Instructive Review of Computation of Electric Fields using Different Numerical Techniques

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There are different numerical techniques for computing electric fields. These numerical techniques enable the designer to study the problems that are difficult to be solved by analytical methods. This paper attempts to give an instructive review of different numerical techniques in electric field analysis. These techniques involve finite difference, finite element, boundary element, charge simulation, finite element with variable field intensity and Monte Carlo methods. The merits and limits of the various methods are outlined. Some examples are given in which the field computations using different numerical techniques are compared. As an example the most convenient technique applicable to electric field computation within the tank of power transformers is introduced.

INTRODUCTION

UNTIL NOW electric and electromagnetic fields have been regarded as probably the most abstract and difficult part of the undergraduate electrical engineering curriculum. This is largely due to the fact that such fields cannot be visualized directly [1].

Electric fields can be computed using various methods with different precision. However, for insulation of electrical equipment a more accurate electric field prediction is required. With the advent of computing power and numerical techniques in recent years, it has become practical to use different techniques to compute the electric fields. Such numerical techniques enable the designer to solve problems that are difficult, and use of analytical approach with many empirical factors is impossible.

The aim of this paper is to give a review of the application of different numerical techniques in the electric fields computation that is useful for an undergraduate course. This material should enable the graduate student to use these analyses in graduate research as well as later in the workplace. The authors will also give their personal views on the field evaluation of transformers.

In two-dimensional (2D) analysis, if the field is not time varying, the electric potential V in the actual space is satisfied by Poisson's equation [2]:

$$\nabla^2 V = -\rho/(\epsilon_r \epsilon_0) \qquad (1)$$

where ρ is the electric charge density, ϵ_r and ϵ_0 are relative and absolute permittivity of the free space. In order to solve partial differential Eqn. 1, a number of boundary conditions must be imposed. Analytical solutions for practical boundary conditions are difficult, if not impossible and therefore, numerical techniques are necessary.

Each technique has its own merits and drawbacks and one technique cannot be generally preferred to others. Based on the proposed problem, the most convenient technique must be selected. Different numerical techniques, so far used for the electric field analysis, are briefly reviewed and the most convenient technique is then suggested for electric field evaluation within the interior space of power transformers.

BOUNDARY CONDITIONS

There are two following types of boundary conditions that are considered in the electric field evaluation:

1. Boundary between conducting and dielectric materials.
2. Boundary between different dielectric materials.

From the electrical point of view, one of the following conditions may be satisfied on the boundaries of the first type:

a. The electric potentials of all points on the boundary are known. This is possible if the available conductor is connected to a fixed potential source (Dirchlet condition).
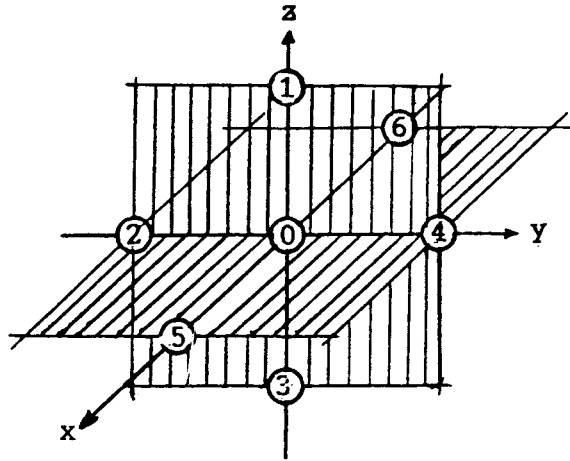
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Fig. 1. Point 0 and six adjacent points.

b. The whole charge on the boundary surface is known while the electric potential over different points is unknown. This is the case when the available conductor is not connected to a fixed potential (floating). Normally in such a case the whole charge on the surface boundary is equal to zero (Neumann condition).

FINITE DIFFERENCE METHOD

Finite difference method (FDM) is the oldest technique in the field computations that was introduced by Gauss. Then Boltzman published it in his notes in 1892. However, because of the large amount of computations required, the extensive use of the FDM dates back only to the event of the computer.

Electric potential V over any region depends on the (x, y and z) coordinates and its derivatives. Potential of any point may be given versus the electric potential of the adjacent points using Tylor's expansion. For instance, consider Fig. 1 having point 0 and six adjacent points. The coordinates of these points are:

$$P_0(x_0, y_0, z_0), P_1(x_0, y_0, z_0 + h), P_2(x_0, y_0 - h, z_0),$$
$$P_3(x_0, y_0, z_0 - h), P_4(x_0, y_0 + h, z_0),$$
$$P_5(x_0 + h, y_0, z_0), P_6(x_0 - h, y_0, z_0)$$

For simplicity, the difference between the origin and the adjacent points is taken to be h. Potential of an arbitrary point $P(x, y, z)$ may be calculated versus potential of point P_0 as follows:

$$V(x, y, z) = V(x_0, y_0, z_0) + [(x - x_0)V_x(x_0, y_0, z_0)$$
$$+ (y - y_0)V_y(x_0, y_0, z_0) + (z - z_0)V_z(x_0, y_0, z_0)]/1!$$
$$+ [(x - x_0)^2 V_{xx}(x_0, y_0, z_0)$$
$$+ 2(x - x_0)(y - y_0)V_{xy}(x_0, y_0, z_0)$$
$$+ 2(x - x_0)(z - z_0)V_{xz}(x_0, y_0, z_0)$$

$$+ (y - y_0)^2 V_{yy}(x_0, y_0, z_0)$$
$$+ 2(y - y_0)(z - z_0)V_{yz}(x_0, y_0, z_0)$$
$$+ (z - z_0)^2 V_{zz}(x_0, y_0, z_0)]/2! + \dots \quad (2)$$

where

$$V_i = \partial V/\partial i, \quad V_{ii} = \partial^2 V/\partial i^2, \quad V_{ij} = \partial^2 V/\partial i \partial j,$$
$$i, j = x, y, z, \quad h = (x - x_0) = (y - y_0) = (z - z_0)$$

