

Motivating Engineering Students by Using Modern Case Studies

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The use of case studies to motivate engineering students is long established. In order to enthuse today's school and college students it is suggested that case studies be presented which show the application of relatively simple mathematics to the forefront technologies. The first example is of an aeroplane taking off where Newton's second law of motion is used in a modern context; a simple and a more refined model offered. The remaining examples are in the area of digital image processing and show the crucial importance of matrix algebra in this modern application.

INTRODUCTION

THE NEED to attract more of the most mathematically able school students to study undergraduate engineering degree programmes is ever more urgent. Part of the difficulty in persuading more young people to pursue their study of mathematics beyond the compulsory stage is their perception, which is shared by the general public, that much of the mathematics to which they have been exposed is irrelevant to modern life. A consequence of the failure to attract sufficient numbers of more able students into continuing with mathematics is that too few of them enter engineering degree programmes with a sufficiently strong mathematical background, yet such a background is required in order to underpin many state-of-the-art engineering applications.

Contrary to popular belief mathematics is rich in its applications in the modern world and much of the technology upon which we have come to rely has been designed with mathematical methods at the heart of the process. There is a need, then, to get this message across at a sufficiently early stage to convince school students that mathematics is a relevant and living subject, with exciting applications in areas of activity which impinge on their daily lives. Armed with mathematical tools students can look forward to a wide range of careers in emerging technologies and modern application areas.

This paper aims to examine one way in which that message can be transmitted.

MAKING A CASE FOR CASE STUDIES

Case studies have long been used as a means of motivating students at all levels to acquire

mathematical techniques and expertise. However, much of the effort has, unfortunately been less effective than it might have been. The difficulty with introducing case studies into the curriculum is getting the balance right. If a case study relies on too much detailed mathematical technique then it can detract from the application and weaken the impact which it seeks to make. On the other hand, too simplistic a mathematical model can offer a false prospectus and mislead students into believing that the only mathematics which they require in their studies is at a low level.

Further, if an unrealistic application of mathematics is offered to the students then they will see through the smokescreen and the exercise will have been counter-productive. If mathematics is applied to a situation to which it has not been applied hitherto, to which it is not currently being applied and to which it is highly unlikely to be applied in the future then the students will get the impression that the mathematicians are desperately seeking applications in order to justify the presence of their subject in the curriculum.

On the other hand, a carefully-chosen case study can act as a real motivator to students and help to convince them that mathematics does have a valuable, indeed essential, role in the advancement of technology and is relevant to their highly technologically-developed world. Every aspect of a young person's life is becoming more dependent on technology: games-stations, hi-fi and mobile telephones are but three obvious examples of a life dominated by the programmable silicon chip. If we are to interest today's schoolchildren in mathematics then the applications which we present to them must relate to that world: cooling liquids and trains going uphill are no longer good enough, however worthy they may be. Presenting modern examples of the interface between mathematics and engineering can have the effect of exciting the more mathematically able students to consider engineering as a career and to maintain their appetite

* Accepted 4 May 1999.

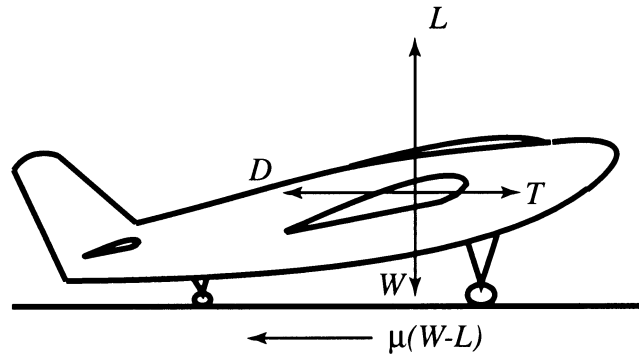


Fig. 1. Forces acting upon an aircraft during take off.

for mathematics in their engineering studies. It can also convince intending students of engineering that mathematics is an important component of their studies, not a necessary evil to be endured.

