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Continuum Mechanics in a Restructured Engineering Undergraduate Curriculum*

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The framework of the revised undergraduate engineering curriculum at Texas A&M University consists of the basic conservation principles and their application to engineering science. The conservation principles are presented in a way consistent with the Kolb learning cycle and an active approach to teaching. Utilizing these principles, a unified pedagogical process is developed, which can be applied to cover topics traditionally taught under statics, dynamics, fluid mechanics, thermodynamics, heat transfer, solid mechanics, materials science and electrical circuits. Using this instructional framework, the students can analytically solve simple engineering problems, while they learn how to formulate complex problems at early stages of their undergraduate education. Since their technical ability to analyze such complex problems is limited during their second or even third year of the undergraduate curriculum, the utilization of computer software enables them to numerically solve advanced problems in the above engineering topics. In particular, the use of the finite-element method as an enhancement tool to solve solid mechanics and heat transfer problems in a sophomore year course on continuum mechanics and more advanced problems in stress analysis in a junior level course on solid mechanics is discussed in this paper. Students use computer software to formulate and solve boundary value problems in a variety of structures, to verify analytical solutions for simple structural problems, and finally to test the various assumptions that permit approximate analyses in solid mechanics.

INTRODUCTION

IN 1987, the American Society for Engineering Education presented a study that voiced concerns about the quality of undergraduate engineering [1]. Through the support of the National Science Foundation (NSF), many schools across the nation have addressed these concerns and are working towards obtaining a higher quality education. In 1990, NSF funded their first two NSF Engineering Education Coalitions, which involved fifteen different schools. By 1995, the number had risen to eight distinct coalitions and over forty participating schools.

Each coalition follows a different course of action to respond to these concerns. (The induction year is listed beside each coalition name.)

The Synthesis Coalition, 1990, integrates teamwork with hands-on laboratory experiences and examples of 'best practices' from industry [2].

The Engineering Coalition for Schools of Excellence in Education and Leadership, 1990, emphasizes more design work in the curricula [3].

The Southeastern University and College Coalition for Engineering Education, 1992, is committed to a comprehensive revitalization of undergraduate education for the 21st Century [4].

The Gateway Coalition, 1992, alters engineering education from a focus on course content to a focus on the broader experience in which individual curriculum parts are connected and integrated [5].

The Greenfield: The Coalition for New Manufacturing Education, 1994, is creating curriculum that teaches proactive manufacturing engineering to advance the manufacturing enterprise beyond that of foreign competitors [6].

The Engineering Academy of Southern New England, 1995, creates a set of engineering courses, curricula, and workforce training programs, which are focused on the integration of design and manufacturing, teamwork, and hands-on manufacturing [7].

The Southern California Coalition for Education in Manufacturing Engineering, 1995, concentrates on a long-term systematic reform of undergraduate manufacturing engineering [8].

Texas A&M University and six other schools formed the 'Foundation Coalition' (FC) in 1993 [9]. The six other schools include Arizona State University, Maricopa Community College District, Rose-Hulman Institute of Technology, Texas A&M University-Kingsville, Texas Woman's University, and University of Alabama. This educational group is unique in that it has a goal to redesign the undergraduate engineering curriculum by providing beginning students with a foundation in engineering problem solving, design, and teamwork integrated with fundamentals of math and science. Recently

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two more universities, University of Massachusetts-Dartmouth and University of Wisconsin, have joined the FC, while many more schools have joined as affiliates.

While the goals of each coalition are different, schools within a single coalition might also follow a different approach to succeeding. Texas A&M University is unique in the course of action it is taking. This five-year NSF program, 1993–98, has challenged Texas A&M in designing a current educational system, which takes advantage of modern teaching methods and technology to provide the finest engineering education available with current budgets. In particular, the restructuring of the sophomore undergraduate engineering curriculum has been based on an earlier NSF funded effort, which introduced the teaching of the basic engineering disciplines as special cases of a general framework based on conservation laws. The conservation of mass, linear momentum, angular momentum, energy, and charge are all introduced as accounting principles under a unified mathematical methodology. Concepts like physical system, flux, and accumulation are first defined in general terms and later specialized for the physical quantity of interest. Topics traditionally taught under statics, dynamics, fluid mechanics, thermodynamics, heat transfer, solid mechanics, materials science and electrical circuits are now covered as applications of these conservation principles.

The freshman year is almost identical for degrees in aerospace engineering, agricultural engineering, biomedical engineering, chemical engineering, civil engineering, computer engineering, electrical engineering, industrial engineering, mechanical engineering, nuclear engineering, ocean engineering, petroleum engineering, and radiological health engineering. Students are first exposed to conservation principles and methods of teaming in addition to physics in the freshman engineering classes.

ENGR 109: Engineering Problem Solving and Computing discusses professional ethics, engineering problem-solving environment (economics, political, technical social) requirements and methodologies.

ENGR 111: Foundations of Engineering I gives an introduction to the engineering profession with emphasis on computer applications and computer-aided design tools and introduces the skills utilized in teamwork.

ENGR 112: Foundations in Engineering II continues the development of skills in problem solving, design, and teamwork and gives the students their first introduction to accounting and conservation principles in engineering sciences.

Most departments have a sequence of five engineering courses in the sophomore year curriculum in addition to departmental courses. The courses in the sophomore year sequence are:

ENGR 211: Conservation Principles in Engineering Mechanics, which discusses conservation principles in engineering and their application to the modeling of mechanical systems and structures and particle and rigid body mechanics.

ENGR 212: Conservation Principles in Thermal Sciences teaches theory and application of energy methods in engineering and uses conservation principles to investigate ‘traditional’ thermodynamics and internal flow fluids.

ENGR 213: Principles of Materials Engineering describes properties of materials using a unified approach and discusses chemical and crystalline structures and characteristics of metals, polymers, and ceramics.

ENGR 214: Conservation Principles of Continuum Mechanics explores continuous media using a unified approach and teaches conservation laws, fundamental concepts, and examples of their use (heat conduction, Newtonian fluids, linear elastic solids, axial bars, torsion, shear and moment diagrams, and beam bending).

ENGR 215: Principles of Electrical Engineering discusses the fundamentals of electric circuit analysis, AC power, and electronics.

The reader is referred to Table 1, which provides the sequence of classes for the redesigned freshman and sophomore level described above and contains references to corresponding textbooks. An Internet web page [15] also provides the reader with more information about the program. Details on the implementation of the restructured sophomore engineering curriculum can be found in [16]. Using this framework, the students learn how to solve simple engineering problems and formulate complex problems at early stages of their undergraduate education.