If h tends to a very small value, the terms containing the third- and higher-order derivatives may be ignored and Eqn. 2 can be rewritten as follows:

$$V_1 = V_0 + hV_z(P_0) + 0.5h^2 V_{zz}(P_0)$$
$$V_2 = V_0 - hV_y(P_0) + 0.5h^2 V_{yy}(P_0)$$
$$V_3 = V_0 - hV_z(P_0) + 0.5h^2 V_{zz}(P_0)$$
$$V_4 = V_0 + hV_y(P_0) + 0.5h^2 V_{yy}(P_0)$$
$$V_5 = V_0 + hV_x(P_0) + 0.5h^2 V_{xx}(P_0)$$
$$V_6 = V_0 - hV_x(P_0) + 0.5h^2 V_{zz}(P_0) \quad (3)$$

$$\sum_{i=1}^{6} V_i = 6V_0 + h^2[V_{xx}(P_0) + V_{yy}(P_0) + V_{zz}(P_0)]$$
$$(4)$$

Solution of Eqn. 4 and substituting from Eqns. 1–2 yields:

$$V_0 = [V_1 + V_2 + V_3 + V_4 + V_5 + V_6 + h^2 \rho_0/\epsilon]$$
$$(5)$$

where ϵ_0 is the electric charge density at point 0. As shown in Eqn. 5, there is a linear relationship between the potential of point 0 and the potentials of the adjacent points. For the cases where difference between the point 0 and the adjacent points is not the same or these points are within different insulating materials, and also for rotating fields in electrical machines, similar equations can be derived [3, 4].

In the FDM, the proposed region is discretized using the equations similar to Eqn. 5. Dimensions of the meshes must be such that the approximation is acceptable. The vertexes of the meshes are nodes on the boundary of the region and their potentials are known or they correspond to point 0 in Fig. 1, enclosed by the other nodes. For the latter note (not on the boundary), equations similar to Eqn. 5 can be written versus the potentials of the adjacent nodes. If the number of such nodes is equal to n, n linear algebraic equations with n unknown values of the node potentials are obtained. Potentials of the points inside the meshes may be determined using different interpolation techniques.

The FDM is not capable of calculating electric field directly at different points on the proposed region. When the potentials of the nodes are obtained, a numerical derivative evaluation technique is used to calculate the electric field intensity: $E = -\nabla V$.

FINITE ELEMENT METHODS

Among the various numerical techniques, the finite element method (FEM) has a dominant position because it is versatile, having a strong interchangeability and can be incorporated into standard programs [5, 6].

FEM is based on this fact that the physical systems stabilizes at the minimum level of energy. The general equation of energy in an electric field is:

$$W = 0.5\left[\int_v \nabla v(\epsilon\nabla v)dv - \int\rho vdv - \int_{S'}\rho_s vds\right] \quad (6)$$

where

v is the volume of the proposed region
ρ is the electric charge density within volume v
S′ is the boundary surface with Numman conditions
ρ_s is the surface electric charge density within S′.

Since Eqn. 1 describes the electric potential distribution in the real systems, based on the minimum energy level theorem, it is concluded that Eqn. 1 minimizes the energy presented by Eqn. 6.

In the FEM, the volume of the proposed region is divided into M small polyhedron elements where their sides form a grid with N nodes. The potential function is then approximated by:

$$V(r) = \sum_{i=1}^{N} f_i(r)V_i \quad (7)$$

where r is any point on the proposed region. $f_i(r)$ is called the shape function having the following features:

a) $f_i(r)$ is equal to zero anywhere, except on the subregion W_i. The sub-region W_i consists of the elements where node i is one of their vertices.
b) $f_i(r)$ is continuous on the boundaries due to W_i, and a polyhedron inside each element.
c) any $f_i(r)$ is equal to unit at the location of node i and zero at the other nodes:

$$f_i(r_i) = 1 \quad \text{for} \quad i = j$$
$$f_i(r_j) = 0 \quad \text{for} \quad i \neq j$$

V_i in Eqn. 7 is equal to the potential of node i. Substituting Eqn. 7 into Eqn. 6, the approximate energy is presented by W^* which is minimized under the following conditions:

$$\partial W^*/\partial V_i = 0; \quad i = 1, 2, \ldots, N \quad (8)$$

Since W^* is a quadratic function of V_i, applying conditions 8 leads to the following linear algebraic equations:

$$GV = A \quad (9)$$

where V is the known vector with elements V_i; A is the known vector obtained from the volume charge density in the proposed region and bound-ary conditions; G is the non-singular square symmetrical matrix.

Solution of this system of equations gives the values of V_is and hence an approximate distribution of the potential can be determined based on Eqn. 7.

Electric field intensity within each element is obtained using the gradient expression as follows:

$$E_m = (-\nabla V)_m = -\sum_{i=1}^{N} V_i.f_i^m \quad (10)$$

Often the first derivative of f_i is non-continuous. Therefore, reduction of the maximum size of the elements and tending to zero, leads Eqn. 7 to the real distribution of the potential. In spite of this, no continuity of the field intensity on the boundaries of the elements remains in force. If function f_i is considered as a complete n-order polynomial, better results can be obtained. If h presents the maximum size of the elements, reduction of h can reduce the potential error with ratio $(\Delta h)^{n+1}$; and electric field is continuous and its error is reduced by ratio $(\Delta h)^n$ [7].

Fig. 2 shows the meshing and equipotential lines determined using the FEM. The FEM could be also used where the permittivity of the proposed region is not constant. In such a case, it is necessary to replace $\epsilon(r)$ with ϵ, which shows the position dependency. Methods have been introduced to consider the floating electrodes with unknown potential or different insulating materials [7]. There are several reports for automatic meshing of the proposed region [9–11].

BOUNDARY ELEMENT METHOD

If distribution of electric charge for every region (including boundary surfaces) is known, electric potential and field intensity for each point can be computed using Coulomb's law of Gauss's law [2]. In practice, Laplacian equation is normally used. This means that electric charge is enclosed only inside the boundaries of the proposed region and volume charge density inside the region is equal to zero or negligible. The electric potential and field intensity are:

$$V(I) = \int\int_s \rho_s/(4\pi\epsilon R)ds \quad (11)$$

$$E(I) = \int\int_s \rho_s(4\pi\epsilon R^2)ds.\hat{a}_R \quad (12)$$

where I is the proposed point, S is the summary of all boundary surfaces, ρ_s is the surface charge density over surface s, R is the distance of the differential elements from point I and $\hat{a}_R$ is unit vector along R directed from differential element ds to point I.