Engineering colleagues can make a real contribution by suggesting forefront areas of their subject which can be amenable to relatively simple mathematical models, to be presented as case studies. They and the mathematics lecturers can work together to produce a portfolio of such case studies for wider dissemination. The examples which follow can be presented at various levels, from school to first-year university.

Example 1: Aircraft take-off distance

As flying has become more popular and larger aeroplanes are developed, longer runways are required in order to accommodate the greater take-off and landing distances. How long a runway is required for a particular aeroplane? Here, we concentrate on a simple take-off. Aircraft designers recommend that an aeroplane lifts off when its speed along the ground reaches a value 1.2 times its stalling speed.

Figure 1 shows the forces acting on an aeroplane. T is the engine thrust, D the aerodynamic drag, L the aerodynamic lift, W the gross take-off weight and μ is the coefficient of rolling friction between the wheels and the runway. Note that the combined normal reaction between the wheels and the ground is $(W - L)$. Applying Newton's second law of motion in the horizontal direction gives:

$$\frac{W}{g} v \frac{dv}{ds} = T - D - \mu(W - L) = F_a \quad (1)$$

where v is the velocity of the aeroplane and s is the distance travelled along the runway. F_a is the net accelerating force. During take-off it is a reasonable assumption, based on experimental measurements, that T is constant, as is W . Both D and L have been shown to be proportional to v^2 so that we may write

$$F_a = F_0 + \left(\frac{F_1 - F_0}{v_1^2} \right) v^2$$

where v_1 is the speed at take-off. The accelerating force at the start is:

$$F_0 = T - \mu W$$

since $v = 0$. The accelerating force at lift-off is:

$$F_1 = T - D$$

since the reaction is zero as the aircraft leaves the ground.

A simple model neglects the drag and friction forces since these have been estimated to account for between 10 and 15 percent of the energy expended. Then

$$T = \frac{W}{g} v \frac{dv}{ds}$$

Separating the variables and integrating from the start to lift-off we obtain:

$$\int_0^{s_1} ds = \frac{W}{Tg} \int_0^{v_1} v dv$$

where s_1 is the distance travelled before the aircraft takes off, and hence:

$$s_1 = \frac{W}{2Tg} v_1^2$$

Remember that:

$$v_1 = 1.2v_s$$

where v_s is the stalling speed. It can be calculated from the formula:

$$v_s = \sqrt{\frac{2W}{\rho S C_{Lmax}}}$$

where C_{Lmax} is the maximum coefficient of lift, ρ is air density and S is the wing area.

Consider a typical executive jet aeroplane for which $W = 48000$ N, $T = 14000$ N, $S = 29$ m², $\rho = 1.225$ and $C_{Lmax} = 1.2$.

The stalling velocity is:

$$v_s = \sqrt{\frac{2 \times 48000}{1.225 \times 29 \times 1.2}} = 47.45 \text{ ms}^{-1}$$

Hence the lift-off velocity is:

$$v_1 = 56.9 \text{ ms}^{-1}$$

Finally, the length of runway required before lift-off is:

$$s_1 = \frac{48000}{2 \times 14000 \times 9.81} \times (56.9)^2 = 565.8 \text{ m}$$

A more sophisticated model uses equation (1). Separating the variables and integrating, as before, produces

$$\int_0^{s_1} ds = \frac{W}{g} \int_0^{v_1} \frac{v dv}{F_0 + \left(\frac{F_1 - F_0}{v_1^2}\right) v^2}$$

so that:

$$s_1 = \frac{W}{2g} \left(\frac{v_1^2}{F_1 - F_0}\right) \ln\left(\frac{F_1}{F_0}\right)$$

