MATHCAD: Teaching and Learning Tool for Reinforced Concrete Design*

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MATHCAD is a sophisticated computation and presentation tool, which is versatile, easy to use, and accessible. It holds strong potential as a learning aid for education and training. This article demonstrates the use of MATHCAD to supplement and enhance traditional teaching and learning methods both inside and outside the classroom. The paper focuses on the topic of reinforced concrete column design. By using the presentation and programming features available in MATHCAD, interactive teaching and learning devices in reinforced concrete design have been produced.

INTRODUCTION

MATHCAD [1] is an efficient learning environment for technical topics such as reinforced concrete design. Its computational and presentation capabilities not only lend themselves to the solution of mathematically-based problems, but also to the effective communication of both problems and solutions. MATHCAD contains powerful presentation capabilities, which include the use of charts, graphic objects, and animation effects. It can also easily import objects from other application programs, such as images and digital photographs. These capabilities offer significant learning enhancements to students of technical subjects.

MATHCAD makes possible new learning strategies for students and teachers. What-if discussions, trend analyses, trial-and-error analyses, and optimization are all valuable learning activities, which take more time than the traditional technical problem solving approach permits. Taking advantage of the computational power and speed of MATHCAD, instructors and students can quickly cycle through problem scenarios, observing trends in the design behavior of reinforced concrete components.

The proposed paper describes the use of the MATHCAD program as a teaching and learning tool in reinforced concrete design courses. A program for the design of reinforced concrete columns is discussed and demonstrated to show the attractive computational environment of MATHCAD and to illustrate its importance as a teaching and learning tool for civil engineering students.

OVERVIEW OF REINFORCED CONCRETE COLUMN DESIGN

Columns are vertical compression members, which transmit loads from the upper floors to the lower levels and to the soil through the foundations. Based on the position of the load on the cross-section, columns are classified as concentrically loaded, Fig. 1, or eccentrically loaded, Fig. 2. Eccentrically loaded columns are subjected to moment, in addition to axial force. The moment can be converted to a load P and eccentricity e. The moment can be uniaxial, as in the case when two adjacent panels are not similarly loaded, such as columns A and B in Fig. 3. A column is considered biaxially loaded when the bending occurs about the X and Y axes, such as in the case of the corner column C in Fig. 3.

The strength of reinforced concrete columns is determined using the following principles:

1. A linear strain distribution exists across the thickness of the column.
2. There is no slippage between the concrete and the steel.
3. The concrete strain at failure for strength calculations is set equal to 0.003 mm/mm.
4. The tensile resistance of the concrete is negligible and is disregarded.

The strength of reinforced concrete columns is usually expressed using interaction diagrams to relate the design axial load ϕP_n to the design bending moment ϕM_n (Fig. 4). Each point on the curve represents one combination of design axial load ϕP_n and design bending moment ϕM_n corresponding to a particular neutral-axis location. The interaction diagram is separated into a tension control region and a compression control region by the balanced condition at point B. The balanced condition occurs when the failure
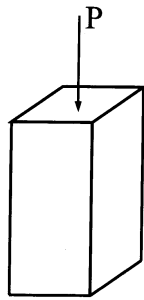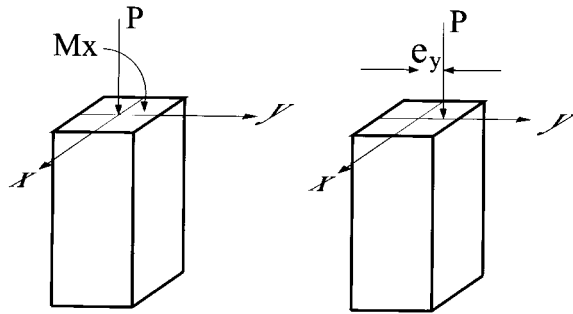
Fig. 1. Concentrically loaded columns.