In practice ρ_s is unknown and it seems that solving Eqns. 11–12 is impossible. However, to
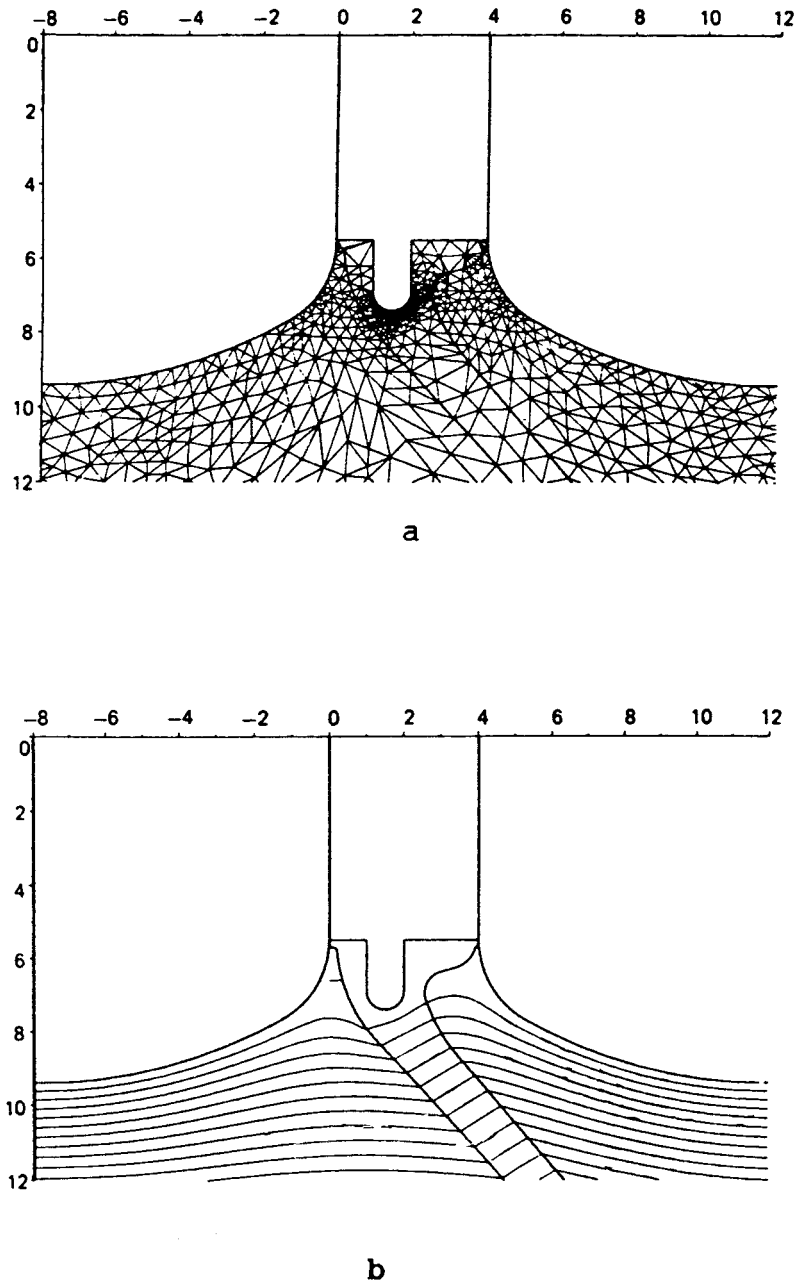
a

b

Fig. 2. Computation of electric field using FEM: a) meshing, b) equipotential lines [4].

overcome this difficulty, the BEM may be employed. In this method, the boundary surface is divided into N elements.

Figure 3 shows a typical boundary elements due to a flat boundary surface on xy-plane. Then taking into account the small dimensions of the boundary elements, a general form may be considered for the surface charge density on element $j(\rho_{sj})$. This general form is often a polynomial with unknown coefficients.

For instance, if it is taken to be a quadratic polynomial as follows:

$$\rho_{sj} = a_{1j} + a_{2j}x + a_{3j}y + a_{4j}xy \qquad (13)$$

substituting ρ_{sj} in Eqns. 11 and 12 gives:

$$V(I) = \sum_{j=1}^{N}\left[\iint_{sj}\rho_s/(4\pi\epsilon R)ds\right]$$

$$= \sum_{j=1}^{N}\left\{\iint_{sj}[a_{1j} + a_{2j}x + a_{3j}y + a_{4j}xy]/(4\pi\epsilon R)ds\right\}$$

$$\qquad\qquad\qquad\qquad\qquad (14)$$

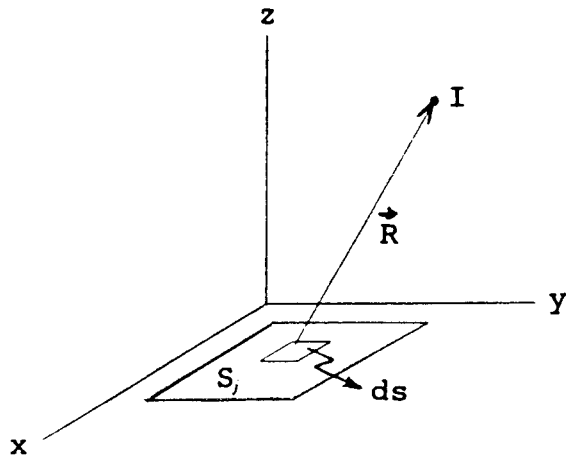$$E(I) = \sum_{j=1}^{N}\left[\iint_{sj}\rho_s/(4\pi\epsilon R^2)ds\hat{a}_R\right]$$

Fig. 3. A flat quadrangular boundary element.