First, we use the following relationship, which has been determined experimentally, to determine the drag coefficient C_D :

$$C_D = \quad 0.02 \quad + \quad kC_L^2$$

profile drag + lift-dependent drag

where

$$C_L = \frac{\mu}{2k}$$

Then

$$F_0 = T - \mu W$$

$$F_1 = T - D = T - \frac{1}{2} \rho S C_D v_1^2$$

For the aeroplane already considered, we take $k = 0.04$, $\mu = 0.04$, which are typical values. Then

$$C_L = \frac{0.04}{2 \times 0.04} = 0.5$$

$$F_0 = 14000 - 0.04 \times 48000 = 12080 \text{ N}$$

$$F_1 = 14000 - \frac{1}{2} \times 1.225$$

$$\times 29(0.02 + 0.04 \times (0.5)^2)(56.9)^2 = 12275 \text{ N}$$

Finally,

$$s_1 = \frac{48000 \times (56.9)^2}{2 \times 9.81(12275 - 12080)}$$

$$\times \ln\left(\frac{12275}{12080}\right) = 650.5 \text{ m}$$

This means that the distance estimated using the simple model was about 13% too short. Parameter values for other types of aeroplanes can be obtained from, for example, [1].

Example 2: Matrices and digital image processing

In this example we show how relevant and interesting examples can be used to motivate techniques in linear algebra, and in particular matrix notation and matrix multiplication. These topics can appear rather abstract when they are met for the first time in a maths class, but with modern digital technologies such as mobile phones and digital television, linear algebra has become as important a tool as calculus for the engineer.

The Internet is now a part of many people's daily lives. Young people, especially, are attracted to it and to related new technologies. They can use the Internet to visit the web-sites of their favourite pop groups or sporting heroes and heroines, and routinely use it to obtain information for college projects. It is an easy matter to download high quality graphic images to grace their bedroom walls or to enhance their assignments. Their familiarity of the process involved in obtaining images means that many will have heard of file types such as *gif* or *jpg*. They will recognise these as the suffices appended to the filenames, as in *spice.jpg* and *wimbledon.gif*. Many home computers are now sold with digital scanners so that the users can, for example, scan their favourite photographs, display them on the computer, and e-mail them to their relatives around the world. What many will not know is that a range of mathematical tools are required to produce these files. Furthermore, a number of mathematical techniques to which they may have been introduced in their secondary school years, and which may have seemed rather dull and lacking in application at the time, form the basis of these tools. Armed with these tools, young people should be aware that career prospects in companies involved in image processing are excellent.

An image stored by a computer is generally a *digital image*. This is an image which can be represented by a matrix. Consider, as a very simple example, the mathematical representation of the symbol π . Very crudely we can picture the symbol as in Fig. 2.

We could represent this image of π by a 3×5 matrix such as:

$$B = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 & 1 \end{pmatrix}$$

Note that we have divided the image into a number of picture elements, or *pixels*, in this case fifteen of them. We have then assigned an integer to each pixel, 0 if the field is to be shaded black, and 1 otherwise. In this example there are just two *levels*, black or white, and so each element in the matrix is the number 0 or 1.

An alternative way of thinking about each row is to draw a graph where the y values are either 0 or 1. So the second row, that is the row vector (1 0 1 0 1), can be graphed as shown in Fig. 3.



Fig. 2. A simple digital representation of π .

This sort of representation will be helpful when we look at how matrices can be used to ‘process’ the image.

Images such as black-and-white photographs can be ‘digitised’ in the same way. The image is divided into $m \times n$ elements where, in practice, m and n are large integers. Each element is assigned a number which represents the intensity of the image in that area. Often a 64-, 128- or 256-point grey scale is used so that a range of grey levels can be represented ranging from black at one extreme to white at the other. A digital image is then represented by an $m \times n$ matrix, whose elements are numbers in the range 1–64, 1–128 or 1–256 depending upon the number of grey levels used. There are computational advantages in selecting matrices with sizes and number of grey levels which are powers of 2, as in $2^6 = 64$ and $2^7 = 128$. For example a black-and-white television image is typically a 512×512 matrix whose elements are integers in the range 1–128. The 256-point digital image represented by the matrix

$$B = \begin{pmatrix} 0 & 50 & 100 & 150 \\ 50 & 100 & 150 & 200 \\ 100 & 150 & 200 & 225 \\ 150 & 200 & 225 & 255 \end{pmatrix}$$

is shown in Fig. 4. Each row could be graphed as was shown in Fig. 3.