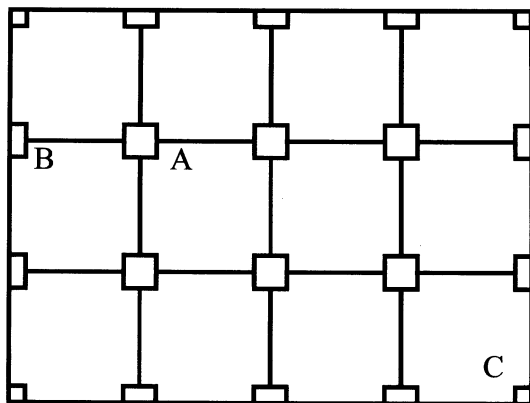Fig. 2. Eccentrically loaded column.

Fig. 3. Uniaxially and biaxially loaded column.

develops simultaneously in tension (i.e., steel yielding) and in compression (concrete crushing).

The manual design of reinforced concrete columns, is usually performed using two hand-computation procedures. The first one consists of checking whether the point (P_u, M_u), which is defined by the factored axial load P_u and the factored bending moment M_u, is inside, outside, or on the interaction diagram $\phi P_n - \phi M_n$. The strength of the column is not adequate if the point (P_u, M_u) is outside the curve $\phi P_n - \phi M_n$. The closer is the point to the curve, the more economical is the design.

The second procedure consists of computing the design axial load ϕP_n and the design bending moment ϕM_n which correspond to the ultimate load eccentricity e_u. The strength of the column is adequate if the obtained values of ϕP_n and ϕM_n are larger than or equal to the values of P_u and M_u, respectively.

Further details on reinforced concrete column design can be found elsewhere [2–5].

MATHCAD PROGRAM FOR REINFORCED CONCRETE DESIGN

A MATHCAD program has been written to automate the manual design of reinforced concrete columns. The program, which totally emulates the manual design procedure, consists of the following computational steps.

Step 1

The first step consists of reading the following input data (Fig. 5):

1. The number of steel layers NSL.
2. The area of steel in each layer As_j $(As_j, j = 1, \ldots, \text{NSL})$.
3. The distance d_j between each layer and the top column fiber $(d_j, j = 1, \ldots, \text{NSL})$.
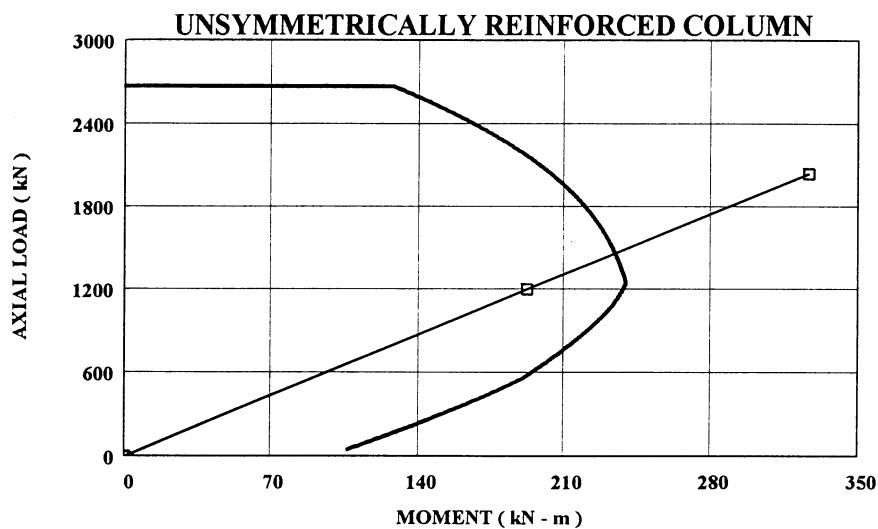4. The dimensions b and h of the column.

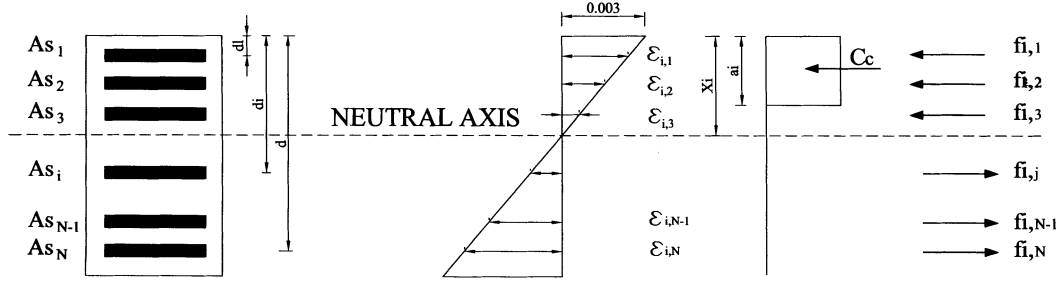
Fig. 4. Column interaction diagram $\phi P_n - \phi M_n$.

