similar differential equations and boundary conditions for one-dimensional ideal

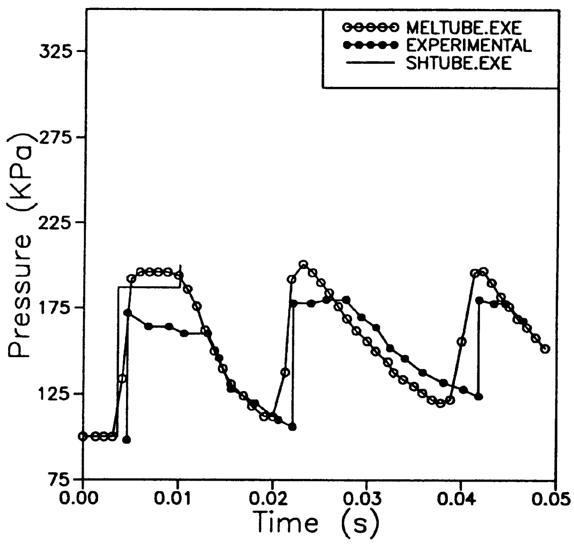

Fig. 7. Air shock tube short duration, Station 20.

The comparable variables for the hydraulic analog

	Equations of gas flow	Equations of the hydraulic analogy
Energy equation	$c_p T + v^2/2 = c_p T_0$	$h + v^2/2g = h_0$
Speed of sound	$c = (kRT)^{0.5}$	$c = (gh)^{0.5}$
Mach number	$M = v/c$	$Fr = v/c$
Continuity equation	$\frac{\partial \rho}{\partial t} + \frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho v)}{\partial y} = 0$	$\frac{\partial h}{\partial t} + \frac{\partial(hu)}{\partial x} + \frac{\partial(hv)}{\partial y} = 0$
Density ratio	ρ/ρ_0	h/h_0
Pressure ratio	P/P_0	$(h/h_0)^2$
Temp. ratio	T/T_0	h/h_0

gas and free surface water flow [12], comparable variables for the analog were obtained when the specific heat ratio k, where $k = c_p/c_v$ set was equal to 2.0. The comparable variables for the analog are shown in the table.

In this study, the hydraulic analogy was easily utilized for studying non-steady flow phenomena by constructing a simple and economical hydraulic shock tube. This hydraulic shock tube device used water as the medium to simulate unsteady one-dimensional air flow.

The analog device at the Emil Buehler Laboratory

The hydraulic analogy device developed at the Emil Buehler Aerodynamic Analog Laboratory at Texas A&M, was an independent standalone shock tube. The hydraulic shock tube was 1 m in length, with a rectangular cross section of 0.14 m by 0.114 m, and an open top. It was made of clear lucite of wall thickness of 1.27 cm, and possessed the same ratio of characteristic dimensions as the University of Illinois facility. There were two regions in the tube, one with a high water level representing high pressure gas and one with a lower water level representing low pressure gas. The two regions were separated by a thin metal gate. The ends of the hydraulic shock tube were closed, with the diaphragm approximately 0.74 m from the lower pressure end of the tube.

A transparent grid of equally spaced lines with major increments (horizontally and vertically) of 0.025 m and minor increments (horizontally only) of 0.001 m was placed on the front side of the hydraulic shock tube to allow for visual measurement of varying water heights. Additional equipment used to collect data included a large screen television, a personal computer, and a SVHS video camera and video cassette recorder.

Hydraulic analogy experiments

The driven region contained an initial water level of 0.025 m with the driver region at 0.05 m. After the diaphragm was quickly lifted manually, the two regions produced an interface, a hydraulic jump (or shock wave) on the driven side, and expansion waves on the driver side.

The experiments consisted of filming the simulation with the video camera. Since the entire tube could not be filmed with high resolution, the experiments were filmed in eleven divisions of 0.1 m length each, as shown in Fig. 8. Later the video tapes were re-examined to record the water heights. During re-examination, a special feature of the video cassette recorder allowed data to be collected for 1/30$^{\text{th}}$ of a second time intervals. Also recorded were front and top views of the experimental technique. This flow visualization allowed the use of colored dyes to emphasize the interface region, the low pressure region, and the high pressure region.

Program ONDISTTW.BAS

Once experimental data were obtained, computational results were generated from the program ONDINSTW.BAS (EXE). This user-friendly program generated water heights and velocities as functions of time for one-dimensional, non-steady flow of free surface water. It also created spreadsheet files containing the computational results.

