

An Experimental Facility for Compressible Flow*

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This paper describes a low-cost, quiet and portable facility for the demonstration of compressible air flow successfully developed by a group of ten mechanical engineering undergraduate students during a two-week project module of their in-house practical training programme. The facility is able to produce a flow of maximum Mach number of about 1.27 through a converging-diverging nozzle. The phenomenon of supersonic flow such as choking and the formation of shock waves can be demonstrated. The facility developed is now being used as an undergraduate project experiment. Due to the quietness and portability of the facility, it is also well suited for classroom demonstrations.

SUMMARY OF EDUCATIONAL ASPECTS OF THE PAPER

1. The paper describes new equipment useful in mechanical and aeronautical engineering courses.
2. The level of students involved in the use of the equipment is typically third-year undergraduates in mechanical engineering.
3. The equipment described is extremely low cost and portable, and is comparatively quiet, yet capable of producing a supersonic stream of Mach number of about 1.27. It is probably the first of its kind in terms of cost, portability and performance.
4. The arrangement can be devised for course work to demonstrate compressible flow phenomenon and to study the flow through a converging-diverging nozzle from low to high speeds, including the choking phenomenon and the determination of shock wave movement inside such a nozzle based on pressure measurements.
5. A suitable accompanying text is *Fluid Mechanics*, third edition, by Frank M. White, McGraw-Hill (1994).
6. The rig developed has been used by third-year undergraduates. It is found that the experiment successfully reinforces students' understanding of the compressible flow theory. The students were amazed by the high speed such a low cost and compact rig can achieve.

INTRODUCTION

IN GAS FLOWS of engineering interest, the speed of flow is often sufficiently high that the

flow becomes compressible. At such high speeds, the flow exhibits some distinct features such as the choking phenomenon and the formation of shock waves. Compressible flow has thus been an important topic in engineering curriculum, particularly in the aerospace engineering.

Traditionally, compressible flow experiments are carried out in a supersonic wind tunnel. Such experiments are costly, noisy and the durations for taking measurements are usually short, if the flow is not continuous. Prior to measurements, preparation is often required to charge the air reservoir that produces a high total pressure for the air stream. Since the supersonic wind tunnel is costly and bulky, it is seldom used by undergraduate students to study the compressible flow phenomenon.

The cheaper and less bulky alternative available is the bench-top type compressible flow rigs. They can be purchased from educational equipment suppliers. These rigs usually employ a multi-stage centrifugal compressor driven by a motor to deliver the flow through a converging-diverging nozzle. The noise level is still quite high due to the multistage compressor operation. With several sets operating, the students must wear ear protectors. Very often, the highest speed they can produce is slightly over Mach 1 and the flow quickly becomes subsonic in the diverging section. As a result, they do not provide measurement capability in the diverging section. The instrumentation of these rigs is usually the primitive U-tube manometers. The size and weight of such bench-top unit are still considerable and the unit cannot be portable. The cost of a set of the bench-top unit, although much cheaper than the wind tunnel, is of the order of US\$10,000.

The aim of the present task is to develop a compressible flow facility which is low-cost, quiet and portable, yet able to produce a supersonic flow

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Table 1. The project schedule

Duration	Activities
3 days	Project familiarisation, planning and designing
4 days	Fabrication
2 days	Testing and final modifications
2 days	Preparations of report and oral presentation

high enough to demonstrate the development and movement of a shock wave that is often part of a related undergraduate curriculum. The development involves different engineering processes including the design, fabrication and testing of the facility. It thus falls neatly into the objectives [1] of the Project module of the in-house practical training programme devised in the School of Mechanical and Production Engineering at the Nanyang Technological University for students to gain first-hand experience in design, manufacture, construction and assembly of parts and testing of the final product that neither lectures, tutorials nor laboratory sessions can offer. The in-house practical training programme, previously reported in references [2] and [3], is an intensive ten-week compulsory training programme undertaken by the students at the end of their second year of study. It includes a two-week project module in which students are divided into groups of about ten and each group is required to work on a different mechanical engineering related product right from scratch to finish. The students are motivated by the fact that the project module poses a challenge for them to put the knowledge they learn in producing a practical engineering product which they can physically feel and derive satisfaction if the project is successful. For this reason, they are often enthusiastic and contribute good effort and teamwork. Examples of some products previously developed in the project module which have been published in the literature are a man-powered aircraft [4] and a smoke-wire flow visualisation rig [5].