The rapidly changing technology in today’s society has presented engineering educators with the difficult task of balancing an undergraduate curriculum emphasizing specific tools, which may have a short lifespan, with a curriculum based on the teaching of fundamentals that is independent of changes in technology [17]. The restructuring of the undergraduate curriculum at Texas A&M University that utilizes the conservation principles framework reflects the belief that teaching the fundamentals early on is the cornerstone of engineering education. This belief has been substantiated through various evaluation procedures described in greater detail in [18, 19]. Retention data observed for students in the old curriculum and students in the new redesigned curriculum show that women and ethnic minorities retain at a higher rate and that a majority of men maintain about the same retention rate. Results of standardized critical thinking tests show that students participating in the new curriculum performed at significantly higher levels at the end of the year than did the students in the old curriculum. However, the students’ technical

Table 1. A typical freshman year and sophomore year engineering curriculum (Aerospace Engineering).

		FRESHMAN YEAR			
	First Semester	*Cr Hr*		*Second Semester*	*Cr Hr*
ENGL 104:	Composition and Rhetoric	**3**	**CHEM 107**:	Chemistry for Engineers	**4**
ENGR 111:	Foundations in Engineering I	**2**	**ENGR 112**:	Foundations in Engineering II	**2**
MATH 151:	Engineering Mathematics I	**4**	**MATH 152**:	Engineering Mathematics II	**4**
PHYS 218:	Mechanics	**4**	**PHYS 208**:	Electricity and Optics	**4**
Directed Elective		**3**	Directed Elective		**3**
KINE 199:	Physical Activity	**1**	**KINE 199**:	Physical Activity	**1**
		SOPHOMORE YEAR			
	First Semester	*Cr Hr*		*Second Semester*	*Cr Hr*
AERO 201:	Introduction to Aerospace Engineering	**3**	**AERO 320**:	Numerical Methods	**3**
ENGR 211:	Conservation Principles of Engineering Mechanics [10]	**3**	**ENGR 213**:	Principles of Materials Engineering [12]	**3**
ENGR 212:	Conservation Principles in Thermal Science [11]	**3**	**ENGR 214**:	Conservation Principles in Continuum Mechanics [14]	**3**
MATH 251:	Engineering Mathematics III	**4**	**ENGR 215**:	Principles of Electrical Engineering	**3**
Directed Elective		**3**	**MATH 308**:	Differential Equations	**4**
KINE 199:	Physical Activity	**1**	**KINE 199**:	Physical Activity	**1**

ability in mathematics is limited during their freshmen and sophomore years, and the students can actually solve only simple problems, even though they can formulate more involved engineering problems. An additional difficulty inherent in the presentation of more abstract topics to undergraduate students is motivation to study such topics, since it is more difficult to draw examples compatible with students' experiences. The two drawbacks of the fundamentals first approach, i.e. student motivation and limited mathematical tools available, can be addressed using novel instructional methodologies and technology in the classroom. The instructional methodologies include teaming, collaborative learning and the Kolb learning cycle, [20–24], while technology heavily relies on the availability of computer-based tools for numerical simulations [25, 26].

There has been increasingly strong evidence in the field of education that active teaching and also teaming and collaborative learning pay dividends in terms of learning [27, 28]. In the development of the Foundation Coalition courses, and in particular the introduction of conservation principles in the engineering curriculum, attention has been focused in two areas. One area is active teaching through the Kolb learning cycle [29], and the second area is using technology in a collaborative learning environment. The use of technology is compatible with the realization that each student has a different learning style [30], and the curriculum is presented in such a manner that each learning style is addressed. Consistently implementing the Kolb learning cycle [20, 21, 29, Appendix], coupled with additional active teaching methods, enables the instructor to motivate the introduction of the abstract notions associated with the conservation principles early on in the undergraduate curriculum [32].

The second focus area corresponds to the most significant change in the classroom that has been introduced at Texas A&M University, which allows computer software to become a real instructional tool. The introduction of computer technology in the classroom is facilitated with two personal computers for every team of four students, thus promoting extensive teaming and collaborative learning. Class meetings include teaming exercises that utilize computer software, where the students solve the example problems as a team, but are individually responsible for knowing the solution from the team [18, 26]. These types of activities are given after a formal lecture presentation of the material, thus allowing the application of a concept and traversing across several learning styles. For faculty, this is a new way of looking at the teaching and learning process [33] that requires giving up some control of the classroom, while investing more time in lecture preparation. Primarily, the availability of computer technology in the classroom facilitates numerical simulations of complex engineering problems, which the students can formulate but are not capable of solving by hand.

In the present work the authors make use of well-established software technologies to assist the learning process in undergraduate continuum mechanics courses. These courses establish the structure of engineering using the conservation laws and the second law of thermodynamics, and they provide a framework for a problem-solving methodology. General-purpose finite-element software is utilized in a sophomore-level course where the conservation principles are applied to continuum mechanics (solids and heat transfer) and a junior-level solid mechanics course where structural problems are solved. The integration of material taught in the two courses is achieved through the use of the same software in both courses. Even though restructuring of undergraduate mechanics curricula by integrating with mathematics, physics, as well as computer software, has been undertaken by several engineering schools [25, 26, 34–36], the current approach emphasizes the use of technology and active teaching of continuum mechanics, within the unified conservation principles framework in a collaborative learning environment.

The following sections go into more detail on the material outlined above. Next section describes specific topics covered in a sophomore-level course on continuum mechanics and a junior-level course on solid mechanics with applications to structures. A useful technique for formulating continuum mechanics problems is briefly discussed. In the last section follows with applying the problem-solving framework and utilizing computer software to solve a variety of problems. Examples with heat conduction in a simple bar and then in a more complex object, in particular a pizza, are presented. The stresses in a cantilever beam are found for a variety of boundary conditions by using both simple beam theory and finite elements. Finally, the appendix discusses the Kolb learning cycle by explaining each of the learning styles. This section also provides feedback from students regarding the implementation of the Kolb cycle in the context of introducing the basic conservation laws in continuum mechanics.