Comparisons between experiments and computations

The computational and experimental data were compared at the eleven experimental stations. Shown in Figs 9–12 are experimental and numerical results at various stations. Experimental and theoretical data were compared in a wave pattern as shown in Fig. 13. The theoretical results were derived from continuity and momentum equations for hydraulic jumps and characteristics. It is apparent that while the overall agreement between the numerical and experimental data is convincing,

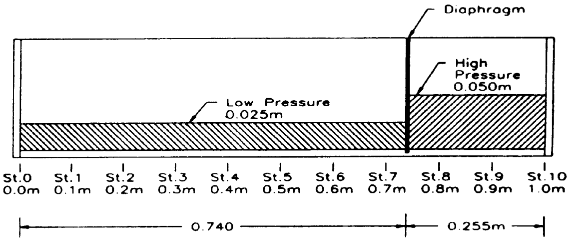

Fig. 8. Front view of the hydraulic shock tube.

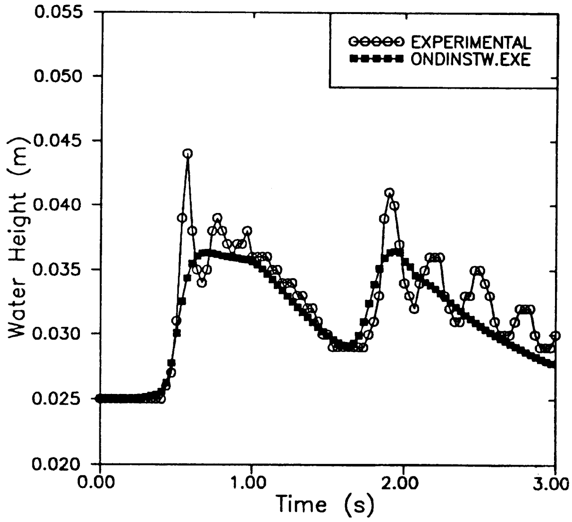

Fig. 9. Hydraulic shock tube, Station 4.

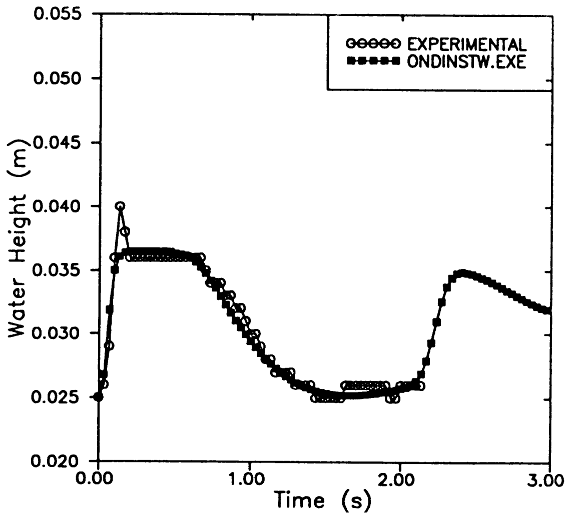

Fig. 10. Hydraulic shock tube, Station 7, low pressure side of diaphragm.

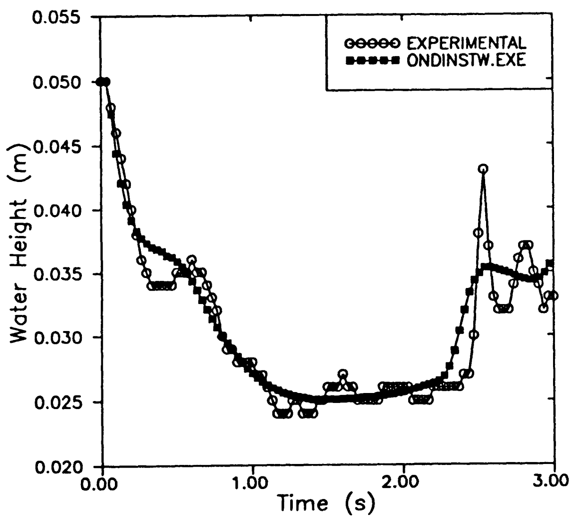

Fig. 11. Hydraulic shock tube, Station 8, high pressure side of diaphragm.

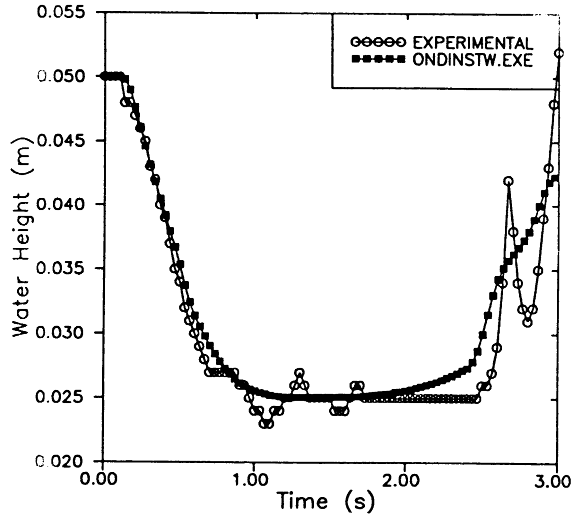

Fig. 12. Hydraulic shock tube, Station 9.

