Efflux Time from Tanks with Exit Pipes and Fittings*

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The efflux time experiment is simple to construct, simple to operate, and very useful for teaching the fundamentals of fluid flow, friction loss, and data analysis in an engineering laboratory course that addresses fluid mechanics. Students have the opportunity to study the effect of various critical parameters and dimensions on the time to drain a tank, analyze the data, and draw meaningful conclusions.

INTRODUCTION

THE DEVELOPMENT of cost-effective teaching tools is an important issue for all institutions of higher learning particularly in light of recent cost-cutting initiatives. Departmental budgets leave little room to maintain or even construct any apparatus that could be used to demonstrate and teach abstract concepts. These abstract concepts will simply remain in the student's mind only as memorized abstract equations and Greek mathematical symbols unless students can see these concepts in action, and be able to understand the physical significance of the equations and symbols. Add to this the pressure on students to carry a full course load, work full-time to pay tuition, compete for a limited number of opportunities after graduation, and the onerous task of completing the senior-level chemical-engineering unit-operations laboratory can become an unpleasant experience for all.

There are simple solutions. While instructors are lecturing at the university level, it is important for them to continually bring students 'back to reality'. That is, instructors need to demonstrate the simple fact that abstract concepts do indeed have physical significance that can be measured and observed. (The authors constantly remind students that employers do not sell abstract concepts and mathematical equations; rather they sell tangible products and valuable services. Employers want students who can relate concepts to the physical world.) One way to demonstrate the physical significance of abstract concepts is through the simple efflux time experiment as described below.

Even though the efflux time experiment at NJIT is part of the fluid mechanics experiment repertoire in the senior-level chemical engineering laboratory, it does have broader appeal to other engineering disciplines. Mechanical and civil engineering students also study fluid mechanics, usually with a laboratory course. This particular cost-effective experiment is suitable for all three engineering disciplines. Furthermore, many schools are integrating simple engineering experiments into lower-level and introductory engineering courses to provide the students an initial experience to the fields of engineering and its application, prior to having students make a firm selection of their preferred field of study. The efflux time experiment is one that should be considered.

APPARATUS

The efflux time experimental apparatus involves a supported tank with some means of gauging or monitoring liquid level (preferably a simple closed-end Plexiglas tube with a 1-inch diameter nipple at the discharge), a 1-inch diameter quick-opening valve (such as a knife gate or ball valve), and a series of pre-cut threaded end-connection exit pipes (and/or tubing lengths, of various lengths and diameters) with reducing couplings and adapters to fit the exit pipes to the quick-action valve. All parts can be obtained off the shelf from an industrial supply house such as McMaster-Carr Supply Co. or W. W. Grainger, Inc. A simple schematic diagram is shown in Fig. 1.

Students will also need a method of monitoring liquid level as a function of real time. This can be done with a simple stopwatch, or with a differential-pressure gauge connected to a transmitter with a local read-out on a strip-chart
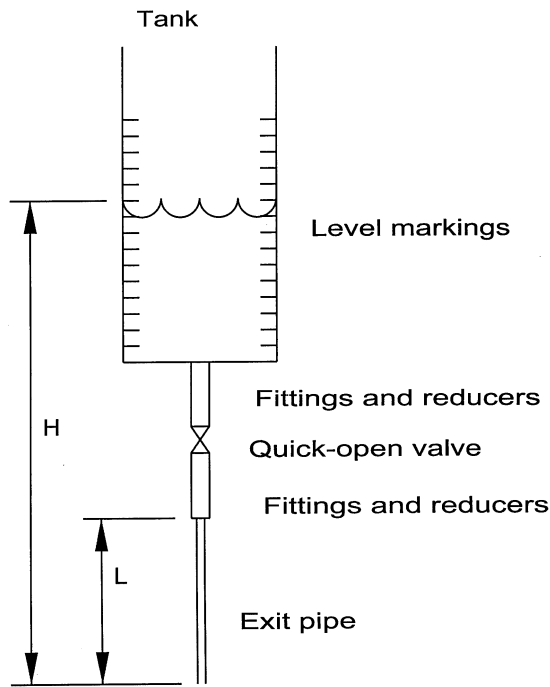
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Fig. 1. Schematic diagram of apparatus.

recorder. If a stopwatch is used, then the tank or a level gauge should be marked at regular intervals. Several chronometers can also be ganged together such that they can all be started simultaneously, and then stopped individually as the liquid level falls past the interval marks on the level gauge.