$$= \sum_{j=1}^{N} \left\{ \int\!\!\int_{sj} [a_{1j} + a_{2j}x + a_{3j}y + a_{4j}xy]/(4\pi\epsilon R^2)ds\hat{a}_R \right\}$$

$$(15)$$

It is clear that V(I) and E(I) are linear functions of the coefficients of the polynomials. By using the following definitions:

$$P_{1j} = \int\!\!\int_{sj} ds/(4\pi\epsilon R)$$

$$P_{2j} = \int\!\!\int_{sj} (xds)/(4\pi\epsilon R)$$

$$P_{3j} = \int\!\!\int_{sj} (yds)/(4\pi\epsilon R)$$

$$P_{4j} = \int\!\!\int_{sj} (xyds)/(4\pi\epsilon R)$$

$$f_{1j} = \int\!\!\int_{sj} ds/(4\pi\epsilon R^2)\hat{a}_R$$

$$f_{2j} = \int\!\!\int_{sj} (xds)/(4\pi\epsilon R^2)\hat{a}_R$$

$$f_{3j} = \int\!\!\int_{sj} (yds)/(4\pi\epsilon R^2)\hat{a}_R$$

$$f_{4j} = \int\!\!\int_{sj} (xyds)/(4\pi\epsilon R^2)\hat{a}_R$$

Eqns. 14 and 15 become:

$$V(I) = \sum_{j=1}^{N} \sum_{k=1}^{4} P_{kj}a_{kj} \qquad (16)$$

$$E(I) = \sum_{j=1}^{N} \sum_{k=1}^{4} f_{kj}a_{kj} \qquad (17)$$

In the next stage, the number of unknowns function P_{sj} is selected on each boundary element. Then, based on the boundary type where sj is part of it, the boundary conditions equation for any selected point is formed using Eqns. 16 and 17. Therefore, a system of algebraic linear equations is obtained which finally produces the coefficients of the polynomial. P_{kj} and f_{kj} are numerically or analytically obtained by integration. Hence the surface charge density distribution on all boundary surfaces is known. Finally the electric potential and field intensity can be determined using Eqns. 16 and 17.
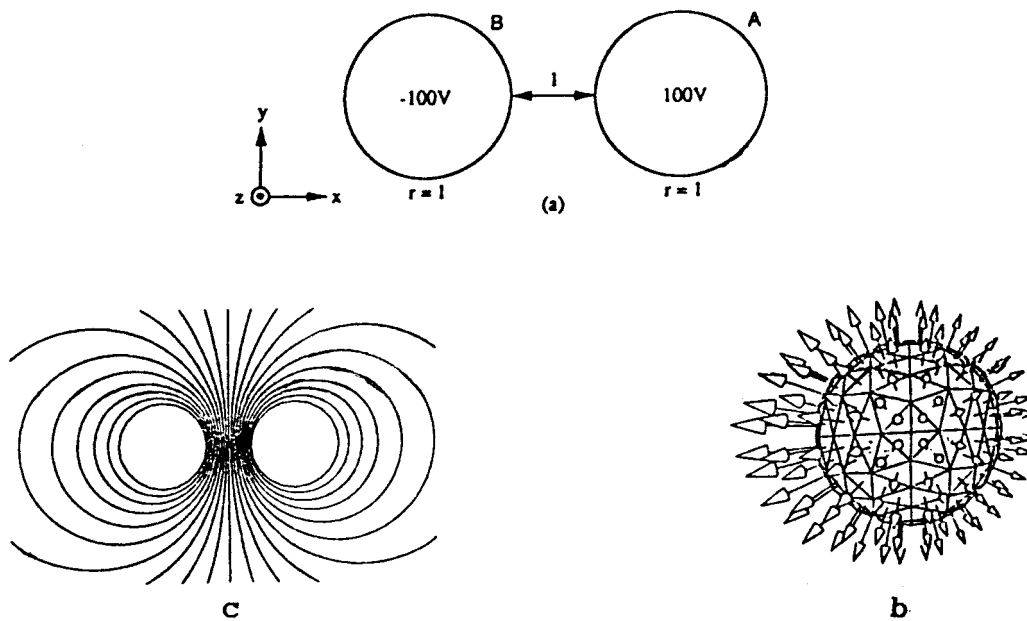

Fig. 4. Electric field calculation using BEM: a) actual electrical system, b) electric field density vectors on the sphere, c) equipotential countors [12].

Figure 4 presents a typical problem solved by the BEM, where the electric field between the two spheres having the same radius r = 1 is obtained. Electric field intensity has been shown with a suitable vector. In addition, the equipotentials between two spheres have been calculated by computation of the electric potential in different points.

A method has been given in [13] for the curved-shape boundary elements in order to model practical surfaces with the desirable accuracy. Boundary elements with full axial symmetry have been presented in the literature [14–16]. Input and output data processing methods in the BEM have been introduced in [12, 17, 18].

CHARGE SIMULATION METHOD

This method is similar to the BEM. The difference between charge simulation method (CSM) and the BEM is the simulation of the surface charge existing on the boundary surfaces. In the BEM the surface charge density function on the different surface boundaries are estimated, while in the CSM the surface charge density is substituted by a set of discretized linear charge distribution. The substituted linear charge distribution is such that the electric potential and field intensity versus their charges are analytic known functions. Charge distribution on an infinite length line with a constant density, on a finite line, on a circle etc., are examples of the charge distribution. Different distribution types and their equations have been given in [4] and [19].

Electric potential and field intensity equations due to the above mentioned charge distribution, for the points on the charges, have singularity. To overcome this singularity, the position of the replaced charge simulation is considered outside the proposed space and normally inside the electrodes. The exact position and the replaced charge distribution type are arbitrarily selected based on the experience. Charge value or their linear charge density is computed such that the boundary conditions are satisfied on some surface boundaries, as described in the following part.

Electric potential and field intensity due to the replaced charge simulation in different points are linear functions of the charge value or charge density:

$$V_j(r) = P_j(r)q_j \qquad (18)$$

$$E_j(r) = f_j(r)q_j \qquad (19)$$

where $P_j(r)$ is the potential factor, $f_j(r)$ is the field intensity factor, r is the relative position of the proposed point and q_i is the jth replaced charge distribution. $P_j(r)$ and $f_j(r)$ are different for different charge distributions. There is the following relationship between them:

$$f_j(r) = -\nabla P_j(r) \qquad (20)$$

Since there is a linear relationship between the electric potential (and field intensity) due to the replaced charge distribution in different points and charge value or charge density, for a set of such charge distribution, the superposition theorem can be applied to calculate the electric potential and field:

$$V_j(r) = \sum_{j=1}^{N} P_j(r)q_j \qquad (21)$$

$$E_r(r) = \sum_{j=1}^{N} f_j(r)q_j \qquad (22)$$

In the CSM, the number of points selected on the surface boundary is equal to the replaced simulation charges. Depending on the selected points, Eqns. 21 and 22 are used and the boundary conditions for individual points are considered. These equations are linear functions of q_j. Since the exact position of the charges and the selected points are known, P_js and f_js in Eqns. 21 and 22 are exactly calculated. Then only q_is are the unknown values of the above linear equations. Therefore, computations of the electric potential and field intensity are possible using Eqns. 21 and 22.

