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A Pole Balancing Cart on an Unmodelled Terrain*

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A pole balancing cart comprises a wheeled or track-guided vehicle which can move under its own power in the fore-aft direction, and a pole pivoted at its lower end which can swing smoothly in the fore-aft plane like an inverted pendulum. The cart is able to balance the pole in a near upright position while it moves across an unmodelled terrain through sensing the inclination of the surfaces and noise compensation. The cart with an inverted pendulum is a classical example in the control of unstable systems. This paper describes the mechanical, electrical, and electronics design of such a pole balancing cart.

NOMENCLATURE

θ	Inclination of pole relative to cart (forward = positive)
α	Angle of pitch of cart (equal to angle of slope, dive = positive)
σ	Inclination of pole relative to true vertical (forward = positive), $\sigma = \theta + \alpha$
$\dot{\sigma}$	Derivative of σ
$\ddot{\sigma}$	Derivative of $\dot{\sigma}$
δ	Desired inclination of pole relative to position where $\sigma = 0$ (forward = positive)
X	Linear position of cart along runway surface (forward = positive)
$\dot{X}$	Derivative of X
F_L	Horizontal force on the cart (forward = positive)
M	Mass of the cart alone without the pole
l	Half length of the pole
V_s'	Controller output signal, equal to current amplifier input signal
K_p	Proportional gain for σ
K_v	Derivative gain for σ
K_{px}	Proportional gain for X
K_{vx}	Derivative gain for X
g	Acceleration due to gravity

INTRODUCTION

THE POLE BALANCING CART involves the balancing of a pole (also called an inverted pendulum) mounted on a vehicle, which is required to manoeuvre on the runway as shown in Fig. 1. The vehicle must remain within the region A (of unknown inclination) for 60 seconds, move across the region B at the shortest possible time, and then stay in region C for another 60 seconds. The pole should remain upright at all times.

The inverted pendulum has always been an interesting control theory problem for the academics. The pendulum is inherently unstable and has been used as a verification model for various control algorithms, such as the state space method [1], nonlinear control [2], artificial neural network [3], adaptive [4], fuzzy [5], and genetic algorithm [6]. Furthermore, the multivariable nature of the problem makes it a very suitable example to illustrate state space design techniques [7–8].

The pole balancing cart was designed and built by an undergraduate mechanical engineering student in Nanyang Technological University (NTU), Singapore as a final year project to fulfil the partial requirement for the degree of Bachelor of Engineering. The objective of this paper is to describe the mechanical, electrical and electronics design of the pole balancing cart. Figure 2 shows the actual prototype of the pole balancing cart.

MECHANICAL AND ELECTRICAL DESIGN

The pole balancing cart comprises a cart as shown in Fig. 3 that uses a small direct-current (DC) motor for propulsion. The motor drives all the four wheels through a gearbox and a discrete component drive train. The cart has an axle, which can rotate in the fore-aft plane. The cart uses a four-wheel-drive traction system. Other components on the cart include the circuit board, current amplifier, gyroscope, power battery and the pole axle. A single-ended 12-volt signal battery with an artificially created ground of 6 V is used to supply

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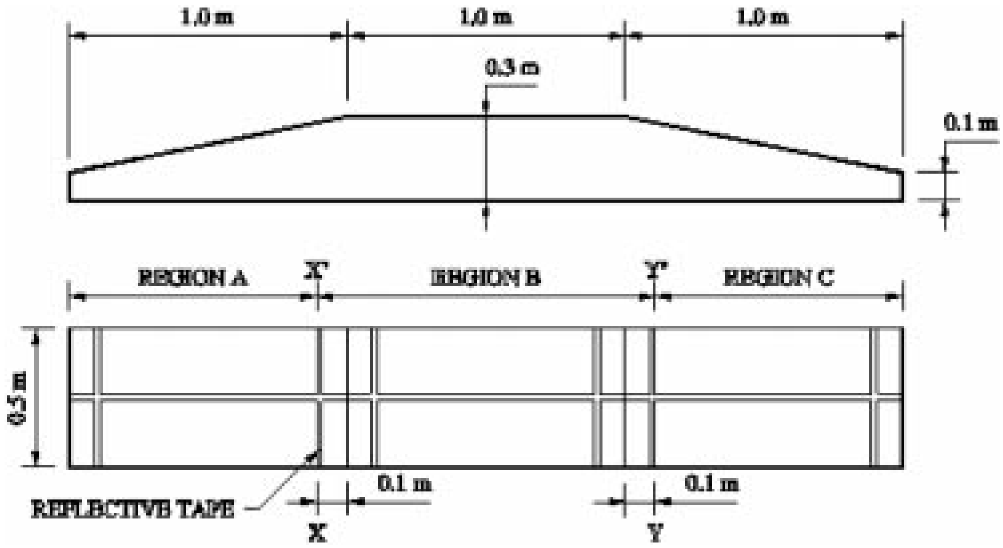