A very graphic way of bring home to students just how useful such ideas can be is to refer them to information about how the American Federal Bureau of Investigation, the FBI, is digitising and thereby storing electronically, its vast database of fingerprints. They have been collecting fingerprints since 1924 and now have over 200 million cards and tens of thousands of new cards arrive daily. The only way that it is possible to search this massive record in order to try to match fingerprints found at the scene of a crime is to use computers and to accomplish this each fingerprint image must be digitised. Information on the image coding mechanisms in use is readily available on the Internet. Students could start, for example, by looking at the site [2] but other sites can be found by searching. It soon becomes evident that

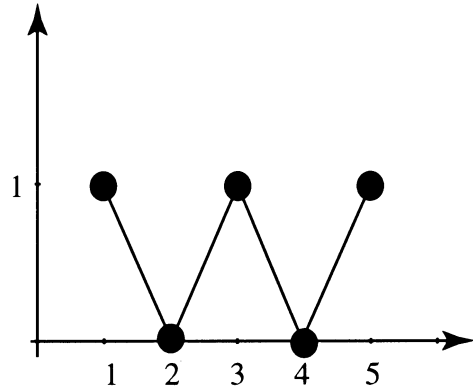


Fig. 3. A graph of the row vector (1 0 1 0 1).

this technology relies very heavily on the mathematical techniques of linear algebra.

Sometimes it is desirable to remove sharp edges from an image. One way in which an image can be ‘smoothed’ is to replace a matrix element by the average of some of its neighbours. For example the four element row vector:

$$(2 \ 1 \ 4 \ 2)$$

graphed in Fig. 5 can be smoothed to that shown with a dotted line by replacing each element by the mean of the four elements. For what will follow it is crucial to note how this can be achieved by post-multiplying the row vector by the matrix

$$\begin{pmatrix} \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \\ \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \\ \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \\ \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \end{pmatrix}$$

so that

$$(2 \ 1 \ 4 \ 2) \begin{pmatrix} \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \\ \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \\ \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \\ \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \end{pmatrix} = \left(\frac{9}{4} \ \frac{9}{4} \ \frac{9}{4} \ \frac{9}{4}\right)$$

This kind of operation is the basis for many techniques so vital for effective digital image processing.

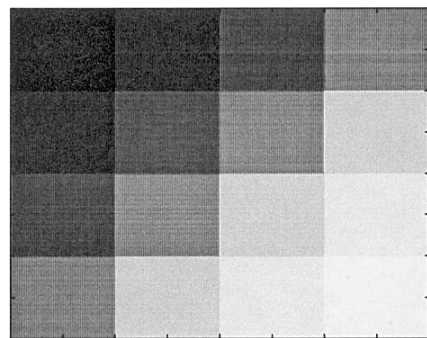


Fig. 4. A digitised image having several levels of grey.

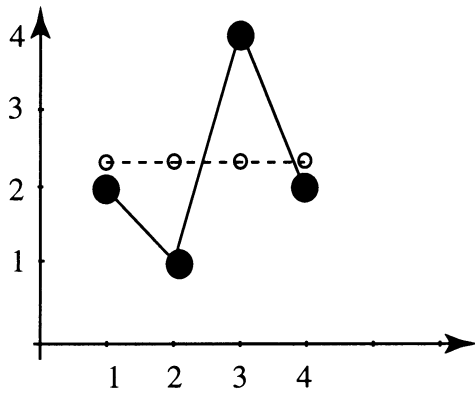


Fig. 5. A row vector can be smoothed by matrix multiplication.