Fig. 5. Reinforced concrete column strains and stresses.

5. The yield strength of steel f_y, the concrete compressive strength f_c', and the steel modulus of elasticity E_s.
6. The factored load P_u and bending moment M_u.
7. If the factored bending moment M_u is less than the minimum bending moment M_{min}, M_u is set equal to M_{min}.

The minimum bending moment M_{min} is computed using the following equation:

$$M_{min} = 0.1 h P_u \qquad (1)$$

Step 2

In the second step, the plastic centroid Y_p, the reinforcement ratio ρ, and the parameter β are computed. The plastic centroid of the column cross section is computed using the following equation:

$$Y_p = \frac{\sum_{j=1}^{NSL} As_j f_y d_j + 0.85 f_c' b \frac{h^2}{2}}{\sum_{j=1}^{NSL} As_j f_y + 0.85 f_c' bh} \qquad (2)$$

The reinforcement ratio ρ is determined using the following equation:

$$\rho = \frac{\sum_{j=1}^{NSL} As_j}{bd} \qquad (3)$$

Finally, the parameter β is computed using the following equation:

$$\beta = 0.85 - 0.05 \frac{f_c' - 27.6}{6.895} \ (f_c' \text{ in MPa}) \qquad (4)$$

Step 3

The iterative procedure starts by selecting the first position of the neutral axis X_i ($X_i = i + d_1$, with $i = 0$).

Then, the parameter a_i (depth of the compression block) is computed using the following equation:

$$a_i = \beta X_i \qquad (5)$$

Step 4

The strain $\varepsilon_{i,j}$ in each reinforcing steel bar is determined by the linear strain distribution to ensure the strain compatibility (Fig. 5). The strain $\varepsilon_{i,j}$ is computed using the following equation:

$$\varepsilon_{i,j} = 0.003 \frac{X_i - d_j}{X_i} \qquad (6)$$

On the other hand, the stresses $f_{i,j}$ in each reinforcing steel bar is obtained using the expression:

$$f_{i,j} = E_s \varepsilon_{i,j} \qquad (7)$$

where $f_{i,j}$ has to be less than or equal to the yield strength of steel f_y.

Using the equilibrium of the internal forces and moments, the design axial load ϕP_{ni} and the design bending moment ϕM_{ni} are, respectively, computed using the following equations:

$$\phi P_{ni} = 0.7 * \left(0.85 f_c' a_i b + \sum_{j=1}^{NSL} As_j f_{i,j} \right) \qquad (8)$$

$$\phi M_{ni} = \phi P_{ni} e$$
$$= 0.7 \left(0.85 f_c' a_i b \left(Y_p - \frac{a_i}{2} \right) \right.$$
$$\left. + \sum_{j=1}^{NSL} As_j F_{i,j} (Y_p - d_j) \right) \qquad (9)$$

The load eccentricity e_i is computed using the following expression:

$$e_i = \frac{\phi M_{ni}}{\phi P_{ni}} \qquad (10)$$

The values of ϕP_{ni} and ϕM_{ni} represent a point on the interaction diagram $\phi P_n - \phi M_n$.

Step 5

The number of iterations (i) is incremented by 1. Then, STEP 3 through STEP 5 are repeated until the value of (i) reaches the value of h.

Step 6

At the end of the computation process, the design bending moment $\phi M_{n(N)}$ is set equal to zero while the design axial loads $\phi P_{n(N-1)}$ and $\phi P_{n(N)}$ are set equal to the following expression:

$$\phi P_{n(N)} = \phi P_{n(N-1)} = 0.8 \left(0.85 f_c' hb + \sum_{j=1}^{NSL} As_j f_y \right) \qquad (11)$$

STEP 1.