DESCRIPTION OF CONTENT AND PROBLEM FORMULATION IN CONTINUUM AND SOLID MECHANICS COURSES

Continuum and solid mechanics provide the subject matter for a sophomore level course in the core engineering curriculum and a junior level course in the departmental curriculum, i.e., AERO 304: Structural Analysis I for Aerospace Engineering students, MEEN 357: Engineering Analysis for Mechanical Engineers for Mechanical Engineering students and CVEN 205: Engineering Mechanics of Materials for Civil Engineering students. The sophomore level course, (ENGR 214), examines the application of conservation principles to continuous media for which internal changes in the medium are important in describing the system behavior. This particular course is the fourth of the five courses developed for Foundation Coalition [18, Table 1]. Students from most engineering disciplines take this course after they have been exposed to the basic conservation laws at the system level in the first two courses of the sequence. They usually take the continuum mechanics course in parallel with the third course of the sequence, which focuses on properties of matter. The basic conservation laws covered are conservation of mass, energy, linear and angular momentum, while conservation of charge and the second law of thermodynamics are briefly mentioned for completeness. Applications of the conservation laws in areas such as heat transfer, fluid mechanics and the mechanics of solids are introduced. During the unfolding of the course, students come to realize that in order to evaluate the stress in a beam, the velocity in a simple flow, or the temperature in a heat conduction problem, they must use concepts from calculus, linear algebra and differential equations, in addition to the conservation laws and properties of matter. This course, therefore, is the course that brings together and integrates knowledge from undergraduate courses in mathematics, basic conservation laws, and material science.

Emphasis is given to the formulation of complex problems, while explicit solutions are found for one-dimensional (1-D) fluid flow, structural, and heat conduction problems. The students are asked to solve simple problems with closed-form solutions to understand the application of proper equations, boundary and initial conditions. Furthermore, the students are trained to formulate many well-posed problems that are beyond their technical ability to solve analytically. Computer software is employed to first solve simple problems, which re-emphasizes the problem formulation process and teaches the students to use the software, and then it is used to find solutions to more difficult engineering problems.

The junior level course, (AERO 304), studies mechanics of solids concepts with applications to structures. The junior level solid mechanics course is much more applied than the sophomore course. The emphasis is on the behavior of basic structural elements, like uniaxial rods, beams, frames, and thin-walled pressure vessels. One of the challenges addressed through numerical simulation is demonstrating the relationship between the general (and complex) governing equations covered in the sophomore year and the specialized governing equations for these structural elements. The advantages and validity of making simplifying assumptions is demonstrated by comparing stress distributions from the simple beam theory with the much more complex plane stress finite-element analysis. These comparisons also demonstrate the limitations of beam theory. Two examples are given herein.

The students learn a unified approach to formulating problems in continuous media, motivated by the commonality of the underlying principles. This approach involves selecting the following:

1. conservation law(s)
2. constitutive equations
3. kinematic relationships
4. boundary/initial conditions.

Table 2 lists the appropriate equations for both heat conduction and solid mechanics problems. Equations shown in Table 2 are derived in class in both one dimension and two dimensions, and students then learn to work with the equations,

Table 2. The four-component problem formulation technique as applied to heat conduction and solid mechanics problems.

Four-Component Approach to Formulating Continuum Mechanics Problems	Heat Conduction	Solid Mechanics (electrostatics)
1) Conservation Law(s)	Conservation of Energy $\rho \hat{C} \dfrac{\partial T}{\partial t} = -\nabla \cdot \mathbf{q} + \rho \Phi$	Conservation of Linear and Angular Momentum $\nabla \cdot \mathbf{T} + \rho \mathbf{g} = 0; \quad \mathbf{T}^{\mathrm{T}} = \mathbf{T}$
2) Constitutive Equations	Fourier's Law $\mathbf{q} = -k\nabla T$	Hooke's Law $\mathbf{T} = \mathbf{C}\varepsilon$
3) Kinematic Relationships	Continuity of temperature	Strain/Displacement relations $\varepsilon = \frac{1}{2}[\nabla \mathbf{u} + (\nabla \mathbf{u})^{\mathrm{T}}]$
4) Boundary and Initial Conditions	Specified temperature or heat flux	Specified displacements or tractions

$\rho \equiv$ density
$\hat{C} \equiv$ heat capacity
$T \equiv$ temperature
t $\equiv$ time
$\nabla \equiv$ del operator
$\mathbf{q} \equiv$ heat flux vector
$\Phi \equiv$ energy input rate per unit mass
$k \equiv$ thermal conductivity
$\mathbf{T} \equiv$ stress tensor
$\mathbf{g} \equiv$ body force per unit mass vector
$\mathbf{C} \equiv$ elastic stiffness tensor
$\varepsilon \equiv$ strain tensor
$\mathbf{u} \equiv$ displacement vector

mainly in Cartesian coordinates. Through homework, quizzes, and exams, students gain mastery of these equations, but their ability is limited to mainly one-dimensional problems. To further expand the student's ability to comprehend complex problems, computer software is then employed to generate computational solutions to complex problems, and can also be used to demonstrate the physical limitations of engineering approximation.

Direct comparisons between the four-component problem formulation technique given in Table 2 and the Kolb learning cycle (see Appendix for more details) can be easily identified. The 'what' part of the Kolb Cycle is directly related to the mathematical formulation and implementation of the four components. The 'How' part is related to actually solving the mathematical expressions derived from using the problem formulation technique. The 'What if' is addressed by exploring different material properties (component 2) and boundary conditions (component 4) in the problem-formulation framework (Table 2). Finally, the 'Why' part of the cycle is the motivation for the necessity of having all four components in the formulation to establish a well-posed problem in continuum mechanics.

Selecting the four components of the formulation does not yield the solution to the problem at hand. The outcome of the four-part formulation is a well-posed mathematical problem in the form of a set of partial differential equations that need to be solved for the variables of interest. Thus, simply stated, this procedure turns a physical problem into a mathematical one. The strength of the procedure exists on the confidence students gain in their ability to formulate problems that may contain a complex geometry or a variety of boundary conditions, which usually leads to problems above the level of their mathematical ability. Simple 1-D problems are also posed in different ways and used to demonstrate the solution procedure for different types of boundary conditions, but 1-D problems do not carry the complexity necessary to build intuition and interest in the student. As more real-life problems are introduced, the complexity of the problem increases, along with the interest of the students, since they feel they are solving problems with real-life significance.

To enable students the ability to solve real-life engineering problems, therefore carrying the formulation into actual implementation, a finite-element analysis (FEA) software program has been taught in the sophomore level course and utilized at both the sophomore and junior level. In particular, the FEA program FEMAP, from Enterprise Software Corporation, has been used for the results presented here and by students in the two continuum mechanics courses. Successful finite-element analysis closely follows the four components of the problem formulation described above, with the main difference being that the computer software actually solves the differential equations resulting from the conservation laws.