Lastly, there should be an easy method of filling the tank with fixed piping, a garden hose, or a bucket and funnel. In addition a method of collecting the discharge or directing it to a sewer drain must be provided for.

PROCEDURE

The experiment is simple to operate:

1. Attach the desired exit pipe to the quick-opening valve.
2. Fill the tank to a level slightly above the highest mark on the level scale.
3. Reset the timers and start them all simultaneously.
4. Open the quick-action valve.
5. Stop the first timer when the liquid level falls to the first (highest) mark on the level scale.
6. Stop a timer as the liquid level falls past preselected uniformly spaced marks on the level scale.
7. Record the times.
8. After the liquid has drained out, close the valve, and refill the tank.
9. Reset the timers, and repeat the experiment as needed.

The tank is always filled to a level several inches above the top mark to provide enough liquid to hydraulically fill the exit pipe, and to accelerate the flowing liquid to an initial quasi-steady state.

THEORY

The theory describing the efflux time of a tank has been derived by Crosby [1] and by Bird, Stewart, and Lightfoot [2], and further extended to systems with the installed fittings by Hanesian [3]. Under conditions of turbulent flow in the exit pipe, the efflux time, t, can be related to the height of the liquid relative to the bottom of the exit pipe, H, by:

$$t = k_1[H_i^{3/7} - H^{3/7}] \qquad (1)$$

where

$$k_1 = \frac{7}{3}\left(\frac{r}{r_0}\right)^2\left[\frac{0.0791 \cdot L_e \cdot \mu^{1/4}}{2^{1/4} \cdot g \cdot \rho^{1/4} \cdot r_0^{5/4}}\right]^{4/7} \qquad (2)$$

This constant contains the relevant physical properties of the liquid and critical dimensions of the apparatus with the total equivalent length of the exit pipe and fittings represented by the term L_e. (See nomenclature section for definition of terms.) The underlying key goal of this experiment is to teach the student that an abstract term such as L_e (that is 'equivalent' length, not actual length) does have physical significance, a term that can be indirectly measured and observed through proper data analysis. Furthermore, the analysis of a set of data, from different runs where the length of pipe has been changed, can clearly demonstrate the concept of 'entrance effect'. Students can calculate the equivalent length pipe that would have the same pressure drop or flow resistance that is caused by the entrance effect.

The constants and exponents appearing in these equations are a direct result of approximating the Fanning friction factor for flow through a pipe by the Blassius relation:

$$f = \frac{0.0791}{\mathrm{Re}^{1/4}} \qquad (3)$$

If we ignore the change in friction factor and simply use a value that represents the average for the flow regime and pipe roughness, then:

$$t = k_2[H_i^{1/2} - H^{1/2}] \qquad (4)$$

where:

$$k_2 = 2\left(\frac{r}{r_0}\right)^2\left[\frac{f_{avg} \cdot L_e}{g \cdot r_0}\right]^{1/2} \qquad (5)$$

In the case of laminar flow in the exit pipe, efflux time is related to liquid height by:

$$t = k_3 \ln\left[\frac{H_i}{H}\right] \qquad (6)$$

where:

$$k_1 = \frac{8 \cdot L_e \cdot \mu \cdot r^2}{g \cdot \rho \cdot r_0^4} \qquad (7)$$

Generalized model

In the most general sense where the Fanning friction factor is locally approximated by a modified Blassius relation:

$$f = \frac{k_4}{Re^n} \qquad (8)$$

the efflux time is related to liquid level by:

$$t = k_5[H_i^m - H^m] \qquad (9)$$

where:

$$k_5 = \frac{1}{m}\left(\frac{r}{r_0}\right)^2 \left[\frac{k_4 \cdot L_e \cdot \mu^n}{2^n \cdot g \cdot \rho^n \cdot r_0^{n+1}}\right]^{1/(2-n)} \qquad (10)$$

and:

$$m = \frac{1-n}{2-n} \qquad (11)$$

For example, the Fanning friction factor for flow of water, at room temperature, through a 1/2-inch, Schedule 40 steel pipe in the Reynolds number range of 40,000 to 80,000 can be approximated by:

$$f_{ex} = \frac{0.016}{Re^{0.0742}} \qquad (12)$$

and the appropriate value for *m* is:

$$m_{ex} = 0.4807 \qquad (13)$$

The friction factor is easily estimated for a wide range of flow conditions and pipe roughness factors using the model presented by Churchill [4].