$N := h$ *Number of iterations*

$Mmini := \dfrac{0.1 \cdot h \cdot Pu}{1000}$ *Minimum bending moment*

$Yp := \dfrac{\sum\limits_{j} As_j \cdot fy \cdot d_j + 0.85 \cdot f'c \cdot b \cdot \dfrac{h^2}{2}}{\sum\limits_{j} As_j \cdot fy + .85 \cdot f'c \cdot b \cdot h}$ *Plastic centroid*

$y := if\left(Yp \neq \dfrac{h}{2}, \; Yp, \; \dfrac{h}{2}\right)$

$\rho := \dfrac{\sum\limits_{j} As_j}{b \cdot h}$ *Reinforcement ratio*

STEP 2.

$i := 0 \ldots N$ *Starting of iterative procedure*

$X_i := i + d_1$ *Neutral axis position*

$\beta := 0.85 - 0.05 \cdot \dfrac{f'c - 27.6}{6.895}$ *β value for f'c*

$\beta := if(\beta > 0.85, 0.85, \beta)$ *β always ⇐ 0.85*

$a_i := \beta \cdot X_i$ *Depth of compression block*

STEP 3.

$\varepsilon_{i,j} := 0.003 \cdot \left(\dfrac{X_i - d_j}{X_i}\right)$ *Strain for steel layer j*

$f_{i,j} := E \cdot \varepsilon_{i,j}$ *Stress for steel layer j*

$f_{i,j} := if\left(fy < E \cdot |\varepsilon_{i,j}|, \; \dfrac{|\varepsilon_{i,j}|}{\varepsilon_{i,j}} \cdot f_y, \; f_{i,j}\right)$ *Stress $f_{i,j}$ always ⇐ f_y (yield strength)*

$\phi := 0.7$ *Load factor for compression columns (tied)*

$\phi P_i := \dfrac{0.7}{1000} \cdot \left[(0.85 \cdot f'c \cdot a_i \cdot b) + \left(\sum\limits_{j=1}^{Nsl} As_j \cdot f_{i,j}\right)\right]$ *Nominal axial resisting force*

$\phi M_i := \dfrac{0.7}{10^6} \cdot \left[\left[(0.85 \cdot f'c \cdot a_i \cdot b) \cdot \left(y - \dfrac{a_i}{2}\right) + \sum\limits_{j=1}^{Nsl} As_j \cdot f_{i,j} \cdot (y - d_j)\right]\right]$ *Nominal bending moment*

$\phi M_i := if(\phi M_i < 0, 0, \phi M_i)$ *Disregard negative fP_i and fM_i*

$\phi P_i := if(\phi P_i < 0, 0, \phi P_i)$

$e_i := \left|\dfrac{\phi M_i}{\phi P_i + 0.0001}\right|$ *Load eccentricity*

STEP 4.

Repeat STEP 2 through STEP 3 until the value of (i) reaches N.

STEP 5.

$\phi M_N := 0$ *$\phi M_N = 0$ (last point in $\phi M - \phi P$ diagram)*

$\phi P_{N-1} := 0.8 \cdot \dfrac{0.7}{1000} \cdot \left(f'c \cdot b \cdot h \cdot 0.85 + \left(\sum As \cdot fy\right)\right)$

$\phi P_N := \phi P_{N-1}$ *ϕP_N and ϕP_{N-1} located on the horizontal plateau of $\phi M - \phi P$ diagram*

$sort(\phi P) \quad \phi P_i > \phi P_N, \phi P_N, \phi P_i)$ *ϕP_N = maximum value of ϕP_i*

STEP 6.

$Mu := if(Mu < Mmini, Mmini, Mu)$ *M_u always >= accidental moment (ACI Code)*

$eu := \dfrac{Mu}{Pu}$ *External load eccentricity*

Fig. 6. MATHCAD Program.

STEP 7

Draw a line that cuts the $\phi M - \phi P$ diagram in order to check the column strength.

$z := 0 \ldots 2$

$F_0 := 0$	$m_0 := 0$	*First point defined by (F_0, m_0)*
$F_1 := Pu$	$m_1 := Mu$	*Second point defined by (F_1, m_1)*
$F_2 := 1.7 \cdot F_1$	$m_2 := \dfrac{m_1}{F_1} \cdot F_2$	*Third point defined by (F_2, m_2)*

The required column strength is represented by the second point (F_1, m_1). The second point has to be inside or on the $\phi M - \phi P$ diagram for the column to be safe.