Example 3: mathematical techniques for image compression

We showed above how digital images are stored using matrices. It is a simple matter to calculate the number of storage bits needed for different sized matrices and different numbers of grey levels.

For example, consider a single image consisting of 256×256 pixels. Suppose the image is made up of 256 grey levels. Noting that $256 = 2^8$ we see that each pixel requires 8 bits of storage making a total of $256 \times 256 \times 8 = 524,288$ bits. If these are stored in 8-bit bytes, 65 536 bytes will be needed. It is easy to see that storage requirements increase very rapidly as we increase the size of the storage matrix, number of grey levels and the number of images. When colour images are considered the numbers rocket further. Clearly a vast amount of data needs to be stored if the digital image is to be represented directly. For example, a single colour picture stored at the resolution viewed on a television screen requires around 1 million bytes for storage. In order to view an image over the Internet it needs to be transmitted from the web-

site being visited to the user's computer. This takes time, and the larger the file, the longer it takes to transmit it. No one likes sitting in front of a computer screen waiting for what seems like forever for an image to arrive. So, for the technology to be viable it is necessary to reduce the size of these files, or *compress* the image in some way. This is an exceptionally important branch of modern engineering, which like those already mentioned, relies very heavily on mathematics.

To see how compression might be attempted consider the following example.

The solid graph shown in Figure 6 has been drawn by plotting the row vector

$$(1 \ 2 \ 5 \ 4 \ 2 \ 2 \ 4 \ 3)$$

which could have been a row in a digital image as described in the previous case study. The trick when trying to compress an image is to throw away some of the information—that which will cause the least deterioration in the quality of an image. We start by looking for locations where the variation in the image is least. We will now 'process' this row vector in the following way. First we find the mean of the first and second elements, and get 1.5, the mean of the third and fourth elements, to get 4.5, and so on. These results are placed in the second row of a matrix as shown:

$$\begin{pmatrix} 1 & 2 & 5 & 4 & 2 & 2 & 4 & 3 \\ 1.5 & 4.5 & 2 & 3.5 & & & & \end{pmatrix}$$

Now subtract each of the means just calculated from the first of the elements used to calculate it. So we subtract 1.5 from 1, to give -0.5 ; we subtract 4.5 from 5 to give 0.5; 2 from 2 gives 0 and finally 3.5 from 4 gives 0.5. These differences

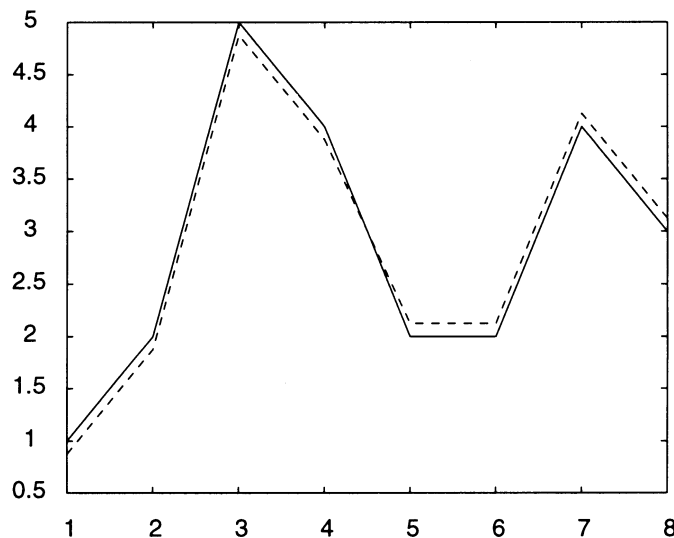


Fig. 6. Graphical representation of an original image and a compressed version.

are placed in the second row of the matrix following the elements already placed there:

$$\begin{pmatrix} 1 & 2 & 5 & 4 & 2 & 2 & 4 & 3 \\ 1.5 & 4.5 & 2 & 3.5 & -0.5 & 0.5 & 0 & 0.5 \end{pmatrix}$$

It is crucial to recognise the importance of the zero here. It represents a place in the image where there is no change from the mean, that is a place of no variation. Note that this is consistent with the original row vector where the value 2 is repeated.