It is clear that in the CSM, boundary conditions are satisfied only in the points selected to write the equations. Before using q_js due to the solution of the equations, it is necessary to study the boundary conditions on other points of the boundary surfaces. When q_js in the CSM is calculated, the boundary conditions on different surface points must be determined. If the accuracy is not enough, number, position and type of the replaced simulation charges and also position of the selected points for writing the equations must be varied in order to obtain sufficient accuracy.

Figure 5 shows a typical problem solved using the CSM. The problem was computation of the electric field between two spheres shown in Fig. 5a. Figure 5b presents the equipotentials lines by substituting the surface charge of each sphere with two point-charges. Figure 5c indicates the corresponding result when three point-charges are substituted for each sphere, in which the accuracy is higher. In addition, two point-charges have been used in Fig. 5d, but the position of charges is different with that of Fig. 5b. Accuracy of the latter case is better than the other two.

FINITE ELEMENT METHOD WITH VARIABLE FIELD DENSITY

In the FEM, the main variable is the electric potential V, while the electric field intensity is normally the required quantity. For example, in the design of high voltage device insulation, it is
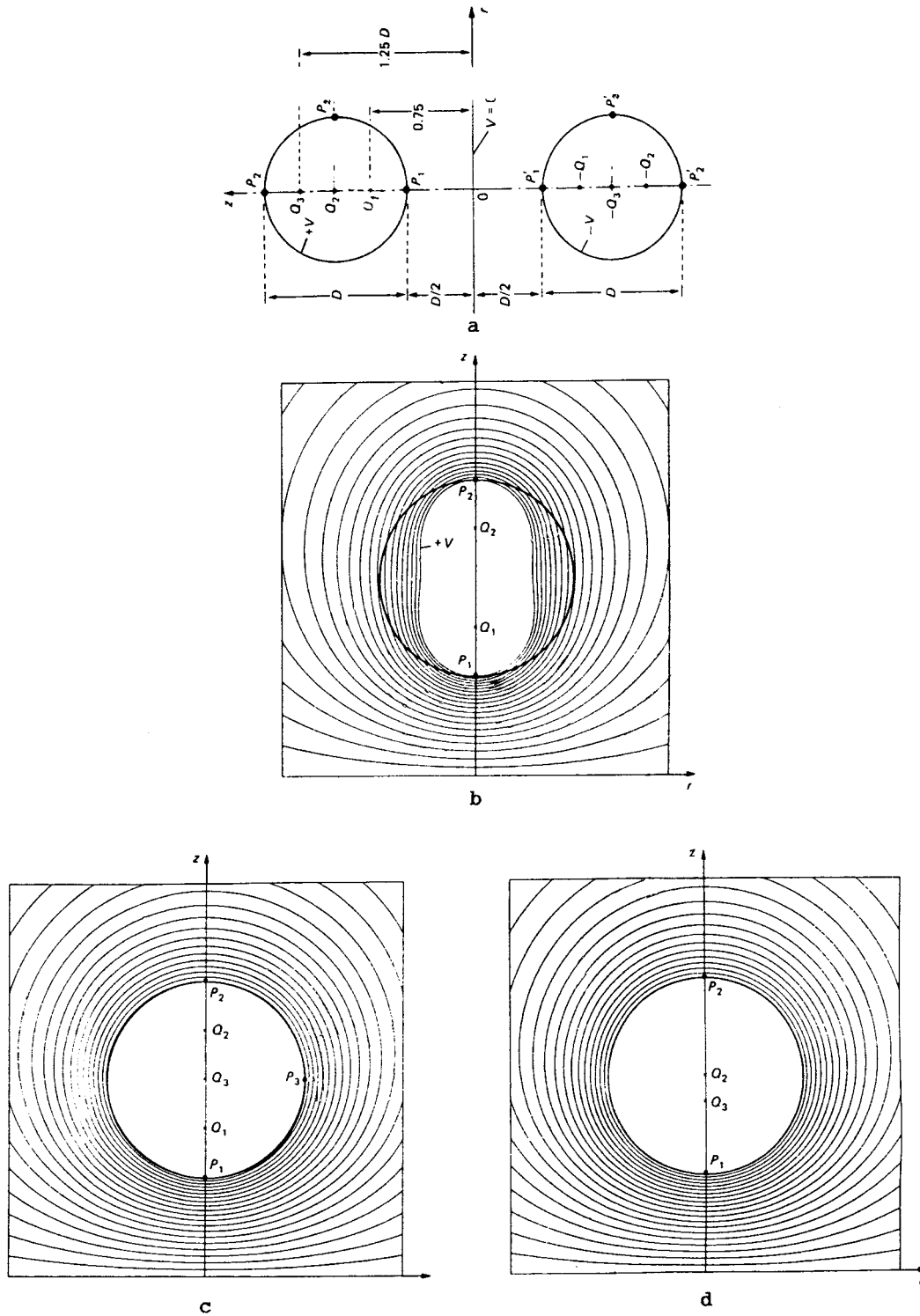
Fig. 5. Calculation of electric field using CSM: a) electric system, b) equipotential when two charge points used for each sphere, c) equipotential when three charge points used for each sphere, d) equipotential when two charge points in different positions used for each sphere.

necessary to have the amplitude and direction of the electric field on the surfaces of the electrodes and insulation. In study of the discharge phenomenon, the path of the force lines is required which can be determined if the electric field distribution in the proposed space is known. In the FEM a

numerical integration technique is used in order to calculate E that normally has error.