In addition to provide the students an opportunity to learn compressible flow phenomenon and develop its associated hardware, the development of a compressible flow facility also serves a dual purpose of producing a practical compressible flow rig that can be used for laboratory investigations

and classroom demonstrations. It was therefore proposed as a project for the project module.

THE PLANNING OF THE PROJECT

Prior to the start of the project, raw materials and some equipment had been purchased for the students. These included a solid cylindrical Perspex rod, a Perspex sheet, silicon tubing, a domestic vacuum cleaner with an electronic speed controller, a scanning valve, a pressure transducer, a DC power supply, a digital meter and an inclined manometer.

The total duration of the project was two weeks, which was equivalent to eleven working days. The project schedule is shown in Table 1. At the beginning of the project, the students were briefed on the theory of compressible flow through a converging-diverging nozzle and the design requirements of the facility.

During the two project weeks, a technician was assigned to help the students in sourcing materials not pre-provided and in particular to train and help the students in the fabrication phase. Teamwork was promoted by letting the students organise and share various activities themselves. Consultation with the supervisor was available at all time. Short discussions with supervisor were also held at the end of each day. In particular, lengthy discussions had been held to finalise the design before fabrication and before the oral presentation on the last Saturday.

The following sections describe the facility developed and the testing results.

THE EXPERIMENTAL FACILITY

The facility primarily consists of three parts—namely the converging-diverging nozzle to produce supersonic flow, the drive unit to induce the flow through the nozzle and the instrumentation to facilitate the pressure measurements. In addition, a barometer is required to measure the ambient pressure for data analysis.

The converging-diverging nozzle was made from a cylindrical Perspex rod. Figure 1 shows the geometry and dimensions of the nozzle. Upstream

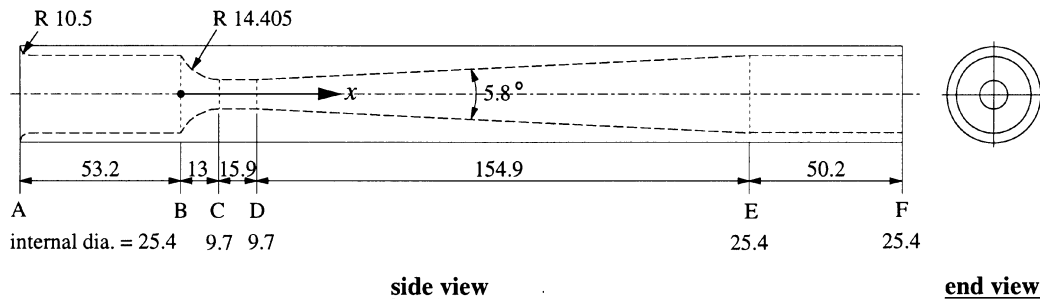


Fig. 1. The converging-diverging nozzle. (All dimensions are in mm).

Table 2. Locations of the pressure tapings ($x = 0$ at B and increases downstream)

Tapping x mm	1	2	3	4	5	6	7	8	9	10	11	12	13
	-26	24.6	33.4	35.2	37.0	39.1	41.1	43.1	45.1	51.1	57.1	63.1	200.9