The process is repeated, by averaging the previously calculated averages in pairs, and then finding the variation from the mean of the first of the pairs. This gives:

$$\begin{pmatrix} 1 & 2 & 5 & 4 & 2 & 2 & 4 & 3 \\ \mathbf{1.5} & \mathbf{4.5} & \mathbf{2} & \mathbf{3.5} & -0.5 & 0.5 & 0 & 0.5 \\ \mathbf{3} & \mathbf{2.75} & -1.5 & -0.75 & -0.5 & 0.5 & 0 & 0.5 \end{pmatrix}$$

To emphasize the distinction between the means, and the differences, the means have now been typeset bold. Finally, one more application of the process yields:

$$\begin{pmatrix} 1 & 2 & 5 & 4 & 2 & 2 & 4 & 3 \\ \mathbf{1.5} & \mathbf{4.5} & \mathbf{2} & \mathbf{3.5} & -0.5 & 0.5 & 0 & 0.5 \\ \mathbf{3} & \mathbf{2.75} & -1.5 & -0.75 & -0.5 & 0.5 & 0 & 0.5 \\ \mathbf{2.875} & 0.125 & -1.5 & -0.75 & -0.5 & 0.5 & 0 & 0.5 \end{pmatrix}$$

In the last row, the number 2.875 is the mean of all the original values and the remaining numbers represent variations from the mean. If the process is reversed we will recover the original row vector. However, and here is the trick, suppose that we throw away one of the non-zero values representing variation, and in particular we throw away the one with smallest magnitude, the 0.1250. This leaves the row vector:

$$(2.875 \quad 0 \quad -1.5 \quad -0.75 \quad -0.5 \quad 0.5 \quad 0 \quad 0.5)$$

Note that, whereas none of the original data values were zero, we now have two zeros. Very efficient techniques exist for storing matrices with a significant proportion of zeros, so-called *sparse matrices*. So, we have thrown away some information and achieved a smaller set of data to be stored. This could be transmitted, for example over the Internet, and it only remains to decode it at the receiving end. This is achieved by reversing the process above. If this is done the restored image vector is

$$(0.8750 \quad 1.8750 \quad 4.8750 \quad 3.8750 \quad 2.1250 \quad 4.1250 \quad 3.1250)$$

Both the original data, and the compressed data (shown as the dotted line) are depicted in Figure 6. Clearly there is some deterioration of the restored image compared with the original, but for many purposes this sort of degradation will not be a

problem. If you had seen these images separately it would be difficult to tell them apart. For example, when looking at a photographic image on a website, or a computer game, fine detail may not be important.

Techniques of linear algebra are seen to be particularly important when it is realised that the operations used to transform the row vector

$$(1 \quad 2 \quad 5 \quad 4 \quad 2 \quad 2 \quad 4 \quad 3)$$

to

$$(2.875 \quad 0.125 \quad -1.5 \quad -0.75 \quad -0.5 \quad 0.5 \quad 0 \quad 0.5)$$

can be achieved by matrix multiplication. To produce the second row the original vector must be post-multiplied by the matrix

$$M_1 = \begin{pmatrix} \frac{1}{2} & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 \\ \frac{1}{2} & 0 & 0 & 0 & -\frac{1}{2} & 0 & 0 & 0 \\ 0 & \frac{1}{2} & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{2} & 0 & 0 & 0 & -\frac{1}{2} & 0 & 0 \\ 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & \frac{1}{2} & 0 \\ 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & -\frac{1}{2} & 0 \\ 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & \frac{1}{2} \\ 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & -\frac{1}{2} \end{pmatrix}$$

