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Core Mathematics for Engineers, Mathematicians and Scientists*

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Our challenge is to develop flexible, robust and coherent curricula that are dynamic with respect to the needs of individual programs, the advances in technology, and the advances in learning theory. The past decade has witnessed a concatenation of forces pressing at an unprecedented rate for curriculum change. These changes, led by the escalating growth in the use of technology for insight, demand a restructuring of the curriculum—content as well as pedagogy. In this paper, I will discuss four major forces promoting change, describe some shortcomings of present curricula, and then suggest a structure for a future curriculum.

FORCES FOR CHANGE

Calculus reform movement

THIS movement, initiated with the panel discussion on 'Calculus Crucial, but Ailing' held during the 1985 Joint Mathematics Meetings, refocused instruction toward engaging students to take responsibility for their own learning. Helping students 'learn how to learn' has become an acceptable goal of mathematics courses. Instructors have altered their roles, decreasing the presentation role and increasing the guide, coach or facilitator role. Small-group work is widespread in mathematics curricula—both in terms of in-class group activities and out-of-class group projects. Real-world applications and hands-on experimentation pervade many curricula today. Use of multiple representation—graphic, numeric, symbolic and verbal—has become standard procedure in the majority of calculus courses. Development of communication skills has become accepted as a legitimate objective of mathematics courses. An increasing number of programs now include an explicit student growth model. This is a model that provides both guidance and accountability for developing essential skills such as reasoning and communication through a sequence of courses. Skills whose development are too important to be left to chance.

The impact of the Calculus Reform Movement is clearly visible when comparing a calculus text published in the late eighties to one published in the late nineties [8]. The impact of the Movement has spread to other mathematics courses, notably linear algebra, differential equations, and now college algebra. Indications of similar reform movements are appearing in the sciences and engineering. For example, Daniel Inman (Virginia Polytech Institute and State University) and Robert Soutas-Little (Michigan State University) have developed a reformed program in mechanical engineering [5].

Technology for insight and teaching

The late nineteen-eighties saw the advent of graphing calculators and desktop computers with sophisticated computer algebra systems (CASs). During the past decade, both the hardware and the software has advanced at an ever-increasing rate bringing into question content issues. Should techniques of integration be dropped from the curriculum now that students have graphing calculators that can integrate symbolically as well as numerically? How should the content of differential equations courses be modified when students have graphing calculators that can plot slope and direction fields as well as symbolically solve first- and second-order linear differential equations? In some instances, technology has completely reversed the order of gaining insight. For example, prior to the late nineteen eighties a great deal of analysis was required in order to sketch the graph of a function. In fact, getting a graph was often the end result. Today, one starts with a calculator or computer plot and then uses the plot to inform the resulting analysis.

Technology has provoked a renaissance in visualization as applied to mathematics [9]. It has brought intuitive and experimental mathematics into the heart of our courses as aids in developing conceptual understanding. Lynn Steen credits technology for reshaping our understanding of mathematics as fundamentally a science of patterns [7]. Are computers primarily tools for developing insight or for computing?

Technology has elevated the importance of numerical approaches in calculus and related subjects. Thanks in large part to CASs, approximation and error-bound analysis is a common theme in today's traditional calculus course. Some reformers, in fact, view approximation and error bound analysis as a backbone of the calculus

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[6]. The iterative capabilities of CASs has contributed to the renewed interest in the sequential approach to the calculus, especially in the use of recursive sequences (sequences whose non-initial terms depend on preceding terms such as in the factorial or Fibonacci sequences). Discrete dynamical systems (i.e. recursive sequences) is beginning to appear as an introductory topic in calculus courses for modeling purposes as well as a forerunner to differentiation and differential equations.

Do more with less

The rapid expansion of information and with it the demand for greater and greater quantitative skills places increasing demands to put more into already overfilled curricula. During the nineteen eighties the Mathematical Association of America's (MAA) Committee on the Undergraduate Program in Mathematics (CUPM) recommended a core program consisting of seven courses [2]. The courses are three semesters of calculus, linear algebra, differential equations, discrete mathematics, and probability and statistics. How to accommodate the important material found in these seven courses into a four course core curriculum became known as the 'seven into four' challenge or more generally the '*n* into *m*' challenge. This challenge was the subject of the Core Curriculum Conference held at the United States Military Academy (USMA) at West Point, NY in 1994 [3]. Successful resolutions of this challenge center on integrating a sequence of topics into a cohesive four semester course rather than considering four one semester courses. More generally, integration of topics rather than of courses provides a model for schools who are faced with an '*n* into *m*' challenge.