Fig. 1. The runway.

the signal. Both the analogue and digital circuits can share the same supply with this arrangement.

The selection of the motor is the first consideration in the design process. Since the analogue controller is to be used, it is decided to use a low voltage permanent magnet DC motor. The initial selection of the motor is based on its physical size, which gives a rough indication of the torque. The motor should also be easily available on the market.

The cart has to be self-powered, thus the use of an external power supply is not considered. The operating voltage and stall current of the selected motor is used to size up the power battery. The stall current of the motor must not exceed the maximum allowable current drain on the battery, and the voltage drop must not be excessive. The

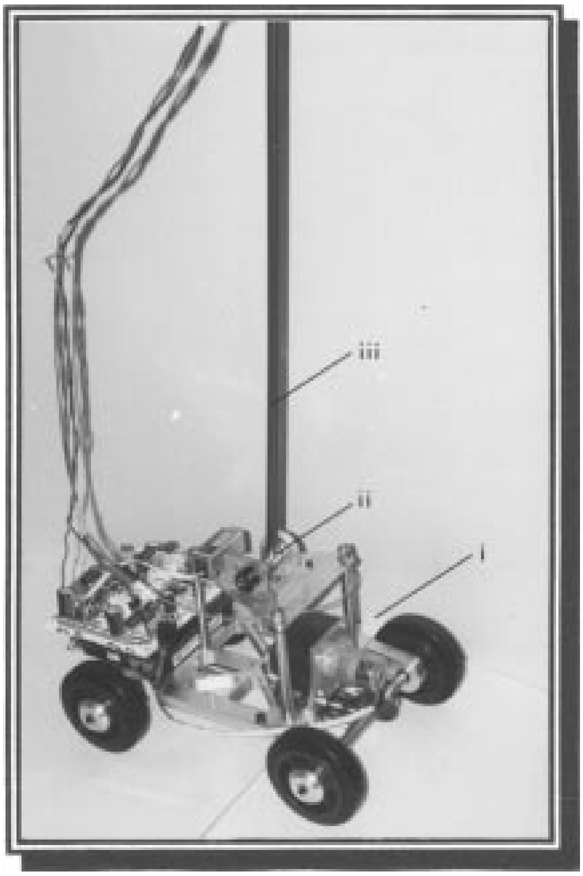

Fig. 2. The prototype of pole balancing cart.

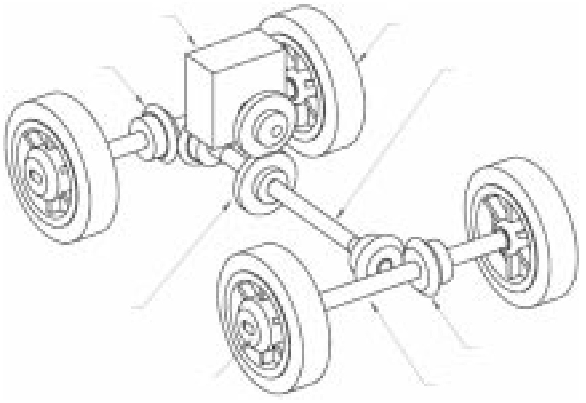

Fig. 3. The four-wheel-drive cart.

ampere-hourage must be adequate for consistent performance at the half peak current for at least 15 minutes in the competition. Two power batteries are used to power the cart. The initial power battery selected during the design phase has a peak current of 40 A. The voltage of the battery at 7 A is 12.3 V for 15 minutes. At 14 A, the voltage is 11.8 V. The batteries weigh about 2.5 kg each. During the experimental testing of the cart, it is found that the battery never exerts the stall current. The typical peak current is about 4 A, which is much below the peak current rating of the selected power battery. Therefore, another type of power battery (Model: Union MX12042) which has a lower peak current rating is used instead. The voltage of the Union battery at 2 A is 12.3 V for 15 minutes. At 4 A, the voltage is 11.8 V. The battery is lighter, weighing about 1.4 kg each. A signal battery (Model: CSB GP1212) is selected. The voltage of the signal battery at 0.72 A is 12.3 V for 15 minutes. The signal battery weighs about 0.56 kg.