There are a number of FEA software packages currently available to engineers for structural and thermal analysis. FEMAP, ANSYS, COSMOS, Pro/Mechanica, and ABAQUS [37–41] are just a few of the widely used FEA programs. FEMAP was initially selected because the Windows-based interface allowed a smooth implementation in the classroom. While each FEA program approximates the solution to a given problem differently, they all require the user to provide a mesh (the problem divided into discrete, or finite, elements), the material properties, mechanical loads, heat sources, boundary and initial conditions. The required input for a successful finite-element analysis closely follows the four components of the problem formulation described above with the student providing the necessary material parameters, boundary and initial conditions, corresponding to steps two and four from the four-component problem formulation technique. The FEA software then completes the solution to the continuum mechanics problem by incorporating the conservation laws and kinematic relations, steps one and three from the four-component step approach, to solve the governing differential equations.

EXAMPLE PROBLEMS UTILIZING THE PROBLEM-SOLVING FRAMEWORK AND COMPUTER SOFTWARE

The example problems covered in this section demonstrate only a small portion of the topics taught in ENGR 214 and AERO 304. The intent of this section is to provide several example problems applicable to these courses and explain how computer software can be incorporated into the instruction process. The examples on heat conduction are from ENGR 214, while the examples on stress analysis are from AERO 304.

Heat conduction in an insulated bar

In the sophomore level continuum mechanics course, heat conduction and solid and fluid mechanics are introduced as applications of the conservation of energy and linear/angular momentum. Two examples of heat conduction will be presented in this work. The first case is a simple 1-D problem consisting of an insulated copper bar with specified temperatures at the ends, which is shown in Fig. 1. The bar has an initial temperature of 0°C. Using conservation of energy and the given boundary and initial conditions, the temperature distribution in the rod under steady-state conditions and the transient temperature response are solved analytically. Next, the students are introduced to the fundamentals of FEA and the basic steps of the FEMAP software, which they use to find the transient temperature response in the bar. The final step here is a comparison between the results from FEA and the analytical solution. This simple exercise allows the students to learn how to deal with the nuances of a new computer program, as well as gain confidence in the results from FEA. It should be noted that the visualization of the temperature along the bar as a function of time in an animated form enhances the learning process.

Study of the cooling of pizza during delivery

A more complex but real-life problem is modeling the temperature profile of a pizza as it cools down during the delivery period. This is an interesting question to pizza distributors as well as pizza consumers and students can formulate such a problem by following the four components of problem formulation mentioned in the previous section. Their limited technical ability in solving differential equations in complex domains, however, precludes them from being able to answer questions like 'How long does it take before my pizza gets cold?' In this three-week project, the four-member student teams are asked to numerically model the temperature profile in a pizza using FEA, from the time it is taken out of the oven until it is consumed. Students enjoy working on such a problem since they can bring their own experiences on the subject, but they are also challenged because they are faced with several difficult decisions: create a correct but simple FEA model, select appropriate material properties for the pizza ingredients and choose appropriate boundary conditions. After some team brainstorming and help from the instructor, it becomes clear that every cross-section of the pizza will have approximately the same temperature profile, and the use of an axi-symmetric model seems appropriate, thus greatly reducing the complexity of the problem. Determining the material constants (e.g. thermal conductivities of bread, cheese, etc.) and the boundary conditions is the most challenging part of this project. Teams must make their own selections and then justify them. Students have very little experience in this area because traditionally such values are simply provided to the students, thus removing their ability to gain intuition about how different materials would behave under similar conditions.

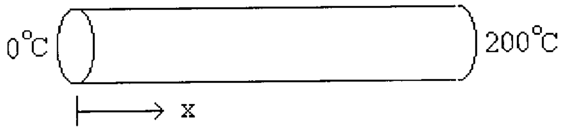


Fig. 1. Insulated bar used for the first heat conduction problem.

During the second week of the project, it is clear that a real experiment is needed to validate the numerical results and the various assumptions made for the material properties and boundary conditions. For this purpose, a laptop computer is dispatched to the pizza vendor and as soon as the pizza is taken out of the oven, the pizza is instrumented with three thermocouples connected to the laptop computer (Fig. 2). The thermocouples are in place as the pizza is loaded into the box, and temperature data from the top, bottom and middle of the pizza's center are collected during the delivery of the pizza. A program, which was written in LabView [42], collects data from the thermocouples using a National Instruments [42] data acquisition device and through the local computer network, the data become available to all teams. The overall goal for each team is to create a finite-element model of the pizza by identifying the mesh, material parameters, and boundary conditions that simulate the temperature experimental data as close as possible, and a grade is assigned based on the simulation. At the end of the experiment, the team with the highest grade is chosen to consume the experiment! The only drawback of the use of technology is that data distribution occurs through the network, and it is no longer necessary for each team to experiment with their own pizza. Finally, each team conducts a presentation during the third week of the project, which utilizes graphical post-processing tools of

Fig. 2. Re-enactment of the physical set-up for the study of cooling of a pizza.

FEA integrated in a Microsoft PowerPoint presentation.

Material parameters and boundary conditions may vary greatly from team to team and some unique approaches to solving the problem have been introduced by the teams. Figure 3 shows a typical axi-symmetric idealization of the pizza, assuming that the box and table (representing the object where the pizza box is placed) are also axi-symmetric. Some teams place convection boundary conditions on the top and side of the pizza, while others model the entire closed box that includes the air between the pizza surface and the upper box cover. In Figure 4, the pizza temperature profile is shown for time $t = 600\,\mathrm{s}$ after the pizza has been removed from the oven, where it was assumed to be at a uniform temperature of 218.3°C (425°F). The figure gives a representation of the temperature gradients in the pizza setup shown in Fig. 3, for typical selection of thermal material properties for the pizza ingredients, taken from a food properties reference [43], and assuming convection boundary conditions around the pizza except the bottom part of the pizza that rests on the table.

The pizza project stimulates students in all portions of the Kolb learning cycle (Appendix). As indicated earlier, the four-component framework addresses the four portions of the Kolb Cycle and this framework is used to help solve the pizza problem. The computer software portion of the project extends the 'How' portion by allowing the students to solve a mathematical problem that is beyond their analytical technical abilities. In addition, the computer software empowers the students by allowing them to explore and evaluate several different boundary conditions and material parameters, which is directly related to the 'What if' portion of the Kolb Cycle. By actually choosing a pizza as the test subject, this application directly appeals to the 'Why' portion as well, motivating the relevance of the conservation of energy, material properties and influence of the environment.