Perform the matrix multiplication to convince yourself that this is so. The third row can be produced from the second by post-multiplying by

$$M_2 = \begin{pmatrix} \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 \\ \frac{1}{2} & 0 & -\frac{1}{2} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{2} & 0 & -\frac{1}{2} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

Finally the fourth row can be obtained from the third by post-multiplying by

$$M_3 = \begin{pmatrix} \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{1}{2} & -\frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

Writing the original vector as P and the transformed vector as T we see that

$$T = PM_1M_2M_3$$

or

$$T = PM \quad \text{where} \quad M = M_1M_2M_3$$

The image is then edited to add further zeros (throwing away variations of small magnitude) producing a new matrix T^* , and then the process is reversed to give P^* , say, simply by postmultiplying by the inverse of M :

$$P^* = T^*M^{-1}$$

When it is necessary to compress a whole image, rather than a single row vector, the matrix operations are exactly the same, although it is usual to apply the averaging and differencing both to columns and rows in a matrix. A very readable and thorough introduction to this process can be found in the excellent paper by Mulcahy [3]. *Matlab* code to implement the process on a computer is also available. A much wider range of material can be found in [4].

The importance of the underlying process cannot be underestimated because it encapsulates the essence of many other mathematical methods. What has happened is that the original data has been processed or *transformed*. In engineering, mathematical transformations are applied to obtain new information about some data which is not necessarily evident when the data is in its original form. We saw this when, by transforming the image data above, we obtained information about the average value, and the variation of the data from that average. There are a number of transformations which can be applied to extract new information from data, among which a group known as Fourier transforms are the most important. These have been around for a long time. Increasing demands from the industry to produce ever faster, and higher quality, compression

mechanisms have led to a proliferation of other transforms. One particular group presently at the forefront is a group known as wavelet transforms which are based on the technique of averaging and differencing detailed above. These are used by the FBI to compress their hundreds of millions of digital images so that they can be stored efficiently, but nevertheless retain sufficient detail to be of use in fingerprint matching.

Example 4: jpeg compression

We have noted already, in Example 2, that students are often familiar with computer files containing digital images. When a still photographic image is viewed over the internet the likelihood is that this image is stored as a so-called JPEG (pronounced 'jay-peg') file.

JPEG is a compression mechanism which works especially well on natural images like scenes or photographs, although it is not designed to compress images containing lettering and line drawings. JPEG stands for the committee who drew up the specification for this compression mechanism: the Joint Photographic Experts Group. When compressing an image using JPEG some of the information in the original photograph or file is thrown away and lost forever. This degrades the quality of the image and if too much information is thrown away the effect will be very noticeable. However, by carefully selecting which bits of information to lose the human eye can be deceived into not noticing any change.

The original digital image is first subdivided into smaller blocks of size 8×8 . A transform called the *discrete cosine* transformation is applied to each block. This transform provides additional information about the frequency content of the image. Loosely, this is information about where values are changing slowly (low frequency) and where values are changing quickly (high frequency). High frequency values are discarded through an operation involving matrix multiplication, resulting in a compressed image file. This can be transmitted



Fig. 7. A jpg image and a compressed version of the same scene.

efficiently over the Internet, and decompressed, albeit with some loss of quality at the receiving end. For example the first image in Fig. 7 is a jpg file with file size of 40KB and the second is a compressed version of size 24KB.

CONCLUSIONS

The case studies offered in this paper are designed to show that applications of mathematics in modern engineering settings are feasible for presenting to young people in schools or colleges in order to excite their interest in mathematics as a living subject with real, and increasing relevance in

the world of today and tomorrow. These exemplars can, of course, be refined and made more sophisticated and are not meant to be the last word in their area of application. A good case study can be re-visited as students mature mathematically and this will help to encourage them to develop continually their mathematical skills.

The generation of suitable case studies requires a closer collaboration between mathematics and engineering lecturers and school teachers. It is vital that we increase the number of mathematically able young people who pursue engineering as a career and we believe that the approach suggested in this paper is one way forward.

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