The closest the second point is to the curve $\phi M - \phi P$, the more economical is the design.

STEP 8

Another method for checking column strength. The method determines ϕP_i and ϕM_i which corresponds to the closest e_i to the ultimate load eccentricity e_u.

sort(e)   *Sort load eccentricities e_i*

$eu = 0.16$

$t(e, eu) := \begin{vmatrix} j \leftarrow 0 \\ \text{while } e_j \geq eu \\ \quad j \leftarrow j + 1 \\ j \end{vmatrix}$ *Find the closest e_i to e_u.*

$t(e, eu) = 171$ *e_{171} is the closest to $e_u \cdot e_{171} = 0.16$*

$\phi P_{171} = 1.217 \cdot 10^3$ *Value of ϕP_N which corresponds to e_{171}.*

$\phi M_{171} = 194.029$ *Value of ϕM_N which corresponds to e_{171}*

Fig. 6. (Continued).

The values of $\phi P_{n(N-1)}$ and $\phi P_{n(N)}$ correspond to the design axial load of concentrically loaded columns (i.e., $e_n = 0$ and $\phi M_{n(N)} = 0$).

At this stage, the interaction diagram is fully determined.

Steps 7 through 9 are concerned with the manual design reinforced concrete columns. In other words, the remaining computational steps deal with checking the strength adequacy of reinforced concrete columns.

Step 7

The eccentricity e_u of the factored load P_u is computed using the following equation:

$$e_u = \frac{\phi M_u}{\phi P_u} \tag{12}$$

Step 8

The strength of the column is adequate if the point defined by P_u and M_u is inside, or on the interaction diagram $\phi P_n - \phi M_n$. The strength of the column is not adequate if the point is outside the curve $\phi P_n - \phi M_n$. The closer is the point to the curve $\phi P_n - \phi M_n$, the more economical is the design.

Step 9

The design axial load ϕP_n and the design bending moment ϕM_n, which correspond to the ultimate load eccentricity e_u, are first determined. The strength of the column is adequate if the obtained design values ϕP_n and ϕM_n are higher than the factored values P_u and M_u, respectively.

TRADITIONAL VERSUS MATHCAD ENHANCED INSTRUCTION

Traditional teaching methods usually involves the time consuming task of the instructor writing detailed problem solutions on the board while students hurriedly copy the solutions into their notebooks. The learning process in the classroom is often suspended while the teacher and the students occupy themselves with transcribing information. This traditional classroom activity can discourage critical thinking, and deprives both the students and the teachers of engaging exchanges with each other about the subject.

A MATHCAD-enhanced teaching method can be successfully integrated into a concrete design course. Figure 6 shows a MATHCAD program developed for the design of reinforced concrete columns. The program is projected directly from the instructor's computer onto a large screen in an appropriately equipped classroom. In the program, different formatting, including various fonts, colors, patterns, and borders are used. The readability of the text exceeds what instructors can produce by hand on the classroom board. The equations look the same as they are written on a blackboard or in a reference book. To free student attention from transcription, students are given a
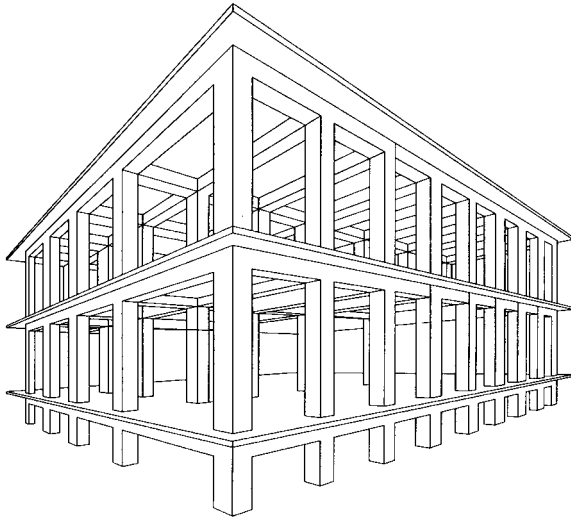
Fig. 7. Program drawings and photographs.

hard copy for taking additional notes. An electronic copy of the MATHCAD program is also made available for the student to review and practice later.