Cantilever beam with different boundary conditions

In the junior level solid mechanics course, a cantilever beam with different boundary conditions and an elementary beam design problem are introduced. After a formal lecture on beam

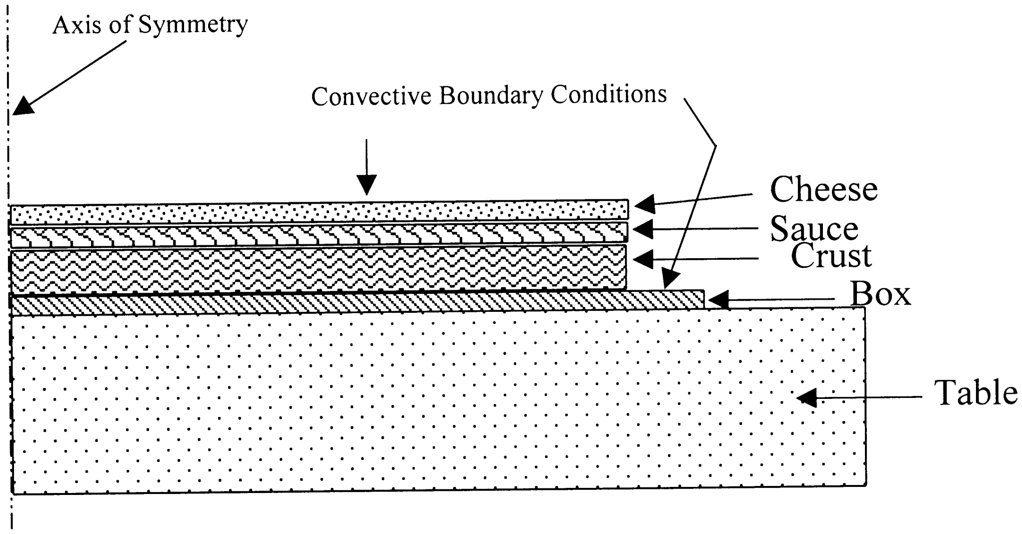


Fig. 3. Axisymmetric model of pizza.

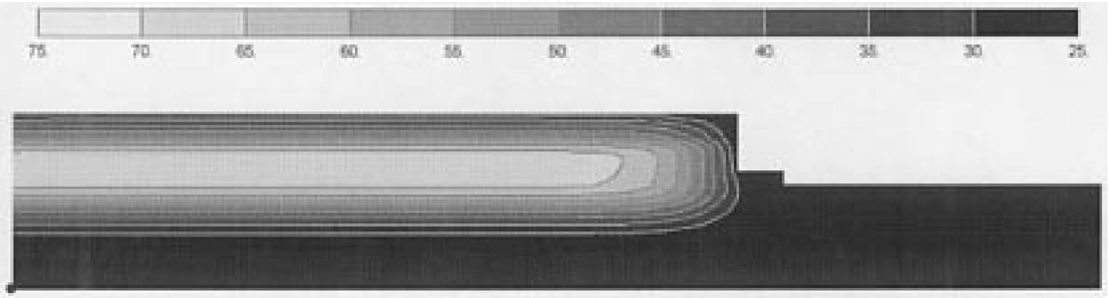
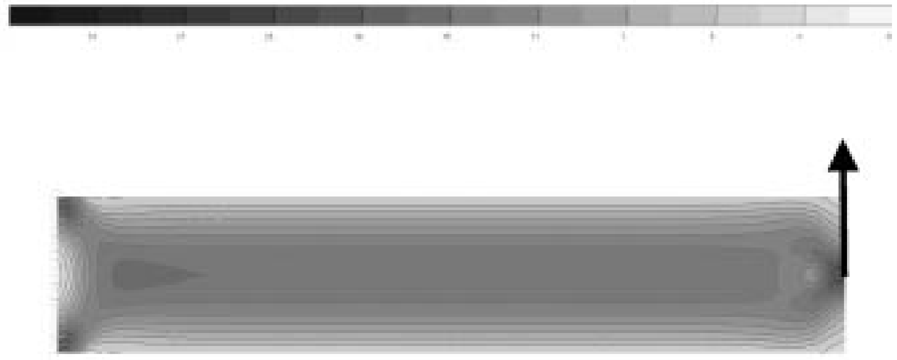


Fig. 4. Temperature profile of pizza at 600 s after it has been removed from the oven.

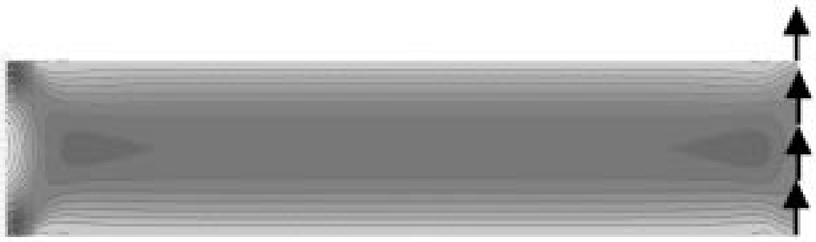
theory, the validity of the assumptions leading to the beam theory equations is examined in a cantilever beam problem. The beam considered is five inches long with a height of 1 inch and a thickness of 0.1 inch. Three different types of loads are considered to illustrate the effect of boundary conditions on the solution:

a. Point load at the right end, $(x = L)$.
b. Constant distributed load at the right end, $(x = L)$.
c. Quadratic varying load at the right end, $(x = L)$.

Note: the total load applied at the right end is one pound for each of the above three cases. The FEA software is utilized to find the stress components across the beam for each of the boundary conditions given. The shear stress contours for each of the three loading cases are shown in Fig. 5. This



Point Load (a)



Constant Distributed Load(b)

Quadratic Distributed Load (c)

Fig. 5. Shear stress distribution on a cantilever beam with various loading conditions.