The cart has to carry all the components mentioned earlier. The wheelbase and the diameter of the wheels are chosen to ensure that the body of the cart does not scrape against the runway while moving from the sloped region to the level region. One important consideration in choosing the size of the wheelbase and its wheels is the proper alignment during the manufacture of the cart. The use of four wheels requires a standard alignment such that all the four wheels are in contact with the ground. A four-wheel-drive traction system is selected as it gives the best directional consistency with maximum transmission of traction force for a given wheel coefficient of friction. A good traction system will ensure that the system maintains its course without falling off the runway.

The gearbox ratio is roughly estimated using force balance based on the weight of the cart, the diameter of the wheel, and the peak power torque. Assuming the coefficient of friction between the wheels and runway is 0.7 based on an estimate for rubber on asphalt [9], the gearbox ratio is obtained by multiplying the coefficient of friction with the weight of cart and the wheel radius, and then dividing by the peak power torque. The final drive torque is obtained by multiplying the gearbox ratio with the peak power torque. A service factor of 1.5 is used for the sizing of the gear train components. An appropriate gearbox and the components of the drive train are then selected.

A final mechanical design is selected after five iterative refinements. One area that needs refinement is the choice of wheel size. Initially, the required wheel size to clear the ramp-over angle is 75 mm. However, at a later stage in the design process, a gear is added near the centre of the chassis due to the following reason. It was found that the original designed speed of the cart was too high. This resulted in excessive current drain. To further improve the effective gain without introducing instability, a 25:1 gearbox was used in place of the 15:1 gearbox. This reduced the initial magnitude of to and fro oscillation during balancing from 6 cm to about 1 cm. In addition, the peak current drain was reduced to about 3.5 A. The gear protrudes below the original belly-line of the cart, thus reducing the ramp-over angle. At the next iteration of the design process, the wheel size has to be increased to 100 mm to ensure that the body of the cart does not scrape against the runway. In the final design, the cart size has a wheelbase of 250 mm, wheel diameter of 100 mm, a 200 mm track, and a weight of 6.5 kg. The final gear ratio selected (Model: Oriental 2GK15K) is 15:1 with an output torque rating of 2.5 Nm, stall linear force of 79 N, a 40 N force at peak power, and a speed of 2.1 m/s at peak power. The motor selected (Model: Mabuchi RS-555SH) has a stall current of 14.5 A and a stall torque of 2.7 kg-cm.

ELECTRONICS DESIGN

The balancing of the pole on the cart as it moves across the runway in Fig. 1 is controlled through a simple conventional proportional derivative (PD) controller. The control requires the input of the inclination of the pole and the linear position of the cart. The inclination of the pole is derived by adding the pitch of the cart, which is measured with a piezoelectric rate gyroscope, and the inclination of the pole relative to the cart's perpendicular, which is measured with a potentiometer.

These two sensors are the source of the noise generated. Theoretically, it is possible to control the inverted pole by feeding back the inclination of the pole alone to the controller. But in practice, due to the presence of the noise in the measurement of the inclination, the cart invariably drifts under such a control system. When the measured inclination is zero, it does not necessarily mean that the pole is at the true vertical. An attempt to take this error into consideration and to maintain the pole at a 'desired inclination' is obtained though the implementation of the control algorithms. The desired inclination is derived from the distance of the cart from its initial position and the velocity at which the cart drifts.