The sketches are annotated with dimensions and notes. By using different drawing entities and varying their color, pattern, and line weight attributes, highly readable drawings are produced to illustrate the computations.

The photograph, which is also shown in Fig. 7, was easily digitized and imported into the program. Photographs and images are rich sources of visual information that can be shared among teachers and students. Images from the field or laboratory bring glimpses of the engineering world into the classroom where they can be shared by all. Existing photos and slides can be digitized using slide and film scanning processes. Digital photographs can be taken with digital cameras and downloaded directly to the computer without the use of film. The interaction diagram $\phi P_n - \phi M_n$,

which is shown in Fig. 4, was easily produced by the program. Like spreadsheets, as soon as a change is made in the input data, the results are updated and the interaction diagram is redrawn. Other types of charts, such as pie and histogram charts, can also be easily generated. As was mentioned previously, interaction diagrams play an important role in the manual design of reinforced concrete columns. The MATHCAD program allows for the determination of an optimum design simply by changing the input data and observing the changes in the interaction diagram.

There are several benefits of a MATHCAD enhanced approach to teaching. The time saved from tedious transcription frees student and teacher for the discussion of concepts, and exploration of alternate problem scenarios, observation of trends, and expansion of the discussion to related topics. Outside the classroom, the instructor uses the same program to quickly generate test questions and solution keys. Trial-and-error solutions are cycled through rapidly. The student can review the classroom material by changing input variables and observing results. Homework assignments can be developed to encourage students to use the program. Making the program available to students, encourages them to learn by exploring on their own. Visual changes of the interaction diagram give students a good control of the design. The time spent using the program to explore problem scenarios posed by the instructor, can lead students to a better understanding of the concepts involved in the problems. Students can learn to write MATHCAD programs using their own way of problem solving.

ILLUSTRATIVE EXAMPLE

This example, which is presented to demonstrate the manual design features of the proposed
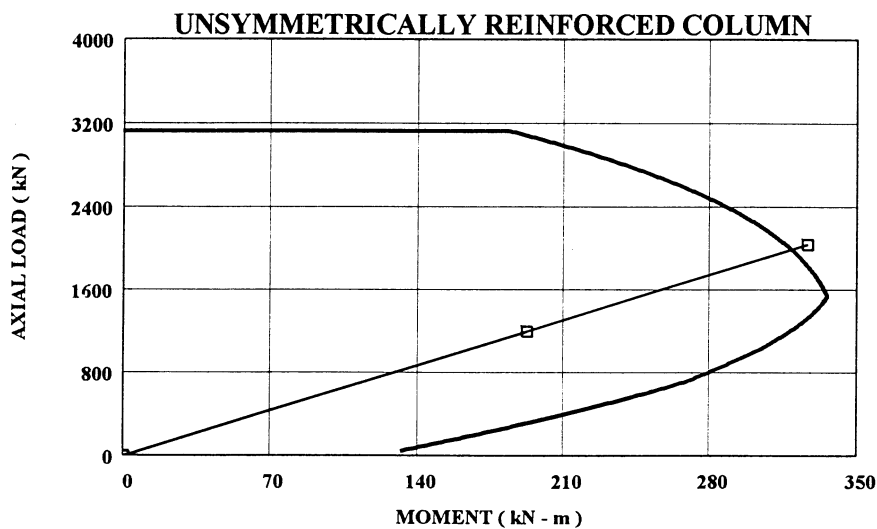