TYPICAL EXPERIMENTAL RESULTS

Typical results for one run are shown in Fig. 2. As expected, the height and apparent rate of efflux decreases with time. It is difficult to assign an exact zero time for the run, but, as we shall see below, this is not necessary. The time recorded when the levels falls to the highest mark need not be accepted as time zero as the error in marking this time is the same as marking any of the time intervals. Furthermore, the slope of the plotted data is important for this analysis, not the
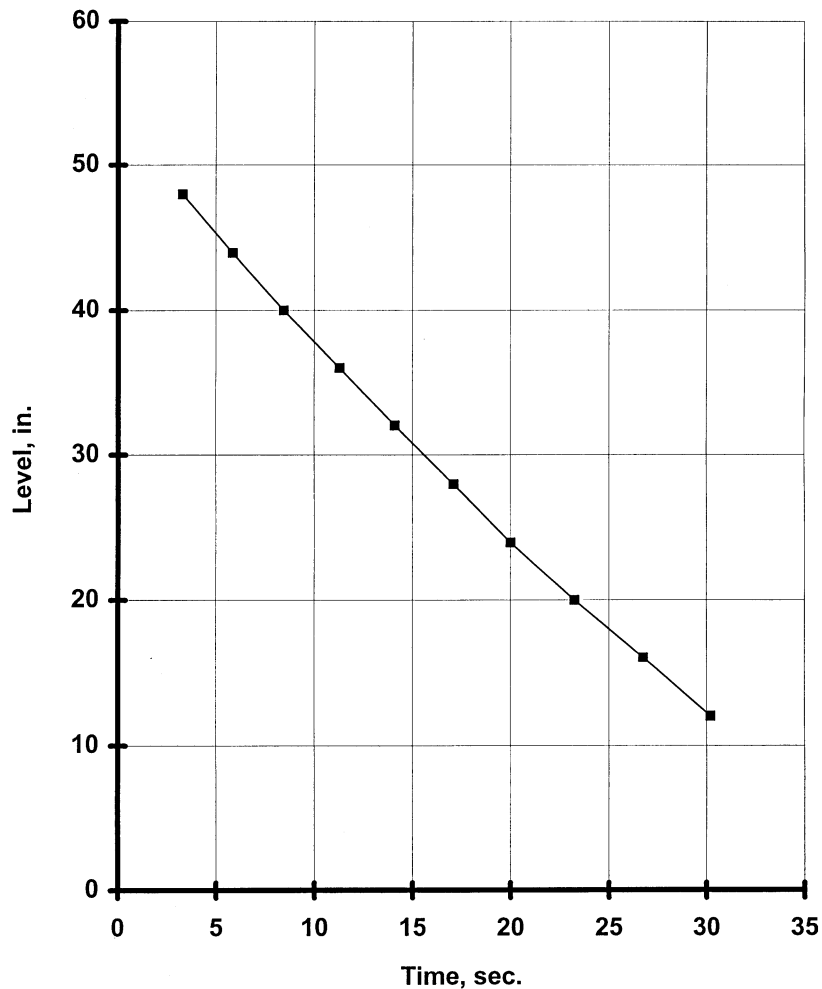

Fig. 2. Typical experimental data.

intercept. Thus the students do not have to adjust all efflux times relative to the time on the first timer.

Data regression for one run

From the run in Fig. 2, it was found that the Reynolds number changed from about 40,000 to 80,000. For this example, the data are plotted as efflux time versus the height raised to the 0.4807 power as shown in Fig. 3. This data should be fit to the best straight line where the intercept defines the best value for time zero, and the slope is the best value for k_5 from which the equivalent length, L_e, is found.

Data set for varying length

The experiments were repeated for a series of pipe lengths. The slopes of the lines were found by linear regression. For each pipe length, an equivalent length is found. Figure 4 shows a plot of the equivalent length versus the actual pipe length, not accounting for the length of the valve and fittings. We should expect to be able to conclude two facts from analyzing this plot after performing a linear regression. First, the intercept of the plot should provide the equivalent length of all valves, fittings, and contractions as flow enters the pipe from the tank. Second, the slope of the line should be unity. That is, every foot of additional exit pipe should increase the overall equivalent length by one foot. Figure 4 does not show this.