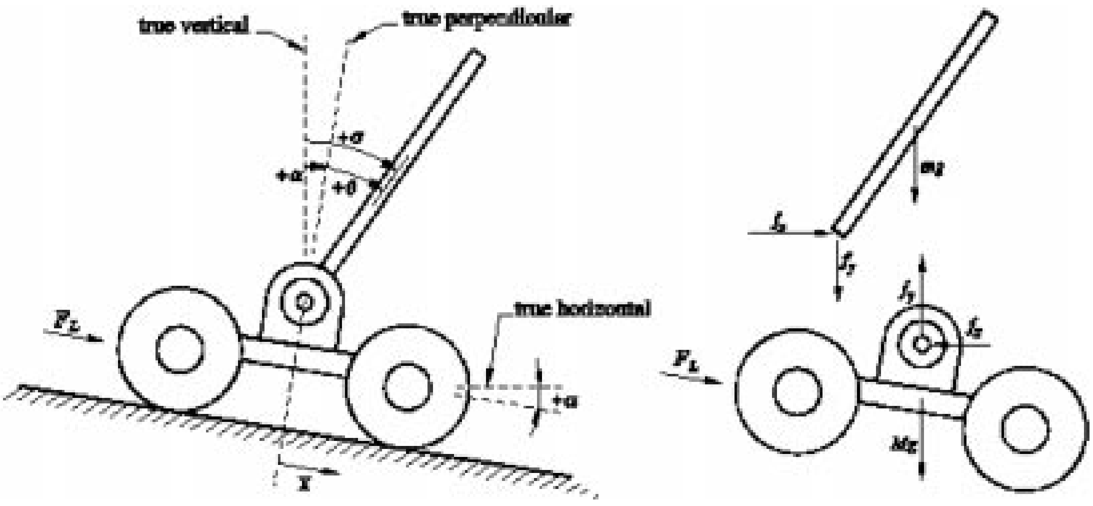

Fig. 4. The model of the cart.

Figure 4 shows a model of the cart. It can be easily proved that the equation of motion [10] is given by:

$$F_L + Mg\sin\alpha = \frac{Mg\sigma - \frac{4}{3}Ml\ddot{\sigma}}{\cos\alpha + \sigma\sin\alpha}$$

The basic pole balancing control is implemented with a PD controller. The feedback signals are as follow:

θ	Inclination of pole relative to cart (forward = positive)
α	Angle of pitch of cart (equal to angle of slope, dive = positive)
X	Linear position of cart along runway surface (forward = positive)

The inclination of the pole about the true vertical is derived as $\sigma = \theta + \alpha$ (forward = positive). The cart is required to transit over a stipulated distance.

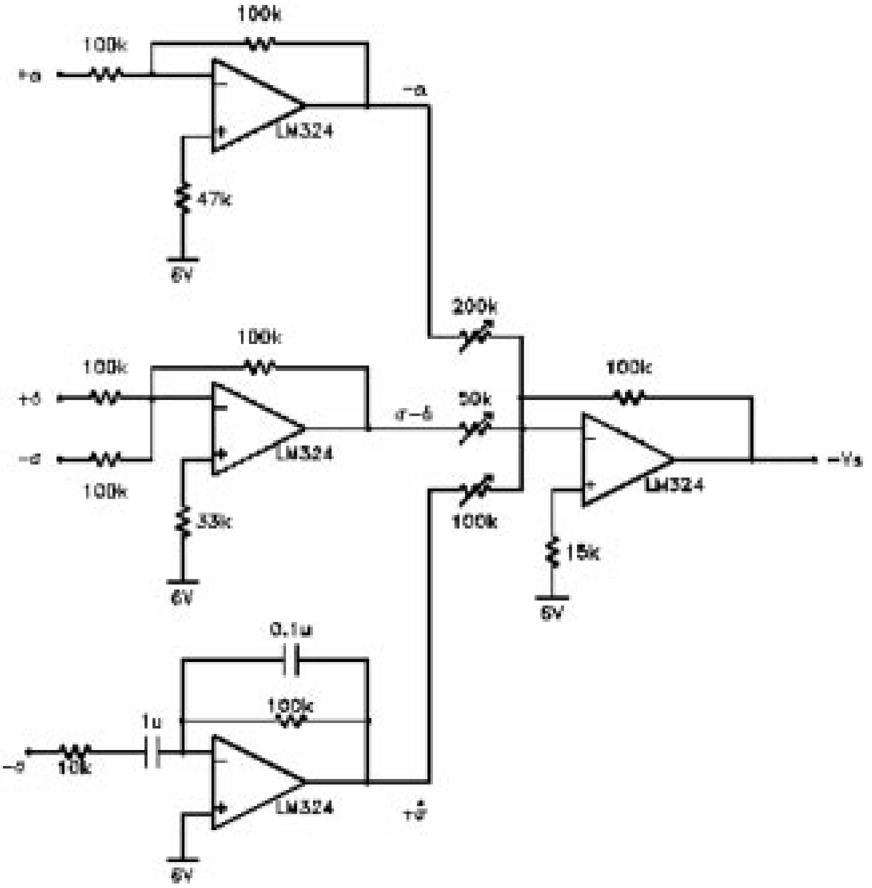

Fig. 5. PD circuit with slope compensation, inverting amplifier, inverting summer and differentiator