of the converging section, there is an inlet portion AB of length 53.2 mm and constant internal diameter of 25.4 mm. The inlet allows the mass flux \dot{m} to be determined, if desired, from a pressure measurement through a pressure tapping installed on it with the aid of careful calibration result. The converging section BC of length 13 mm, has a contour of the shape of a circular arc of radius 14.405 mm with centre on the plane of the throat entrance. This is to ensure the area gradient to be continuous at position C so that the flow is smooth there. The converging section is responsible to accelerate the flow from low subsonic speeds to high subsonic speeds. The throat CD of the nozzle is of length 15.9 mm along with diameter constant at 9.7 mm. The throat is followed by a diverging section of length 154.9 mm and constant diverging angle of 5.8° . The diverging angle is small to avoid possible flow separation under adverse pressure gradient. At subsonic speeds, the diverging section is responsible to decelerate the flow gradually for a smooth pressure recovery. At high speeds, after the nozzle is choked, the diverging section is responsible to accelerate the flow to supersonic speeds up to the location of the normal shock wave, as depicted by the supersonic flow theory [6]. After the diverging section, the flow enters a section EF of constant diameter which can be attached to the inlet of the drive unit via a flange. Machining of the nozzle was done using a conventional lathe machine. Because of the small internal diameter of the nozzle as compared to its length, the nozzle was divided into three sections. Each section was machined and polished separately before they were aligned and joined together. A total of thirteen pressure tapings, of diameter 1 mm were drilled along the nozzle for pressure measurements. The numbering and the locations of the tapings are given in Table 2. Most of the tapings are located at a short distance downstream of the throat because supersonic flow and shock waves are produced in that region.

The drive unit consists of an AC motor attached to a centrifugal vacuum air pump with

an electronic speed controller, which were dismantled from a domestic vacuum cleaner. The speed of the pump can be varied from about 12,000 rpm to 25,000 rpm in order to create different suction pressure. The power of the motor is 1400 W.

To measure the low differential pressure at the inlet of the nozzle, an inclined water manometer is connected to tapping number 1. The inclination of the manometer can be adjusted to control the sensitivity of the manometer, especially at low speeds. At the throat and along the diverging section of the nozzle, the flow speed and hence the differential pressure is high and the pressure tapings are connected to a common strain gauge pressure transducer via a scanning valve, which allows a particular tapping to be selected for measurement. The output of the transducer is calibrated to pressure and is displayed on a digital meter.

For compactness, the drive unit and the instruments are housed in a box, measured merely $35 \text{ cm} \times 30 \text{ cm} \times 25 \text{ cm}$. The weight of the entire rig including the nozzle is less than 4 kg. Figure 2 shows a schematic diagram of the facility. The nozzle can be attached on the air pump before measurements are made. With this set-up, the pressure variations along the nozzle at different flow speeds can be measured easily.

The total cost of the material of the facility was under \$1000. As it is quiet and portable, it can be used in the laboratory as well as in the classroom for demonstrations.

RESULTS AND DISCUSSION

The facility was first subjected to a consistency test. During the test, the motor was switched on and was adjusted to maximum speed. Three sets of pressure measurements along the nozzle were taken at 15 minute intervals. The total running time of the facility was thus more than 30 minutes, which is much longer than the time required to

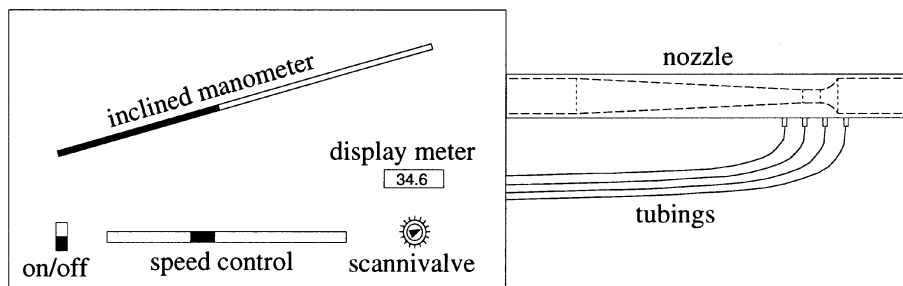


Fig. 2. Schematic diagram of the facility.

perform the compressible flow experiment. The test results showed that the maximum drift in differential pressure was about 1.8%, well within the experimental uncertainties. It can thus be concluded that the facility is consistent. At different motor speed settings, pressure measurements along the nozzle were made.