There is a more accurate FEM technique in which the main variable has been taken E instead of V [20]. If the volume charge density in the proposed space is zero and

permittivity E is constant and isotropic, then the Maxwell's equations for electrostatic fields are as follows:

$$\text{div } D = 0 \tag{23}$$

$$\text{Curl } E = 0 \tag{24}$$

where D is the electric charge density and E is the electric field intensity and:

$$D = \epsilon E \tag{25}$$

Applying Eqn. 24 and using Eqn. 23 and 25 leads to:

$$\nabla^2 E = 0 \tag{26}$$

which is the Laplacian equation. For 2D fields:

$$(\partial^2 E_x/\partial x^2 + \partial^2 E_x/\partial y^2)i$$
$$+(\partial^2 E_y/\partial x^2 + \partial^2 E_y/\partial y^2)j = 0 \tag{27}$$

But a vector will be zero if all its components are zero:

$$\partial^2 E_x/\partial x^2 + \partial^2 E_x/\partial y^2 = 0$$
$$\partial^2 E_y/\partial x^2 + \partial^2 E_y/\partial y^2 = 0 \tag{28}$$

For each component of the field intensity, the FEM is used once and finally the electric field intensity over the whole proposed region is obtained. However, with defined boundary conditions, the equation will have a unique solution. In practice, the boundary conditions on the first-type boundary versus electric potential are known but there is no idea concerning their electric field intensity. In [20], the BEM has been used in order to overcome this difficult; it means that the BEM is employed on the mentioned boundaries then finite element method with variable field density (FEMVFD) is applied.

Analysis has been carried out based on the FEM and FEMVFD and the results have been presented in Fig. 6.

The problem was computation of the electric field in the region between the two cylinders having potentials of 100 V and 200 V. Because of the axial symmetry and its boundaries and infinite length, the field has only a radial component varying in radial direction. Therefore, analysis of the field is possible in two dimensions. Due to the symmetry only one-quarter of the cylinder is used for analysis. Figure 6b shows the meshing for both techniques and Fig. 6c presents the equi-field intensity lines. As seen, in spite of the lower number of meshes in the FEMVFD, the equi-field intensity lines have a better continuity and their accuracy is also higher. In this example, the maximum error in solution by the FEMVFD is about 2% while this error is 9.3% when the FEM is used.

MONTE CARLO METHOD

In the FDM, the potential of every node is equal to the mean value of the potentials of the adjacent nodes. When the distance between the proposed node and adjacent nodes are the same, same weight potentials are involved in the computation of the mean value. Otherwise, the weights will not be equal [4]. But the closer nodes will be heavier. However, the sum of all weights is unity. Generally in the FDM, potential of each node (V_0) versus potential of n adjacent nodes is as follows:

$$V_0 = \sum_{I=1}^{N} W_i V_i \tag{29}$$

where $\Sigma W_i = 1$, V_1 is the potential of the node adjacent to i-th node and W_i is its weight. Value of W_i depends on the relative distance of the i-th node from the proposed node.

The basic equation of Monte Carlo Method (MCM) is similar to Eqn. 29. Therefore, these two methods are similar, except that determination of the adjacent nodes and method of calculation of W_i differs. In the FDM, all nodes are defined after meshing process and at the same time adjacent nodes of every node are determined. In W_i computation, analytical relationships are used. But in the MCM, adjacent nodes are always on the boundaries and their exact positions are determined using a random process and W_is is obtained using probability techniques.

In Fig. 7, S_is show boundary surfaces of the problem. These boundary surfaces are Drichlet type with potential V_i. Calculation of potential at any point such as r_0 is required using equations similar to Eqn. 29. In the MCM, simulation of a random movement is used in order to determine each i node. Any movement begins from r_0 and after successive jumps with variable lengths and in random directions, leads to a S_j.

Conditions governing on every random movement are as follows:

a) All movements begin from r_0.
b) Length of any jump is equal to the minimum distance of the beginning point with boundary surfaces (S_is).
c) Direction of very jump is random.
d) The end of any random movement will reach when the minimum distance mentioned in item b is smaller than that of the predefined value such as 0.

At the end of a random movement, a point on one of the boundaries with the closest distance to the end point is selected as an adjacent point. Finally, potential of point r_0 is calculated as follows:

$$V(r_0) = \sum_{i=1}^{N} V(r_i^*)/N \tag{30}$$
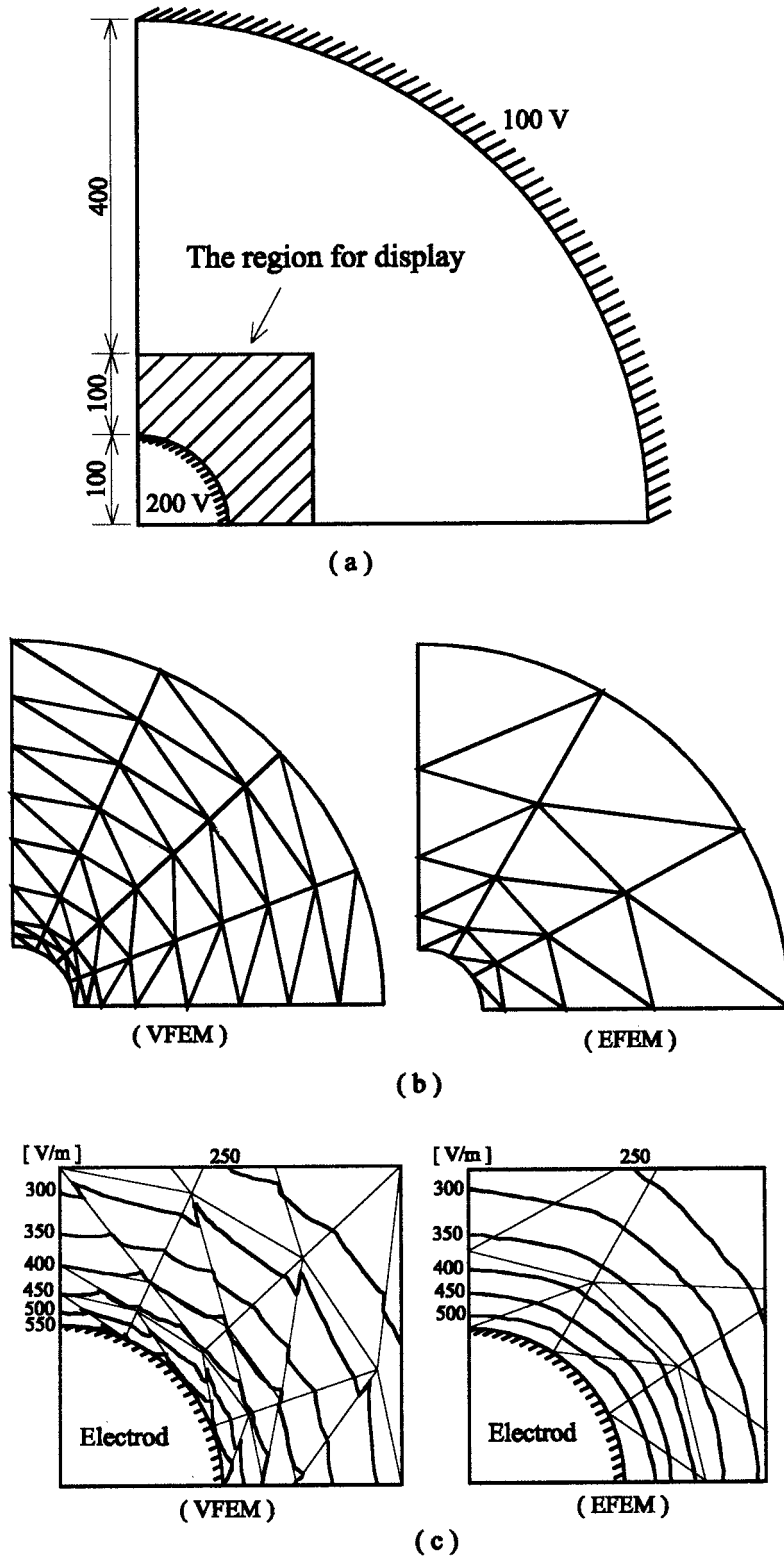
where N is the total number of the random