There are several explanations for this, and it would be a useful exercise to allow the students to perform a sensitivity analysis of their data to find out which variables have the greatest impact on the final results. In this case, it is found that small changes in the tank and pipe diameters, and in the relative roughness factor for the pipe being used, can have a profound impact on the results. Students should go back to the experimental apparatus to confirm the dimensions of the equipment, and to inspect the condition of the pipe lengths. Depending on the age of the equipment and the quality of water that had been used over the years that the equipment was in use, there may be scale build-up in the lengths of pipe. A small amount of scale can in fact make a rough pipe smoother as scale would tend to collect in crevices and so reduce the relative roughness of the pipe.
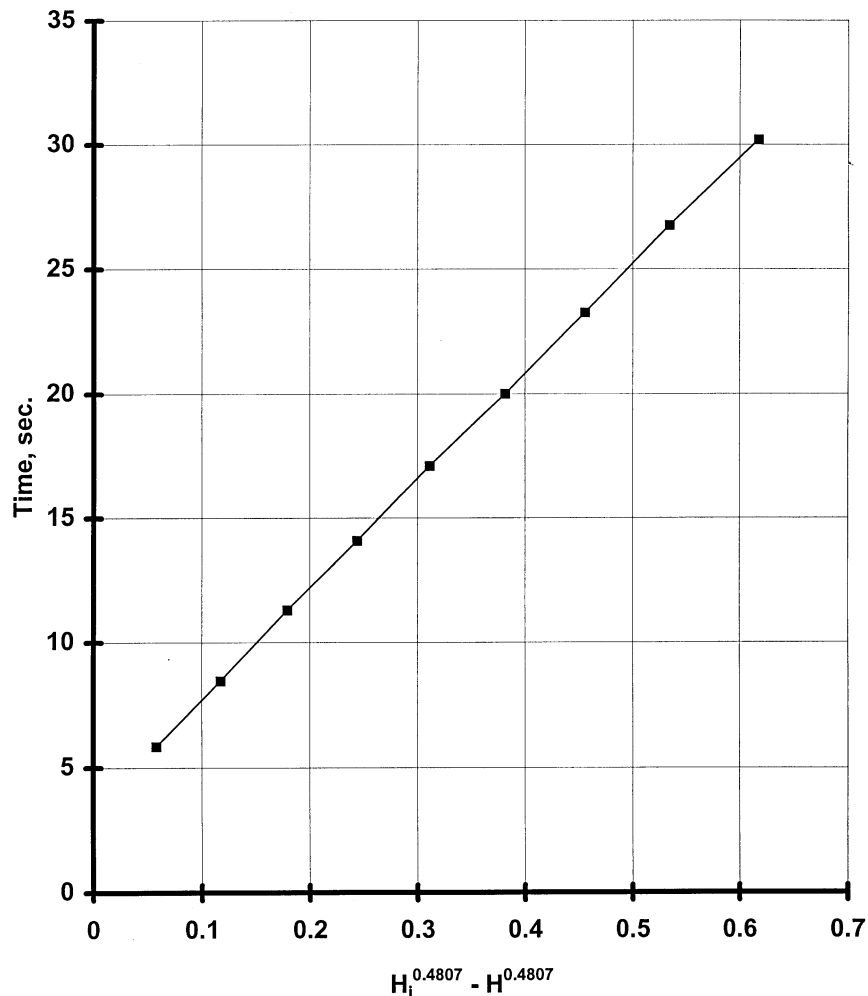