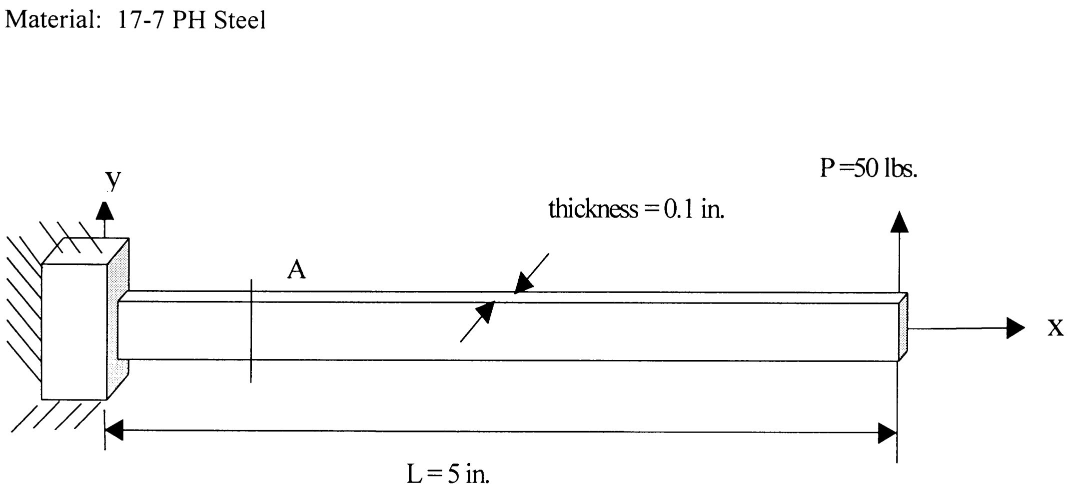


Fig. 6. General specifications of the cantilevered beam.

exercise illustrates to the students the effect of the boundary conditions on the FEA solution. While the total force is the same, the application of this force is different, as shown in the sketches in Fig. 5. This enforces the significance of applying correct boundary conditions. As mentioned before, exploring various boundary conditions applies to the 'What if' portion of the Kolb Learning Cycle. Stress components from FEA are then compared to beam theory results and are shown in Table 3. In the table, results are compared at the middle point, ($x = 0.5L$), along the beam length and at a point near the right end, ($x = 0.95L$). For each loading condition, beam theory predicts the same response, while FEA produces different results near the right end. The results illustrate to the students the variances in the principles behind beam theory and FEA. While certain assumptions were made to derive the equations in beam theory, these assumptions do not exist in the FEA model, which provides the exact solution (within the numerical accuracy of the finite-element discretization). Therefore, it has been verified to the students that the application of the load is important in FEA, but it is irrelevant in beam theory. The use of computer software graphics serves as a very important step in this verification process.

Optimal shape of a cantilever beam

Another example of using FEA software to enhance understanding of mechanics is the optimal design of a cantilevered beam shown schematically in Fig. 6. Given a specified length and thickness, the students must determine the required variation of the beam height in order to minimize weight and prevent yielding based on the von Mises yield criterion.

There are several concepts stressed in this activity:

- optimum design;
- optimization criteria;
- the iterative nature of the design optimization;
- advantages of beam theory versus FEA and vice versa;
- effects of beam theory assumptions on the accuracy of the solution.

The requirements of this activity include:

1. Design the lightest constant-height beam.
2. Design the lightest variable-height beam.
3. Analyze the variable-height beam using plane stress finite elements.

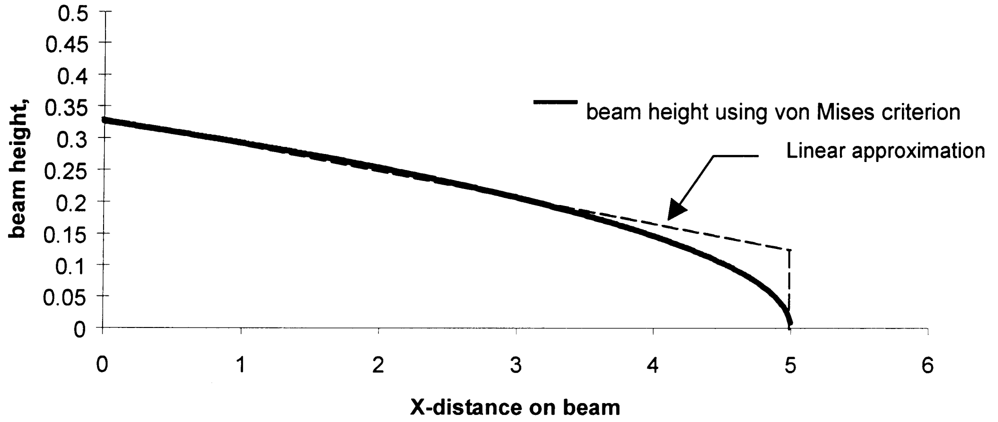


Fig. 7. Bean theory solution for the beam height satisfying the von Mises yield criterion.