The primary resolution of the seven into four challenge is the integrated core curriculum initiated in 1991 at USMA. The program begins with a study of discrete dynamical systems (discrete mathematics) and introduces topics from linear algebra in order to solve systems of recursive sequences using eigenvalues and eigenvectors. The study of long-term effects and equilibrium states introduces the limit concept for sequences. The approximation approach via sequences is then used to develop the calculus. First- and second-order linear differential equations, studied following integration of a function of a single variable, are viewed as the continuous counterpart of discrete dynamical systems. Multivariable calculus and probability and statistics complete the program. A strong focus on problem solving and modeling pervades the program. In addition, the emphasis on student growth in terms of mathematical reasoning, mathematical modeling, scientific computing, writing in mathematics, history of mathematics, and connectivity to partner disciplines, prepares the students to perform the analysis and synthesis required in addressing open-ended problems in engineering, mathematics, and the sciences.

Another aspect to meeting the *n* into *m* challenge involves reducing the time allotted to calculus from three to two semesters. One approach to this is to integrate the treatment of one and several variables [6]. Viewing single variable calculus within the context of multivariable calculus creates an environment that encourages a strong emphasis on generalization and exploration. How do you generalize the concept of real numbers to two dimensions? How do you generalize differentiation to a function of two or more variables? This approach is better suited to the development of confident and competent problem solvers than is the 'add on' approach to topics associated with the tradition of separating the treatment of one and several variables. The integrated approach is particularly appropriate for those students who have studied calculus in high school. These students constitute the large majority of students presently taking calculus in college.

Interdisciplinary cooperation

In 1996 the National Science Foundation (NSF) developed a major initiative to promote interdisciplinary cooperation. Although it is too early to speak of success, there is ample evidence of a growing force to incorporate interdisciplinary cooperation within curricula. Multi-year grants have been awarded, professional associations have formed committees, workshops and conferences are being held, and instructional materials are being published. The April 12, 1999 *Higher Education and National Affairs Bulletin* (p. 4) cites a report of the National Research Council issued in March 1999 that calls for faculty to work together to develop courses that 'illustrate connections among disciplines'.

Building on the Calculus Reform Movement's emphasis on out-of-class group projects, USMA under its NSF Project INTERMATH has formed a national consortium to develop and promote the use of Interdisciplinary Lively Application Projects (ILAPs). These small group projects are developed through the collaboration of faculty members in at least two departments. Because ILAP problems reside in partner disciplines, they help students link different disciplines.

A pollution problem in the Great Lakes provided a setting for an ILAP developed between faculty in the environmental engineering and mathematics departments. An ILAP developed between the physics and mathematics departments asked students to model the effects of an earthquake on a water tower as a spring-mass system. Analyzing the smog in the Los Angles Basin became a chemistry-based ILAP. Faculty from mathematics and mechanical engineering departments collaborated on an ILAP to analyze forces on a bridge. (See [1] for additional examples.)

In addition to linking departments at the student level, ILAPs open up communication links among the faculty. These communication links have led to team-taught courses with faculty from mathematics and environmental engineering as well as between faculty in mathematics and physics.

Interdisciplinary initiatives hold the promise for 'multiple wins' situations. From the perspective of the mathematics department, interdisciplinary efforts help provide relevance. From the perspective of the partner discipline, these efforts provide a source of exposure to students who may not have taken a course in that discipline. Interdisciplinary efforts enrich students' problem-solving skills as they experience the advantages of viewing problems from the perspective of different disciplines. Collaboration between faculty members of different departments can provide building blocks for increased interdepartmental communication and improved interdepartmental rapport. Of course, the strongest argument for interdisciplinary cooperation is that the real world works that way.

SHORTCOMINGS OF PRESENT CURRICULA

A common response to the forces described above is to modify existing course structures rather than create new structures. Time is the crucial factor. If group projects are estimated to take six hours of work per student per project and two projects are assigned in a semester, the syllabus needs to be adjusted to provide for this additional twelve hours. Assuming the idealistic state of two hours of preparation for each hour of class, these twelve hours translate into four class periods. Now add a class for each project for student presentations and discussion and the total is a minimum of six classes for the two projects. Instruction in the use of technology, calculators or computers, can easily take the equivalent of two class periods over the course of a semester. In-class activities and student presentations also require considerable class time. Therefore, adopting the recommendations of the Reform Movement requires reducing course content. This is the 'lean' of the *Toward a Lean and Lively Calculus* [4].

Removing topics from a syllabus is difficult to do for several reasons. Some examples are: sacred cows abound, adjustment to downstream courses need to be made, the flow of the text is disrupted, exercises need to be reselected, treatment of remaining topics may need to be altered, class preparations need to be changed, and the elimination of topics may be viewed as 'dumbing down' the course. In addition, changing a curriculum in an evolutionary manner runs the risk of destroying program coherence. This is particularly true for a multiple course core program. As a result we often attempt to retain too much content while responding to the forces discussed earlier. This, in turn, leads to a more shallow treatment of topics rather than the deeper conceptual understanding advocated by the Reform Movement.

Dependency on a text imposes major difficulties to adjusting an existing course. The greater the dependency, the more the course is determined by the philosophy, approach, and syllabus of the author(s). The flavor, if not the content, of most courses is determined by the text rather than the instructor. Thus, modifying a course usually involves creating disconnects with the text in terms of philosophy, approach, and syllabus. As a consequence, the lack of suitable texts that 'fit my philosophy and objectives' is a major barrier to reform. A further difficulty is that texts are partially out of date by the time they enter the market due to the rate of change propelled by the forces discussed above and the time required to release a new text.