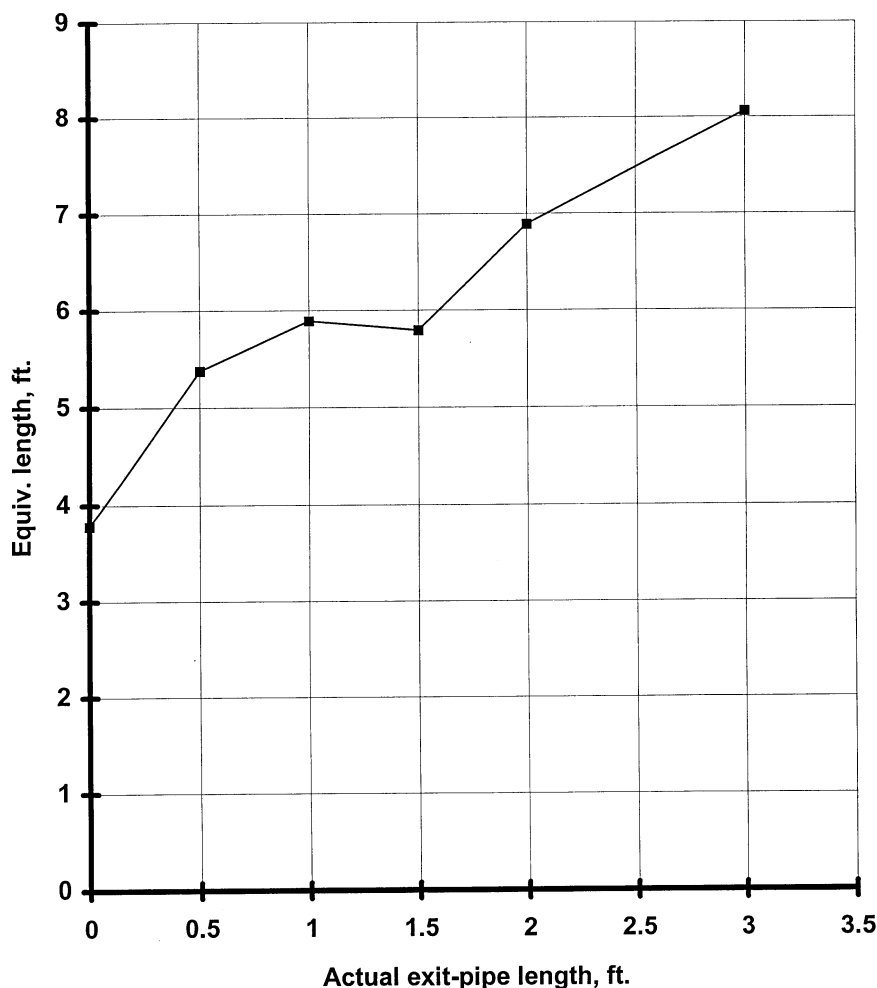Fig. 3. Typical data analysis for turbulent flow.

Fig. 4. Total equivalent length vs. exit-pipe length.

The use of a spreadsheet to analyze experimental data obviously simplifies and eases the process of doing a sensitivity analysis.

Data set for laminar-flow run

Figure 5 shows typical data from a run where the flow was in the laminar regime. Laminar flow is easily obtained by attaching a long length of 1/4- or 1/8-inch diameter plastic or metal tubing. (We have found that we can use old, discarded GC columns where the ends have been cut off and the packing blown out.) The results presented in Fig. 5 predict an equivalent length of 33.7 ft for a 6-m long, 3/16-inch diameter, coiled tube, including the fittings, reducers, and quick-action valve.

EXTENSION OF THE EXPERIMENT

Several other experiments can be set up that would allow the student to investigate the impact of other parameters (physical dimensions or fluid properties). Students can do a series of runs to examine the effect of pipe or tank diameter, or exit valve position. Experiments can be repeated where the thermophysical properties of the liquid are varied (for example, heating the liquid to reduce viscosity, adding a solute such as a biodegradable thickening agent to increase viscosity, etc.).

The apparatus can also be modified to allow the students to study the effect of surface roughness. For example, a series of pipes can be prepared with varying roughness factors. Conventional stainless steel pipe can be electropolished to provide pipe with essentially a smooth bore. Other pipes can have fine threads cut into the inner surface on a lathe to produce pipes with a known, very large roughness factor.

The students also have an opportunity to determine if the flow through the exit pipes is fully developed, turbulent flow. They can also postulate and test various methods for ensuring that the flow at the entrance of the exit pipe (after all fittings) is fully developed.