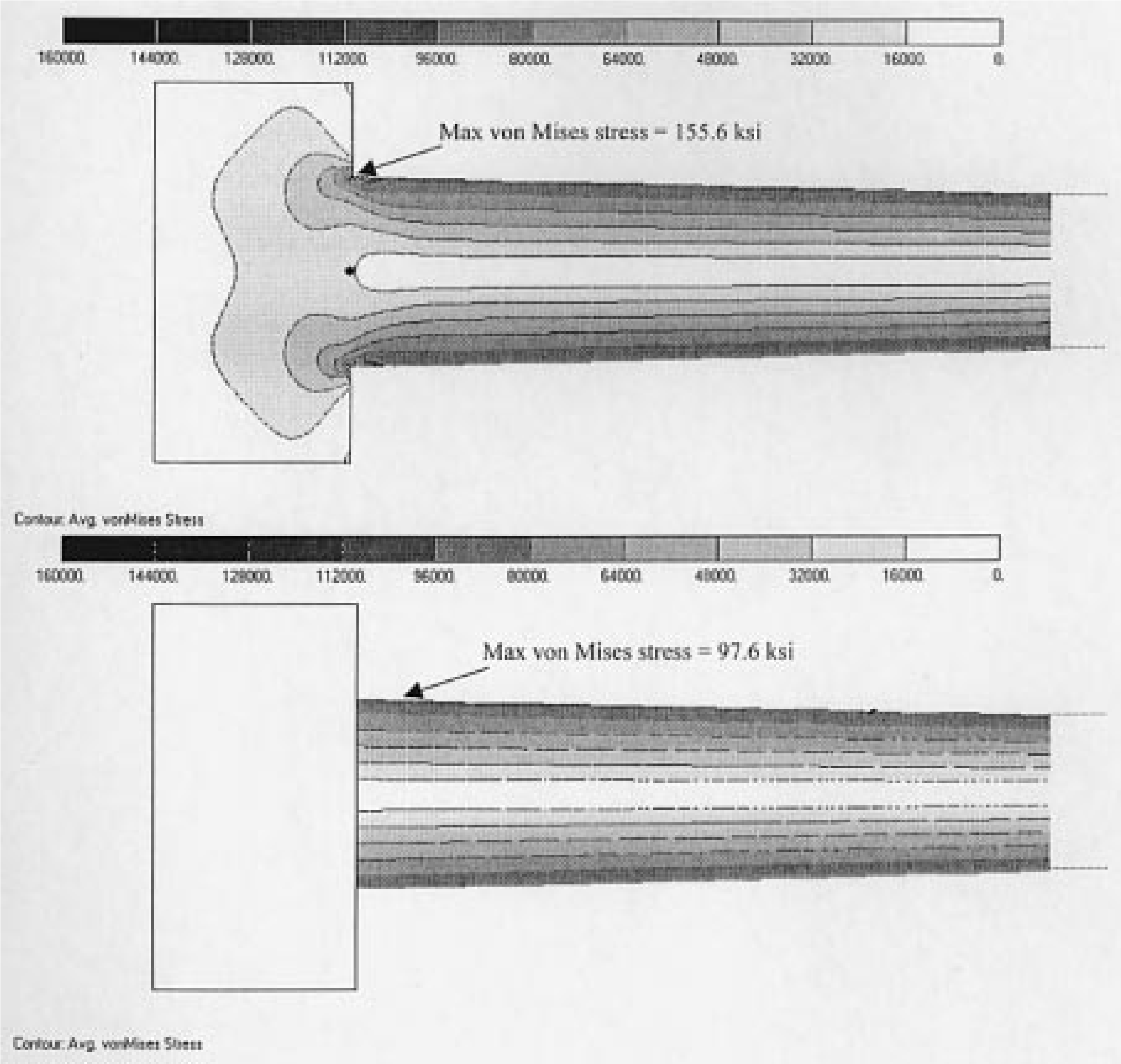


Fig. 8. Comparison of von Mises stress from FEA plane stress (top) and beam theory (bottom).

4. Further optimize the beam design to avoid areas of high stress concentration.

Based on beam theory, the bending stress should dominate the beam design, while the shear stress becomes important near the loaded end. Students at the junior level have no difficulty finding the optimum shape for a beam that satisfies these criteria. Such a shape is shown in Fig. 7. At this point a compromise between a 'perfect' shape and an acceptable linear approximation of the shape, which is shown in Fig. 7, is evaluated. The manufacturing cost is a significant factor in design

Table 3. Comparison of end loading effects on three stress components for FEA and beam theory solutions.

Stress (psi)	FEA Point Load	FEA Constant Load	FEA Distributed Load	Beam Theory
For x = 0.5L and y = h/4:				
Txx	−75	−75	−74.992	−75
Tyy	0.11E-5	0.79E-6	0.89E-6	0.0
Txy	11.20	11.20	11.20	11.25
For x = 0.95L and y = h/4:				
Txx	−10.779	−5.5067	−7.5131	−7.5
Tyy	0.57574	−0.11255	0.00293	0.0
Txy	12.877	11.358	11.165	11.25

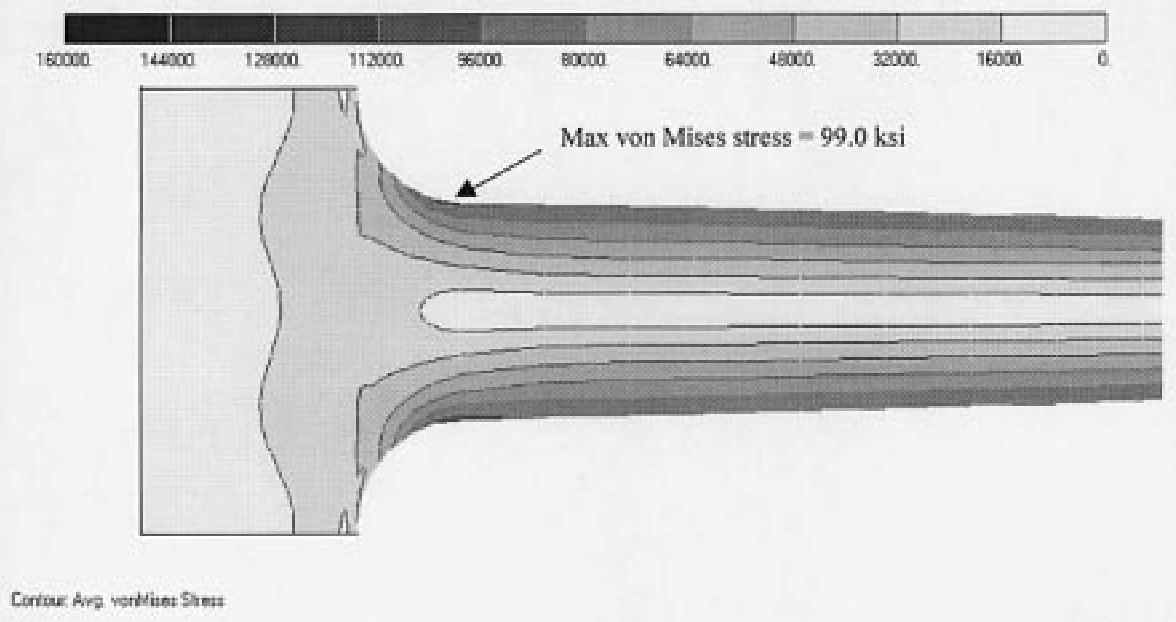


Fig. 9. Effective stress distribution for beam design problem with fillets.

optimization, and the linear approximation might be the best design.

Figure 8 shows a comparison of von Mises stress contours between the FEA plane stress and the beam theory results for an optimized beam. The two sets of results agree very well for most part of the beam. However, beam theory does not indicate any stress concentrations at the support area, while this is apparent with the FEA analysis. With this activity, students come to appreciate the simplicity of the beam theory formulas and see their limitations. The accuracy of beam theory illustrates that simplifying assumptions are not just for the purpose of homework problems. They can be used to obtain simple formulas that capture the essence of a problem. However, if details such as the stresses at the support are required, more rigorous analysis is required.

In addition, students observe close agreements in transverse displacements from the two analyses, while the shear strain distribution in the plane stress analysis validates the claim that transverse shear strain is very small for thin beams. Adding fillets to the support eliminates most of the stress concentration, as shown in Fig. 9.

In summary, while the FEA solution describes more accurately the stress distribution in loaded members, it enhances classroom claims that approximate theories, like beam theory, still have a role in structural analysis. Furthermore, it provides a powerful tool to address the 'how' and 'what if' of the Kolb learning cycle.