Fig. 6. Comparison of computed electric field using FEM and FEMVFD: a) electric system, b) meshing based on the two techniques, c) equi-field intensity lines using both methods.

movements and r_i^* is the adjacent point due to the j-th random movement. In order to eliminate the statistical errors, it is necessary that N becomes large enough and often several thousands.

In Eqn. 30, all weights are apparently the same and equal to 1/N. In fact, it is not so, because taking into account the large number of random movements (N), probability of several reputations of one point exists and for relatively shorter
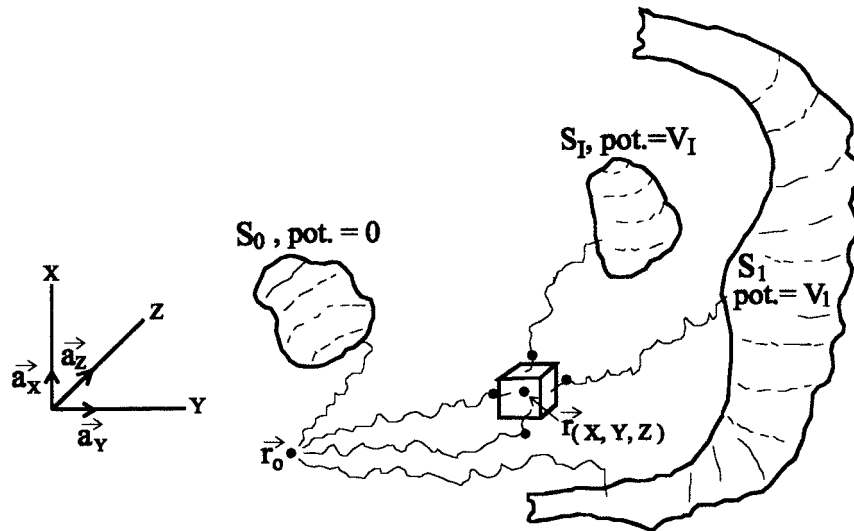
Fig. 7. Representation of the possible random movement paths for reaching from the proposed point (r_0) to one of the boundary surfaces (S_i).

distance from r_0, this probability is larger. Suppose constant potential V_i for any boundary surface, Eqn. 30 can be written as follows:

$$V(r_0) = \sum_{i=1}^{n} N_i V_i \qquad (31)$$

where n is the number of boundary surfaces and N_i is the number of random movements ended to a point at S_i.