From the experimental data, assuming isentropic flow, the local Mach number [6] for subsonic flow and flow upstream of shock wave is given by:

$$\frac{P}{P_{01}} = \left[\frac{1}{1 + \frac{\gamma-1}{2} M^2} \right]^{\gamma/\gamma-1} \quad (1)$$

where γ is the specific heat ratio, P_{01} is the ambient pressure, P and M are the local pressure and Mach number respectively. Here the pressure ratio P/P_{01} can be obtained from:

$$\frac{P}{P_{01}} = 1 - \left(\frac{P_{01} - P}{P_{01}} \right)$$

where $(P_{01} - P)$ is the gauge pressure directly taken from the manometer and the digital meter. For flow downstream of shock wave, the local Mach number is given by:

$$\frac{P}{P_{02}} = \left[\frac{1}{1 + \frac{\gamma-1}{2} M^2} \right]^{\gamma/\gamma-1} \quad (2)$$

where P_{02} is the total pressure downstream of the shock wave.

To determine the Mach number, P_{02} is thus required which can be obtained by considering the nozzle exit condition for which:

$$\frac{P_e}{P_{01}} = \frac{P_e}{P_{02}} \frac{P_{02}}{P_{01}} = \frac{P_e}{P_{02}} \frac{A_1^*}{A_2^*} = \frac{P_e}{P_{02}} \frac{A_1^*}{A_e} \frac{A_e}{A_2^*} \quad (3)$$

where A is the cross-section area of the nozzle; the subscripts 1, 2 and e refer to upstream and downstream of the shock wave and exit of the nozzle; and the superscript * refers to the critical condition. Now from normal shock relationships, we have:

$$\frac{A_e}{A_2^*} = \frac{1}{M_e} \left\{ \frac{1 + \frac{\gamma-1}{2} M_e^2}{\frac{\gamma+1}{2}} \right\}^{(\gamma+1)/2(\gamma-1)}$$

and

$$\frac{P_e}{P_{02}} = \left\{ \frac{1}{1 + \frac{\gamma-1}{2} M_e^2} \right\}^{\gamma/\gamma-1}$$

where M_e is the Mach number at the exit of the

nozzle. Putting into (3) and rearranging, together with $A_1^* = A_t$ where A_t is the throat area, yields:

$$M_e^2 = -\frac{1}{\gamma-1} + \left[\frac{1}{(\gamma-1)^2} + \left(\frac{2}{\gamma-1} \right) \times \left(\frac{2}{\gamma+1} \right)^{(\gamma+1)/(\gamma-1)} \left(\frac{A_t/A_e}{P_e/P_{01}} \right)^2 \right]^{1/2} \quad (4a)$$

For the nozzle used, with $\gamma = 1.4$ for air,

$$M_e = \left\{ -2.5 + \left[6.25 + 0.03562 \left(\frac{P_e}{P_{01}} \right)^{-2} \right]^{1/2} \right\}^{1/2} \quad (4b)$$

This gives M_e . Then P_{02} can be obtained by using (2) at the nozzle exit, i.e.

$$P_{02} = P_e \left[1 + \frac{\gamma-1}{2} M_e^2 \right]^{\gamma/\gamma-1} \quad (5)$$

where $(P_{01} - P)$ and $(P_{01} - P_e)$ are the gauge pressures directly taken from the digital meter. The latter corresponds to pressure tapping 13.

The location of the shock wave must also be determined so that the appropriate equation (1) or (2) can be applied to give the correct Mach numbers. It can be determined by finding the Mach number M_1 just upstream of the shock using the normal shock relationship:

$$\frac{P_{02}}{P_{01}} = \left[\frac{(\gamma+1)M_1^2}{2 + (\gamma-1)M_1^2} \right]^{\gamma/\gamma-1} \times \left[\frac{\gamma+1}{2\gamma M_1^2 - (\gamma-1)} \right]^{1/\gamma-1} \quad (6)$$

or by using the standard normal shock wave table. Having found M_1 , the area at the shock location A_s can be determined from the area relation:

$$\frac{A_s}{A_t} = \frac{A_s}{A_1^*} = \frac{1}{M_1} \left\{ \frac{1 + \frac{\gamma-1}{2} M_1^2}{\frac{\gamma+1}{2}} \right\}^{(\gamma+1)/2(\gamma-1)} \quad (7)$$