CONCLUSIONS

Texas A&M has designed and implemented, with the support of the NSF-funded Foundation Coalition, a sophomore core engineering curriculum consisting of five 3-credit hour courses, using conservation principles as the basis. The curriculum has been adjusted to fit the reduced number of credits and the increased use of technology, teaming, and collaborative learning as part of a longer-range plan to redesign the entire four-year engineering curriculum at Texas A&M. Instructional methodologies utilized in this process include active teaching/collaborative learning and the Kolb learning cycle. Examples of using computer technology in the teaching of the sophomore level continuum mechanics component of this curriculum and a junior level course on mechanics of solids has been discussed in this paper. The use of computer software for both courses has been very constructive and provided the opportunity to demonstrate the 'How' and 'What if' in the Kolb learning cycle. The authors also found it rewarding for the students to be able to visualize and gain intuition of complicated problems in heat conduction and stress analysis, while keeping in touch with the underlying fundamental conservation principles.

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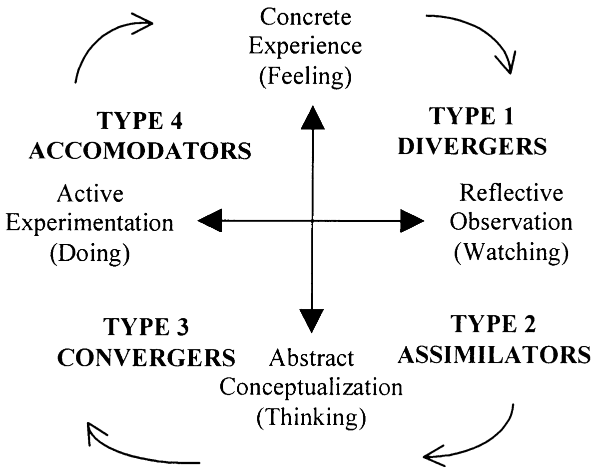

Fig. A1. Learning styles in the Kolb cycle [20].

APPENDIX

Effectiveness of the kolb learning cycle

The Kolb learning cycle identifies four different types of learning styles:

- divergers
- assimilators
- convergers
- accommodators.

They represent the internal structure of the learning cycle that forms a cyclic sequence ranging from concrete experience to reflection to conceptualization to experimentation [19, 20, 28].

The four learning styles in the Kolb cycle are based on the ways people perceive and process information. Research styles studies suggest that this is a more effective learning environment for students and helps them reach higher learning levels because it requires them to function in their non-preferred learning modes, thus promoting individual growth (Bloom's Taxonomy) [20, 30]. Another way to describe the various learning styles is by associating each style with a particular question. For example, divergers like to know the reason behind the subject ('Why?'), assimilators, on the other hand, prefer to be given the facts ('What?'). Similarly, convergers prefer the 'hands-on' approach ('How?'), while accommodators prefer to teach the subject to themselves and explore other ways of solving the problems ('What if?'). The styles are shown schematically in Fig. A1 [20]. As one progresses around the learning cycle from diverger to accommodator, the role of the instructor becomes less dominant, while the role of the student becomes more prominent.

Perceiving information through experience and reflective observation are important characteristics of a **diverger.** A reason must be given 'Why' the material concerns them. The student wants to see how the material fits into 'the big picture'. They seek personal reasoning as they learn the material and truly rely on individual interaction from the instructor. Therefore, a motivated instructor is very important in the learning process for a diverger.

Assimilators, on the other hand, use abstract conceptualization and reflective observation to process data. They take small amounts of material from different places and put it together to solve large, complicated problems. This type is detail-oriented ('What') and prefers to be given a procedural way to solve the problem. Assimilators thrive on organized instructors who show a repetitive way to solve the problems.

Convergers also use abstract conceptualization, but they process the information actively rather than reflectively, as did assimilators. They are a 'hands on' type of students who like to see *'how'* things work. They learn by actually 'doing' and not by listening to how it should be done. Instructors who spend more time helping the student actively experience the lesson and less time giving all of the details behind it are preferred for convergers.

Finally, **accommodators** comprehend information through experience and process it through active experimentation. They prefer learning through self-discovery ('What if') and resent too many procedures to follow. They take the information given to them and then use their imagination to create something brand new. Instructors who teach the basics and allow the students to teach themselves are favored. Accommodators appreciate instructors who interact and serve as a resource but stay away from supervising.

Testing at several campuses across the nation has shown that the majority of students are either assimilators or convergers. Understanding the various types of learning styles allows an instructor to not only meet the learning needs of all students in the classes but also to encourage and enhance learning. All learning styles can be met if the problem is presented in such a way to allow it [19–23, 28, 30, 31].

Students in ENGR 214 were asked to best describe their personal learning style and then realize how they can use this information to improve their own learning style in the context of conservation laws [19]. The findings were similar with the national trends mentioned above, i.e. most students were either assimilators or convergers. For the specific case of energy conservation and heat transfer, divergers prefer justification of the study of heat transfer by mentioning numerous cases in engineering applications where heat transfer is manifested. Assimilators favor an explanation of how energy balance is applied with Fourier's law to solve boundary value problems in heat conduction. Convergers associate well with using computer software for examples and 'hands-on' activities with familiar case studies of heat transfer. Finally, accommodators, taking advantage of the available computer software, like the study of different possibilities, for example trying different insulating materials for house walls or different toppings and boundary conditions in the pizza project.

At the end of the coverage of energy conservation, where a conscious effort was made to address all learning styles, the students in the ENGR 214 class (about 40) were asked to answer questions regarding the implementation of the Kolb cycle in the classroom. Overwhelmingly, the responses were positive. The students liked the motivation of the energy conservation law by drawing examples from engineering applications and everyday examples from cooking to weather. They also liked the motivation of Fourier's law of heat conduction and the necessity of the four components in the formulation of problems. One clear message from the response of the students was that the practical applications helped visualize the information more than simply using abstract equations (balance between 'How' and 'What'). Another response was that giving more views to a specific problem made it easier to understand (revisiting the same problem in a way that addresses different learning styles). A large majority of the students were in favor of 'hands on' labs. Still several students said that computer labs could only do so much. Physical things to touch and construct provide a concrete notion of the subject matter (balance between reality and virtual reality in addressing the 'Why' and 'What if'). A few students felt that there was too much time spent on 'What' and not enough on 'Why' and 'How,' even though there was a deliberate effort to emphasize the learning styles of divergers and convergers. This response represented strong adherence by some students to the different individual learning styles. The above responses reinforced the very important point about having all four learning styles incorporated in the class.

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