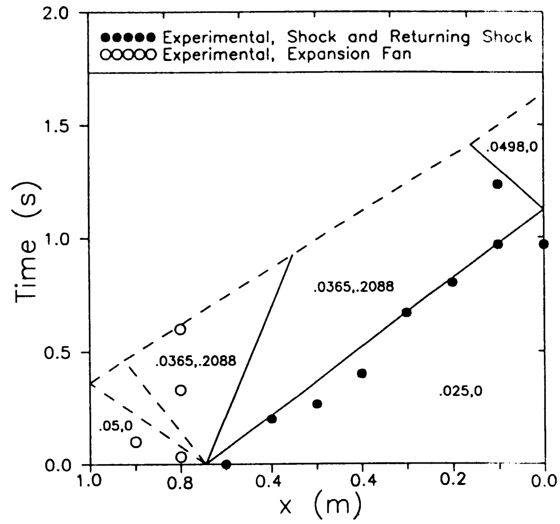

Fig. 13. Hydraulic shock tube wave pattern.

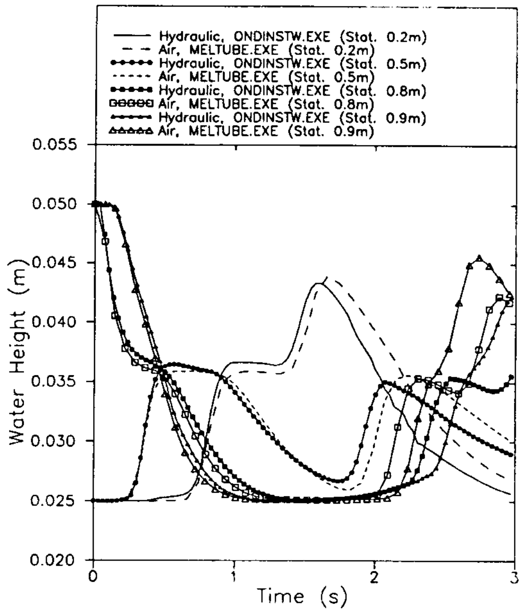

Fig. 14. Comparison of air density ratios and hydraulic water height ratios.

the experimental points exhibit a distinct superposition of shorter wavelength oscillations; these represent water surface ripples (attributable to capillary surface waves, which in steady flow water table experiments are often misjudged as being 'Froude lines' in analogy to Mach lines).

It has been suggested before [13] for steady flow, that the gas density ratio of the compressible gas should give the best correlation to the water heights ratio of the analogy. This is shown in Fig. 14 generated numerically by the program MELTUBE.BAS (EXE) for gas density ratios and the program ONDINSTW.BAS (EXE) for water height ratios.

A time conversion was used with MELTUBE. BAS (EXE) where the reference absolute temperature (290 K) was replaced by the stagnation value of the high pressure gas, $(290 \times 2.639^{(1/3.5)} = 382.66\,\mathrm{K})$. This produced a new time scale. The time conversion equation was:

$$t,w = t,m\left(\sqrt{\frac{(1.4)(287)(382.66)}{(9.81)(.05)}}\right)\left(\frac{1}{6.759}\right)$$

$$t,w = t,m \times 82.79$$

Shown are Stations 0.2, 0.5, 0.8, and 0.9 m. The excellent quantitative agreement supports the selection of the gas density ratio as analog quantity for the water height ratios.

CONCLUSIONS

Stressing the educational objectives of observing transient flow characteristics, emphasis was given to three aspects of one-dimensional non-steady flows, namely:

1. Experimentation with a shock tube;
2. Experiments using the hydraulic analogy;
3. Theoretical analysis applied to both 1 and 2 with computational methods specialized for microcomputers.

After assessing the theories and results obtained in the preceding sections, the following conclusions can be drawn:

- All methods are successful in explaining the basic mechanisms and phenomena.
- The experimental results obtained with the air shock tube, when compared with the idealized operation given by the program SHTUB.BAS (EXE) reveal the difficulties of achieving instantaneous diaphragm openings, hence the initial theoretical shock wave strengths may not be obtained.
- Computational results of shock tube flows by the program MELTUBE.BAS (EXE) compared well with the experimental and theoretical results for the early stages of wave formation. However, the short comings of the homentropic concept were revealed leading to an over estimation of initial shock wave strength and demonstrated the effects of numerical wave diffraction.
- Long range analysis of transient flows as they develop toward cyclic or asymptotic operation can be successfully carried out by numerical calculation. It is limited, however, in accuracy to moderate initial disturbances (pressure ratios less than 3) and in space resolution due to the memory capability restriction of microcomputers.
- The gas density ratio is the preferred selection for correlation with the water height ratios in the hydraulic analogy.
- The hydraulic analogy provides a convenient facility for demonstrating transient flow effects in a significantly expanded time scale. Agreement between the observed flow phenomena of varying water height with time, and computational results obtained with program ONDINSTW.EXE are shown to be satisfactory.

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