Knowing A_s , the shock wave location x_s can be found from the geometry of the nozzle. For the nozzle used, with $\gamma = 1.4$,

$$x_s = \frac{4.85 \left\{ \left[\frac{1}{M_1} \left(\frac{1 + 0.2M_1^2}{1.2} \right)^3 \right]^{1/2} - 1 \right\}}{\tan 2.9^\circ} + 28.9 \quad (8)$$

The variations of pressure ratio and local Mach number in the streamwise direction obtained by using the above procedure are plotted in Figs 3 and 4 respectively. When the motor speed was low, the

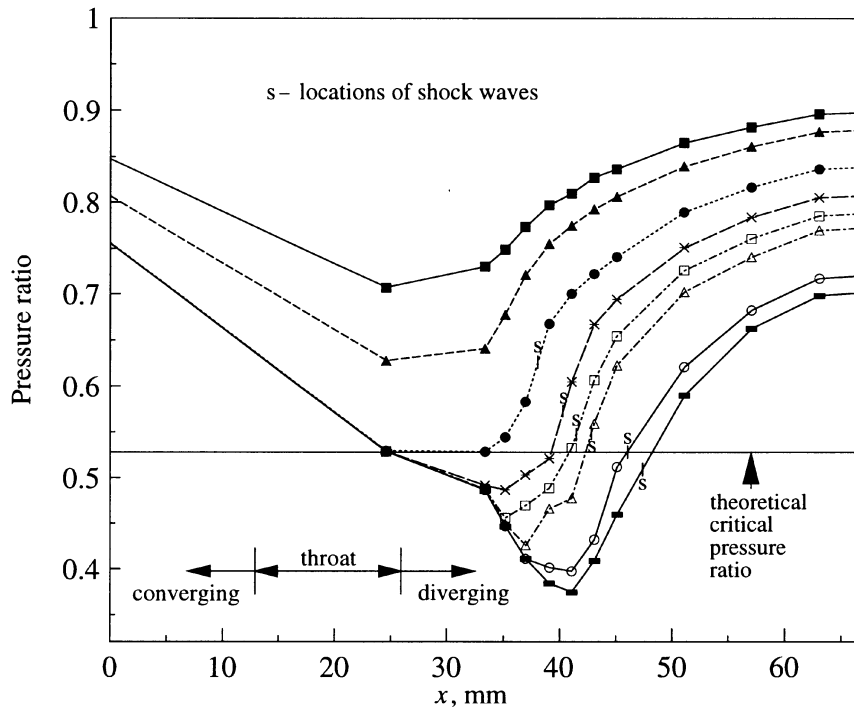


Fig. 3. Variations of pressure ratio P/P_{01} with x .

pressure dropped along the converging section and reached a minimum at the throat after which it recovered along the diverging section. This corresponded to an increase in flow speed along the converging section followed by a decrease in flow speed along the diverging section as depicted by the incompressible continuity equation and as also shown in Fig. 4. As motor speed was increased, the pressure dropped and recovery in the converging

section as well as in the diverging section became more rapid. When the pressure ratio at the throat was reduced to about 0.53, the experimental critical pressure ratio, the Mach number at the throat increased to about 1. At this point, the nozzle was choked.

Beyond choking, at higher motor speeds, the flow condition in the converging section and at the throat became independent on motor speed.