Hence, the control objective is not to regulate the pole such that it is always equal to zero, but rather at a desired inclination. The PD controller generates an output voltage signal V_s to be fed to the cart motor through a current amplifier, where:

$$V_s = K_v \dot{\sigma} + K_p(\sigma - \delta)$$

and

$$\delta = K_{px} X + K_{vx} \dot{X}$$

In addition to the PD signal, the same circuit also adds the slope compensation signal to overcome the cart's weight when it is on a slope. The PD circuit with the slope compensation is shown in Fig. 5. The controller receives three inputs, i.e. the inverting signal α, δ and -σ, and the differentiator for σ, to produce corrective output signal V_s to the cart's motor. The differentiator has an integral cut-off frequency above 15.9 Hz. The value of K_p is set to approximately 5 K_v whose gain is set by the combined gain of the differentiator (gain = 0.1) and PD circuit (gain = 2.5) is approximately 0.25. These values are obtained experimentally.

The σ input to summer and the differentiator is obtained by summing α and θ. Mathematically, the σ is given by $\sigma = K_1\theta + K_2\alpha$. The value of K_1 and K_2 are 4.5 and 0.8, respectively. The θ is measured by a potentiometer and the pitch angle α is measured by a piezoelectric rate gyroscope (Model: Murata GYROSTAR). The gyroscope produces an output proportional to its angular velocity. An integrator is used to convert the angular velocity signal to the angular displacement. The gyroscope is very sensitive to environmental disturbances such as temperature and presence of earthed metallic objects near it. It is also the source of the noise generated. A large part of the noise comes from the 8 kHz oscillator driving the piezoelectric element. An operational amplifier buffer and a capacitor are used to condition the signal. A threshold detector is added to the circuit to eliminate the random bias signal, usually in the order of mV. The incremental X and its derivative $\dot{X}$ are summed with appropriate gain to generate δ. The gain values are determined experimentally, whereas the X value is measured by a multi-turn potentiometer. The potentiometer is an absolute position sensor. For this application, a means to convert the signal to one that is referred about any chosen $X = 0$ position, regardless of its actual output value, has to be provided. The principle being employed is that on command, the current value of the potentiometer is stored. This is called the reference X. A difference amplifier then subtracts this reference X from all subsequent signal output form the potentiometer. This results in the incremental X value which is zero at the point where the 'store' command is given.

The controller's signal voltage V_s' consists of the theoretically required corrective signal voltage V_s, and a friction compensation signal V_f added to generate the torque to overcome the friction of the drive train. The sign of the friction compensation signal is always the same as the corrective signal voltage V_s.

The friction compensation circuit is shown in Fig. 6. Mathematically:

$$V_s' = V_s + \frac{V_s}{|V_s|} \times V_f$$

Furthermore, the friction under high torque (accelerating uphill) is higher than that under low torque (accelerating downhill) as shown in Fig. 7. This is due to the higher reaction of the bushes and bearings on the shafts during high torque.

Mathematically, the controller's signal voltage V_s' is given by:

$$V_s' = V_s + \frac{V_s}{|V_s|} \times V_f - K_f \alpha$$

The magnitude of V_f is set at approximately 1.2 V while K_f is set at approximately 0.3. These values are arrived at experimentally. The controller's signal voltage V_s' needs to be converted to a proportionate motor current I_s to drive the wheel