We must not discount the possibility of pockets of air or vapor under some high flow-rate situations. Immediately downstream of a contraction or fitting, there will exist a vena contracta that may contain trapped air or vapor. The presence of air or vapor may be due to inadequate flushing during the time when liquid is accelerating, or there may
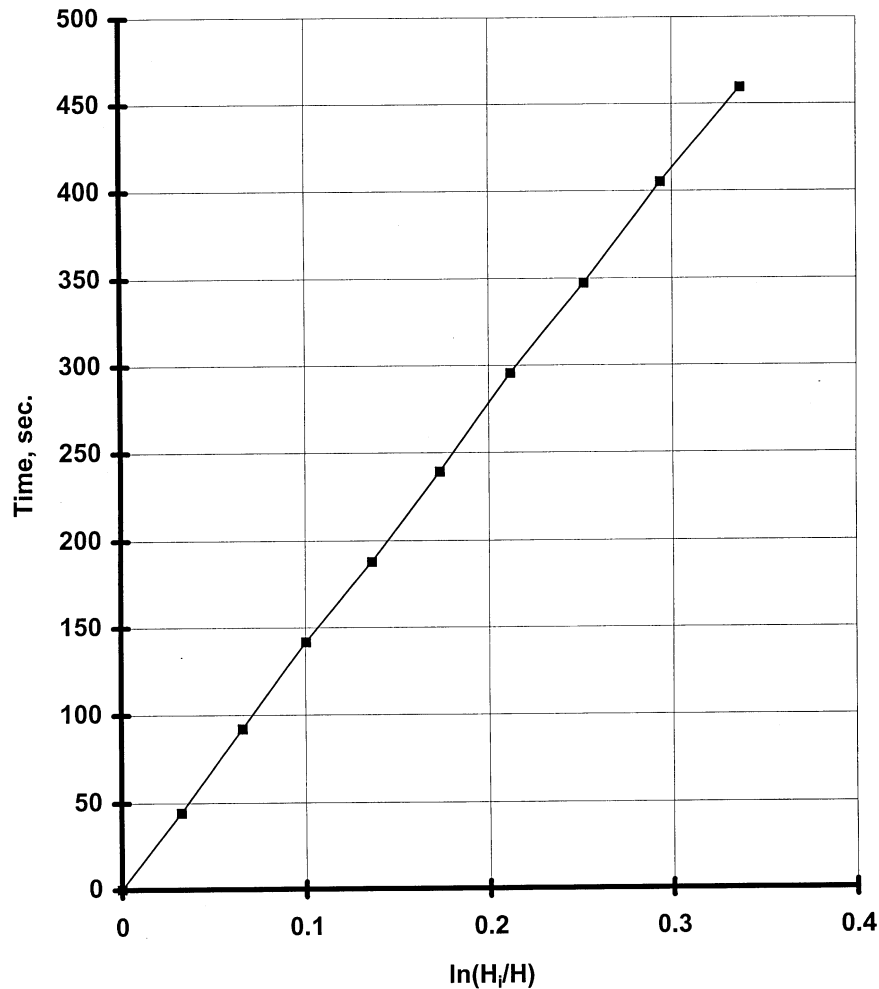
Fig. 5. Typical data analysis for laminar flow.

be a small leak of air into that zone from outside. The presence of this zone would call for a different analysis of the data. Indeed, because our raw data (time required to attain a particular level) seems to be relatively insensitive to the length of the exit pipe, it is quite possible, in our experimental set-up, that air or water vapor is present somewhere in the exit pipe or fittings. An experimental apparatus that uses some type of transparent pipe, such as glass or Plexiglas, would verify this situation.

Still another factor that can be examined is the siphon effect. That is, students can determine the effect of the level of the outlet of the pipe. This can be examined by using a length of garden hose coiled up so that the experiment can be repeated after raising or lowering the outlet of the hose. Students can determine if centrifugal forces have an impact on the drain time by varying the diameter of the coil of hose. They can also examine the effect of the direction of the discharge from the hose. The analysis above assumes that the liquid flows are unidirectional. If the angle of the discharge is changed, even pointing it straight up as in a fountain, the effect of kinetic energy changes can be observed.

In all of the above parametric studies, the students should estimate, or predict, the results in advance. Theoretical analysis may suggest that changes in pipe roughness, for example, would alter the efflux time by only a fraction of a second, not enough difference to be clearly observed with manually operated chronometers. Several repeat runs for a particular set-up would allow students to perform statistical analysis of experimental data, a skill that is gaining ever more prominence and importance in industry.

CONCLUSION

One of the most important lessons that students can learn from running this experiment is this: the method of data analysis depends on the experimental set-up and conditions. Our experience has shown that students are quick to fit any data (usually numerical results without units) to a straight line, calculate the slope and intercept of that best-fit line, report those numbers (usually without units), and finally walk away thinking that they had successfully completed the experiment

and data analysis simply because they indeed did get a computer-generated 'best fit' of a straight line from the data. This experiment teaches the student to carefully consider what data can fit a straight line, allow him or her to assign and appreciate the physical significance to the slope and intercept, and then draw meaningful conclusions. The simplicity of the apparatus and its operation allow students to concentrate more on the data analysis, and less on actual time-consuming physical testing. Thus several runs can be performed in a reasonable time period so that the students have a lot of 'real-world' data to work with.

NOMENCLATURE

d	pipe inner diameter
f	Fanning friction factor
g	acceleration of gravity
I	liquid level in the tank
k	constant with collected terms
L_e	equivalent length
m	constant $(1 - n)/(2 - n)$
n	exponent in friction-factor correlation with Reynolds number
r	tank inner radius
r_0	pipe inner radius
Re	Reynolds number $= dv\rho/\mu$
t	time
v	fluid velocity in pipe
μ	fluid viscosity
ρ	fluid density

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