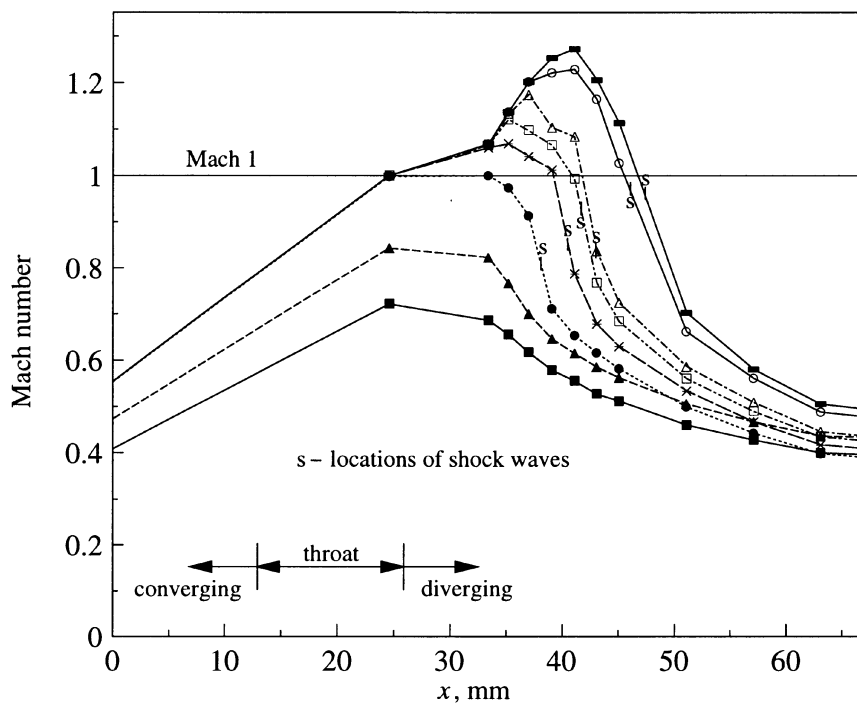


Fig. 4. Variations of local Mach number M with x .

Table 3. Locations of the shock waves

Speed setting	Shock location x_s , mm
-----●-----	38.1
-----×-----	40.6
-----□-----	41.6
-----△-----	42.9
-----○-----	46.0
-----■-----	47.3

The pressure continued to drop after the throat, indicating the occurrence of supersonic flow regions. Further downstream, the pressure dropped to a minimum and then recovered. The pressure recovery was rapid at first and then became gradual. The rapid recovery of pressure in the diverging section showed the formation of shock waves across which energy dissipated rapidly. The locations of the shock waves x_s calculated based on the nozzle exit condition by using the procedure described earlier are tabulated in Table 3 and are indicated by 's' in Figs 3 and 4. After the shock waves, the flow quickly became subsonic. It can be seen in Table 3 and in Figs 3 and 4 that the shock wave moved downstream as motor speed was

increased. At maximum motor speed, the flow speed reached to a maximum Mach number of about 1.27 at about 16.5 mm downstream of the throat.

CONCLUDING REMARKS

A low-cost, quiet and portable experimental facility for the demonstration of supersonic flow phenomenon has been successfully designed, fabricated and tested as a student practical training project. The facility can be used in laboratory classes and can also be easily carried to the classrooms for demonstrations of compressible flow. It is able to show the pressure and Mach number variations along a simple converging-diverging nozzle from subsonic to supersonic speed. With these variations, the choking phenomenon, the development of supersonic flow and the formation and movement of shock waves can all be observed. The results agree well with the compressible flow theory. The highest Mach number achievable is about 1.27, well suited for an undergraduate experiment in the area of aerodynamics.

REFERENCES

1. *Information on In-house Practical Training*, School of Mechanical and Production Engineering, Nanyang Technological University, Singapore (1994).
2. S. Thiruvarduchelvan, An undergraduate in-house practical training project in mechanical and production engineering, *Proc. 1st Australasian Education and Engineering Conference*, Paper 7B5, University of Sydney (1989).
3. A. Ameen, Effectiveness of project module as part of an in-house practical training programme, *Int. J. Mech. Eng. Edu.*, **18**, (1990) pp. 187–192.
4. R. S. J. Palmer and K. Sherwin, A man-powered aircraft as a student project, *Int. J. Mech. Eng. Edu.*, **14**, (1986) pp. 273–280.
5. S. C. M. Yu and L. P. Chua, Construction of a smoke-wire rig for visualisation of complex three-dimensional flow, *Int. J. Engng. Ed.*, **9**, (1993) pp. 467–476.
6. J. D. Anderson, *Fundamentals of Aerodynamics*, 2nd ed., McGraw-Hill (1991).

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