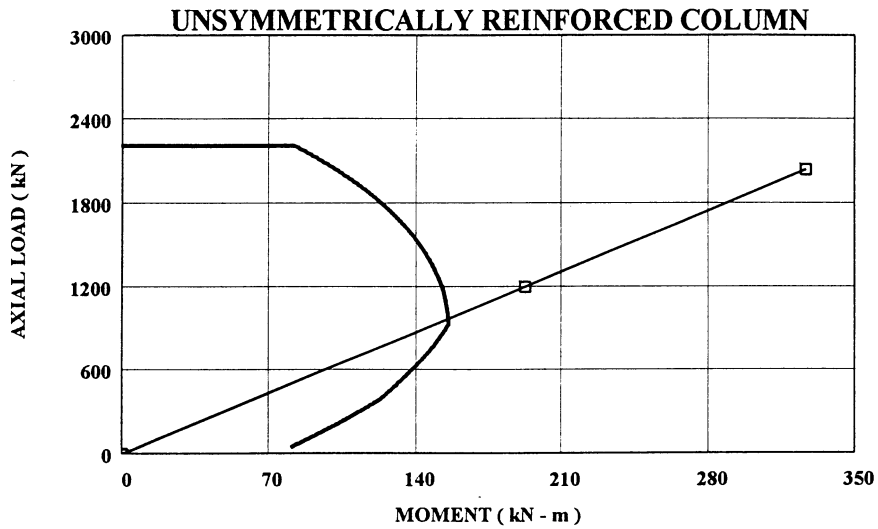Fig. 8. Input data and interaction diagram for first design trial.

Fig. 9. Input data and interaction diagram for second design trial.

program, involves the design of an unsymmetrically reinforced column. The input data for the first design trial is summarized in Fig. 8, along with the corresponding interaction diagram. The results show that the first design trial is not economical since the point (P_u, M_u) is well inside the curve $\phi P_n - \phi M_n$.

The program is easily used to improve the first design trial either by reducing the column cross-section area or by reducing the reinforcing steel area. In the second design trial, the column cross-section dimensions are changed to 400 mm and 350 mm, respectively. The interaction diagram, Fig. 9, shows that the strength of the column is not adequate. Therefore, the selected cross-section dimensions are not acceptable. In the third design trial, the selected column cross-section dimensions are changed to 450 mm and 350 mm, respectively. The interaction diagram, Fig. 10, shows that an optimum design was easily reached after only two trials. This shows the efficiency of

the proposed program in the design of reinforced concrete columns.

CONCLUSIONS

MATHCAD contains tools which can enhance and supplement traditional methods of teaching and learning. The versatility, accessibility, and ease of use make MATHCAD a platform for creating learning modules for technically-based courses. MATHCAD contains the capabilities for traditional classroom computation, but at a greater degree of accuracy, reliability, and presentation quality. In addition, its speed at repetitive tasks, and its programmability, make new learning strategies possible. MATHCAD programs take time for an instructor to develop, but with many benefits in return. By freeing the instructor and student from tedious computation and transcription,
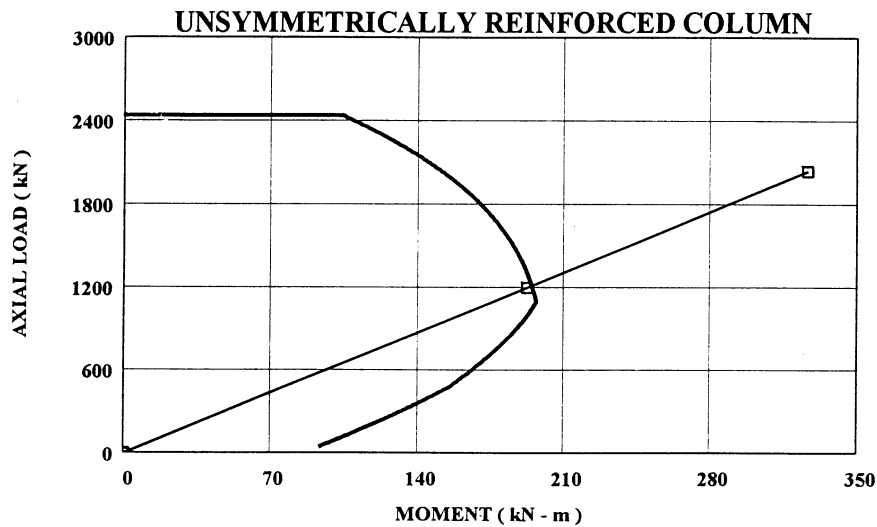

Fig. 10. Input data and interaction diagram for third design trial.

MATHCAD programs create opportunities for meaningful understanding of technical material. A well-designed MATHCAD program can engage both student and teacher, inviting their exploration and discovery of the subject, drawing them deeper into the secrets that it holds.

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