The rapid advances in technology compound the difficulties in finding suitable texts. Two examples will underscore this point. First, within the past year calculators have come on the market that evaluate indefinite as well as definite integrals, solve first- and second-order differential equations, plot slope and direction fields. What does the availability of this technology imply for the treatment of integration? Do we eliminate techniques of integration from our calculus courses? What is the essence of integration and how do we teach it given the present technological tools? Second, because of the available technology, the reform movement in differential equations places strong emphases on the qualitative and numerical approaches. How can the qualitative approach be integrated into the traditional treatment of differentiation? There are presently no texts that address either of these situations. Furthermore, when such texts become available they will probably be out of date with respect to technology's treatment of improper integrals and series.

SUGGESTED STRUCTURE FOR A FUTURE CURRICULUM

I propose the following three-step process for developing a dynamic curriculum that will maintain its integrity and relevance in a rapidly changing technological society:

Step 1. Those who are responsible for the core mathematics curriculum meet with representatives from partner disciplines to identify underlying themes for the core program. For example, change, accumulation, approximation and error-bound analysis, and transformations would probably be included in most lists. Ideally the number of themes would be small, say no more than six.

Step 2. Those who are responsible for the core mathematics curriculum develop a 'glorified' outline for the entire core that clearly reflects the agreed themes. This outline will be the 'text' for the core program. Thus it will include some textual type material including sample in-class activities, exercises, and discussion topics.

Step 3. Instructors 'flesh out' the outline by assigning small individual or group research/discovery projects. These assignments would contain two or three references to books in a library, Internet sources, texts, problem books, or other easily obtained resources. Upon completion of a project, the students would make a class presentation and lead a discussion on the topic. Different groups, in fact the whole class, could be given the same assignment.

Some examples of projects (in calculus) are:

- Conduct a rate experiment (e.g., rolling a ball), collect data, and compute a sequence of average rates of change that approximate the exact rate of change at a specified point. (Derivative)
- Qualitatively develop possible solution curves to a logistic differential equation and discuss the effects of changing parameters. (Derivative)
- Conduct a warming or cooling experiment, collect data, develop a model, and then solve graphically and symbolically. (Derivative)
- Discover Euler's method and then develop an improved method. (Derivative)
- Generalize the integration concept to include finding volumes of solids (solids of revolution, swimming pools). (Integral)
- Generalize proper integration to improper integration in a probability or series setting. (Integral)
- Investigate numerical approximation methods and then develop a 'new' method (say, a quartic approximation). (Integral)

The outline would be developed through debates on what should be included in the program rather than what can be removed from someone else's program. This is a constructive and uniting process. The outline needs to provide sufficient time to allow concepts to be covered to the desired depth. I strongly suggest that a buffer period of at least two weeks length be provided for each semester. This would offer instructors the opportunity to partially tailor their course to their students. The outline could be custom published by the semester and sold to students at a considerable saving over present text book prices.

I believe this structure has several advantages over present course structures. A few of these advantages are:

- The program offers maximum flexibility.
- The themes provide coherence to the curriculum.
- The people responsible for the program develop their *own* program based on their *own* philosophy rather than adapting a program and philosophy of some other author.
- The outline and projects could be easily changed from semester to semester, thus making it easy to keep the curriculum current and the pace reasonable.
- The research/discovery projects directly address the goal of students' learning how to learn on their own.
- The research/discovery projects directly address the goal of learning how to work in groups.
- Students are much more likely to read a resource in order to prepare a project then they are to read their own text. Thus the research/discovery projects directly address the goal of helping students learn how to read in their discipline.
- Research/development projects could easily be crafted to link to partner disciplines as well as to past segments in the core program.
- The program offers instructors the opportunity to take ownership of their course.
- The program offers more growth opportunities for the instructors than do traditional programs.

The proposed structure would require a considerable amount of work to prepare the curriculum materials for the first offerings of the core courses. However, these materials would accumulate in successive offerings resulting in a convergence to a time commitment similar to that in present courses.

CONCLUSIONS

The explosion in the quantity of information available and the spiraling increases in speed of processing and transmitting information is pressuring curricula to place greater focus on helping students learn how to manage information. Students need to learn how to identify, validate, and apply pertinent information in problem-solving situations. In short, the information age places a premium on students learning how to learn on their own. This emphasis is further underlined with the changes in employment scenarios from lifetime careers to sequences of careers. Making curricula relevant to the steadily increasing variability of society and business requires that increased value be placed on interdisciplinary cooperation and student experiences of generalizing results. Finally, the exponential rate of development of technology for insight and teaching demands a level of curriculum flexibility that exceeds the capabilities of a text-based curriculum.

Evolutionary modifications of present curricula are insufficient to create programs for a rapidly changing, technological society. A revolutionary restructuring of curricula is required to create cohesive programs with sufficient flexibility to educate tomorrow's students.

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