Although the MCM was described using the FDM, it is necessary to note that the MCM is itself an independent method having special
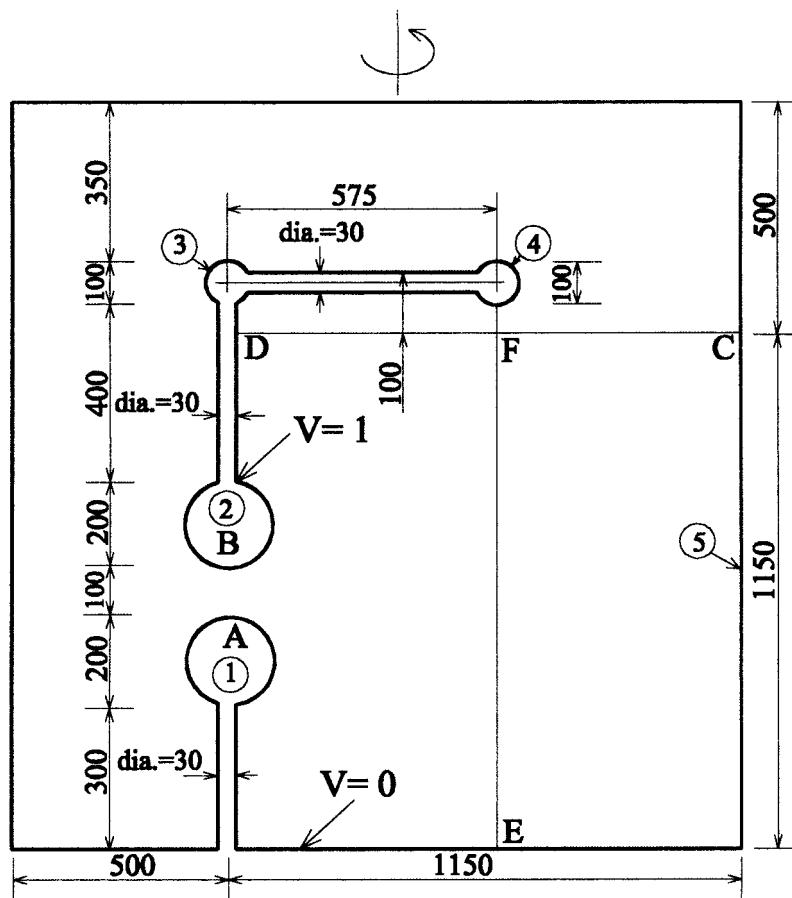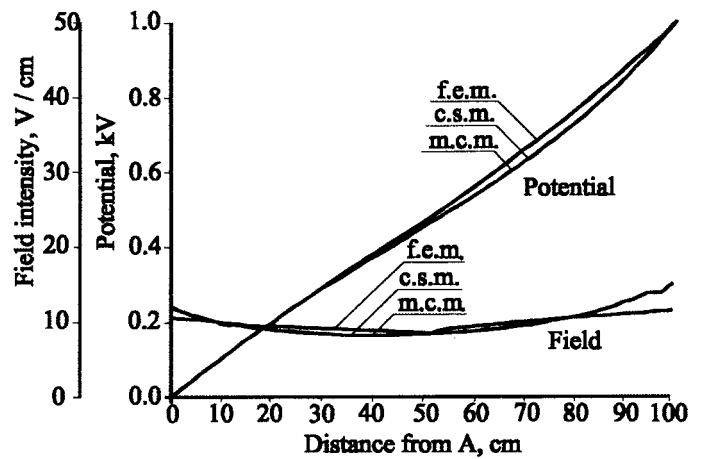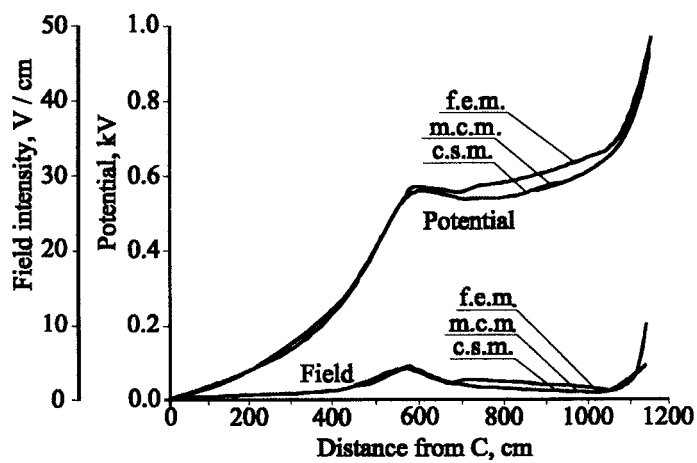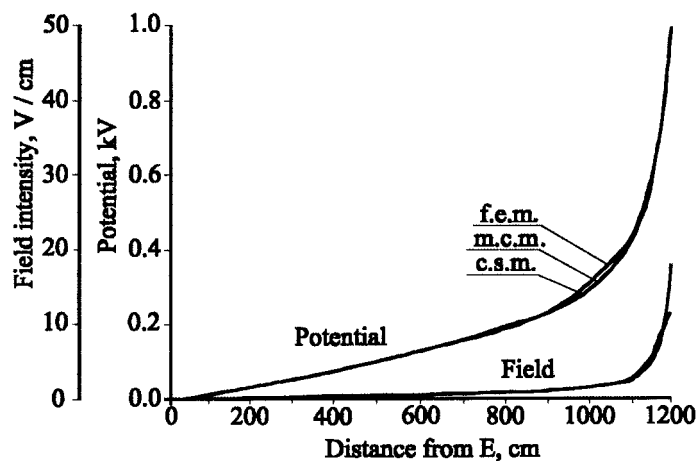

Fig. 8. Geometry of laboratory high voltage electrodes [7].

Fig. 9. Comparison of the computed electric fields of the system shown in Fig. 8 using three techniques: a) along path AB, b) along path CD and c) along path EF [7].

fundamentals [21]. Computation of electric potential using MCM is similar to the microscopic analysis of gas pressure where the random movement of the molecules is simulated. A technique for computation of the electric field over the spaces consisting of different isolating materials has been presented in [22]. In [7, 22], a number of techniques have been introduced to speed up the calculation using the MCM. Figure 8 shows the geometry of the laboratory high voltage electrodes in which the

electric potential and field along line AB, CD and EF have been estimated using the three methods: MCM, CSM and FEM. The results are presented in Fig. 9. It is clear that the accuracy of the MCM is similar to the other techniques. There are about 24000 random movements in this simulation.

COMPARISON OF METHODS AND CONCLUSIONS

Any numerical technique for electric field evaluation has its own merits and drawbacks and it is not generally possible to prefer one technique over the others.

Structure of the power transformer indicates that normally 3D computation of the electric field is required because there is no symmetry in order to ignore one dimension. In addition, numerical technique must be capable of dealing with the narrow insulating or conducting layers.

FDM and FEM in the electric field computation have two drawbacks:

a) In determination of potential distribution using two methods, numerical derivative techniques must be used in order to obtain the electric field intensity. This has considerable error that leads to a large error in the electric field computation. In many cases, the field is required for the design of insulation of electrical equipment. Meanwhile, study of some phenomena such as electrical discharge is possible by electric field computation.

b) In order to prevent a large electric field and its drawbacks, the sharp edges on the different surfaces are avoided. Therefore, the curved surfaces are often preferred. FDM, FEM and FEMVFD have difficulty in modeling such curved surfaces; but CSM and BEM can be easily adapted for such cases. On the other hand, although FDM, FEM and FEMVFD can theoretically compute 3D fields, there are many problems in dealing with this matter. One serious problem is 3D mesh generation and its modification to approach the required accuracy. Generally manual calculation is cumbersome and time consuming and also computer programming is really complicated. Another difficulty is the large size of the coefficient matrix of the system of equations. In these methods number of equations is proportional to the memory required for computation while in other methods (CSM and BEM) this number is proportional with area of the boundary surfaces. It means that in FDM, FEM and FEMVFD, the coefficient matrix has one more dimension than the coefficient matrix due to the other above-mentioned methods. Therefore, more computer memory and longer computation time are required. Hence FDM, FEM and FEMVFD may not be considered convenient techniques for electric field computation.

MCM, at least in electric field computation, is not so common and has no considerable progress in recent years. At the present, application of this method in narrow layer problems is difficult, if not impossible.

In spite of the simplicity of computer programming and high accuracy of the method, in computation of 3D electric fields having narrow layers CSM is confronted with a major difficulty. In this method it is necessary to consider charges within the mentioned layers such that they have enough distance from two sides of the layer. But the thickness is too narrow and such an assumption may not be correct.

Finally BEM is capable of analyzing 3D fields and there are some reports showing its applications to narrow layers. In conclusion, BEM may be considered the most convenient technique for electric field computation within the interior of the power transformer tank [23, 24].

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