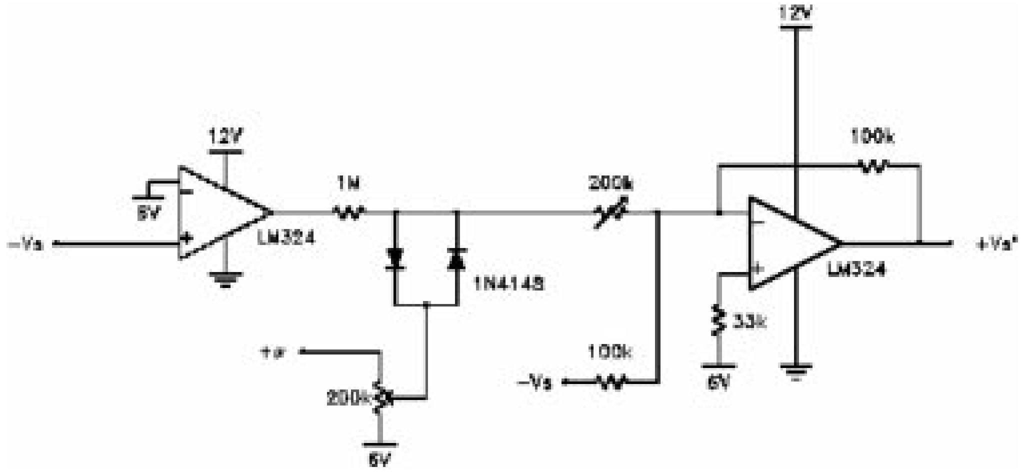

Fig. 6. Friction compensation circuit.

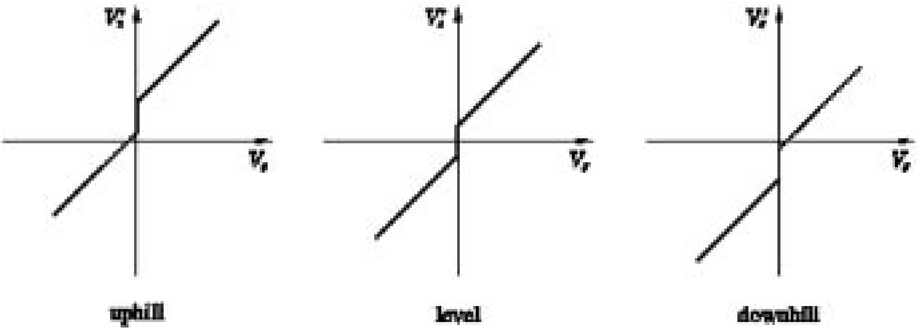

Fig. 7. Slope-dependant friction compensation of V_s'.

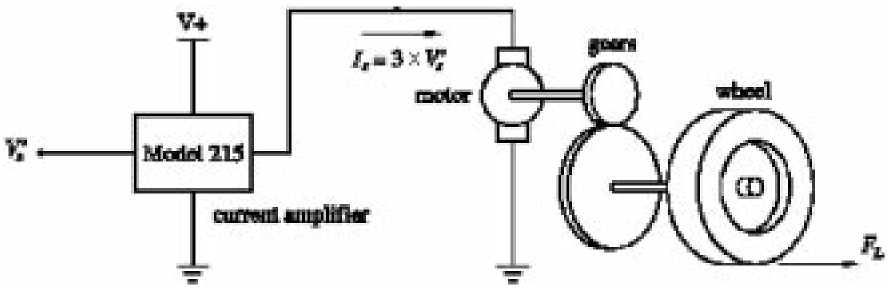

Fig. 8. Conversion of controller signal voltage to motor current.

as shown in Fig. 8. An amplifier (Copley Controls Model 215) with a gain of 3 A/V is selected. The amplifier has a peak current capability of 20 A and a working voltage range of 12 to 80 V. The current capability exceeds the motor's stall current of 14.5 A and the voltage range is suitable for the motor operation at 24 V.

CONCLUSION

A prototype of the *pole balancing cart* has been designed and built by an undergraduate engineering student within five months. Both the mechanical and electronics aspect have been carefully considered by the student in the design phase. A four-wheel-drive cart is selected due to its maximum traction and good directional consistency. The cart is self-powered by two batteries and carries a pole pivoted on the cart. The pole is able to balance itself in a near upright position as it moves across a runway with level and unknown inclination through a simple proportional-derivative controller that takes into account the presence of noise in the measurement of the inclination. The different aspects of the control, the pole balancing, the fast transit, the cart's weight and friction, and the sensor noise problems are carefully taken into consideration in the design of the electronics control circuits. The *pole balancing cart* is effective in engaging students in the mechatronics discipline to solve practical control problems.

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