Cables Under Concentrated Loads: A Laboratory Project for an Engineering Mechanics Course*

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Cables are one of the common structures studied in a first-year engineering mechanics course (statics), since the flexible cable is one of the usual methods of supporting loads. For example, the suspension bridge has been used for many centuries and is perhaps the best example of the use of cables in engineering. In this paper, we describe a simple laboratory experiment, appropriate for undergraduate students, to analyze a cable under the action of a system of concentrated external forces. The shape of the cable is measured using graduated rules. The resultant of the system of applied forces and its line of action, reactions at supports and tensions in the segments of the cable are obtained using three different procedures—experimental, graphical and analytical—with good agreement being found between them all.

SUMMARY OF THE EDUCATIONAL ASPECTS OF THIS PAPER

- 1. This paper proposes to verify the equilibrium of a cable under the action of a finite number of concentrated forces by means of the analysis of a simple laboratory experiment.
- 2. The experimental set-up is composed of very simple elements and only easy experimental measurements—lengths and masses—need be made. The relations between the length, tensions in the different segments of the cable, reactions at the supports and applied loads are analyzed.
- 3. The experimental analysis of the cable is completed and compared with graphical and analytical studies, that allow students of statics to understand the relation between theory and the actual physical behavior of mechanical systems, because understanding both aspects of mechanics are essential.
- 4. The system analyzed is an interesting example to understand the equilibrium of parallel an coplanar force systems, in which all of the lines of action of the forces are parallel, they lie in the same plane, while the vectors representing moments are normal to this plane.
- 5. The laboratory project may be integrated into an introductory engineering mechanics course by considering both laboratory sessions as formal lectures.

6. The experiment described in this paper provides students with not only an understanding of the equilibrium of a cable but also a better understanding of the basic cencepts of statics such as equilibrium equations, free-body diagrams, reaction at supports, resultant of a system of forces and its line of action, and tensions in the segments of the cable.

INTRODUCTION

THE STUDY of the statics of cables can be found in most undergraduate textbooks on mechanics, together with the different topics included in the subjects of physics and mechanics for engineering and architecture students [1–5]. Nevertheless, less importance is given to this topic, since it appears at the end of the syllabus and is generally replaced by the study of structural elements of more common use such as trusses or beams. In addition, the topics dedicated to the study of the statics of cables are rarely dealt with when there is not enough time to cover the whole syllabus. In spite of this, the statics of cables presents some didactic advantages over that of the other structural elements mentioned above. It includes-as in the case of trusses and beams-concepts such as concentrated and distributed loads, moments, support reactions and internal efforts [1]. In addition, it presents the didactic advantage that the concepts can be visualized in the laboratory by

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Fig. 1. Photograph of the experimental set-up analyzed.

means of low cost, easy-to-assemble experiments using simple materials.

Due to a unique combination of resistance, low weight and flexibility, cables are usually used to support loads and transmit forces in building structures (bridges, struts, etc.) or for power transmission in machines and vehicles (chains, belts, etc). Cables are also used to transmit electricity through the power grid and information through the telephone network. In the latter two cases, the only load supported by the cable is its own weight and the shape that the cable adopts is known as catenary [6].

In this paper we present a laboratory project

based on the analysis of an easy-to-assemble, low cost, laboratory experiment to study experimentally the equilibrium of a cable under the action of a finite number of vertical, parallel, concentrated, external forces. We consider that the cable is homogenous, flexible, non-extendible and of negligible weight. In a simple way, the shape of the loaded cable and the reactions at the supports are experimentally measured. The relations between the length, tension in the different segments of the cable, reactions at the supports and applied loads are analyzed. The experimental analysis of the cable is completed and compared with graphical and analytical studies.



Fig. 2. Definition of the parameters of the system.



Fig. 3. Measurement of the horizontal and vertical distances of the cable.

From Newton's laws, if the system is in equilibrium the sum of the external forces vanishes, and the sum of their moments about any point also vanishes. These equilibrium laws must be considered in our analysis.

EXPERIMENTAL SETUP

Figure 1 shows a photograph of the experimental set-up analyzed. In this figure, the shape of the cable under the action of three concentrated external loads can be seen. In order to assemble the experimental setup, a cable (such as a twisted polyamide line used, for example, in a physics laboratory in the mathematical pendulum experiment) is fixed at its ends to two vertical rods by means of right-angled clamps. We considered the particular case in which the support points of the cable lie on the same horizontal level. The generalization to the situation in which the support points are at different levels is immediate. The cable supports three vertical loads acting at different points at which weights of 120, 120 and 160 g are hung. The absolute error of the masses is 0.2 g.

Figure 2 shows the different parameters which serve to characterize the cable in equilibrium: L = 140 cm is the length of the cable, a = 120 cmis the horizontal distance between supports A and B (known as span), $W_1 = 1.176 \text{ N}$, $W_2 = 1.176 \text{ N}$ and $W_3 = 1.568 \text{ N}$ are the vertical loads applied at points P_1 , P_2 and P_3 of the cable, respectively, and $L_1 = 40 \text{ cm}$, $L_2 = 40 \text{ cm}$, $L_3 = 30 \text{ cm}$ and $L_4 = 30 \text{ cm}$ are the lengths of the segments of the cable. The absolute errors of the lengths and weights are 0.1 cm and 0.002 N, respectively. The shape the cable adopts in equilibrium, supported at its ends and subjected to a set of punctual loads at different intermediate points, is called a 'funicular polygon' [2]. Once the cable is in equilibrium, it is a simple matter to obtain the 'funicular polygon' experimentally. The distances x_1 , x_2 and x_3 , and the sags y_1 , y_2 and y_3 , at the load points are measured with the aid of horizontal and vertical rules, as can be seen in Fig. 3. With these data the angles θ_1 , θ_2 , θ_3 and θ_4 , which the different segments of the cable form with the horizontal line, can be easily calculated using the following equations:

$$\tan \theta_1 = \frac{y_1}{x_1} \tag{1}$$

$$\tan \theta_2 = \frac{y_2 - y_1}{x_2}$$
(2)

$$\tan \theta_3 = \frac{y_2 - y_3}{x_3}$$
(3)

$$\tan \theta_4 = \frac{y_3}{a - x_1 - x_2 - x_3} \tag{4}$$

Table 1 summarizes the values of the measured and calculated parameters that characterize the cable in equilibrium under the action of the external loads W_1 , W_2 and W_3 .

EXPERIMENTAL ANALYSIS

Measurement of the reactions at supports

It is possible to experimentally measure the modulus R_A and R_B of the reactions at the supports. To do this, we detach one of the ends of the cable and tie it to a pan (Fig. 4) previously weighed on a balance. On the vertical bar, we put a small pulley around which the cable is passed (point *B* of Fig. 4). Next, weights are successively put on the pan till the segment P_4B reaches its original length L_4 . To do this, it is only necessary to make a little mark on the cable in order to check that the mark stays just at the top of the pulley.

Table 1. Experimental measurement results of the horizontal distances x_1 , x_2 and x_3 , and vertical distances y_1 , y_2 and y_3 , and calculated values of the angles θ_1 , θ_2 , θ_3 and θ_4 .

$x_1 = 32.0 \pm 0.1$ cm $x_2 = 39.2 \pm 0.1$ cm $x_3 = 28.4 \pm 0.1$ cm
$y_1 = 24.0 \pm 0.1$ cm $y_2 = 32.0 \pm 0.1$ cm $y_3 = 22.0 \pm 0.1$ cm
$\begin{array}{l} \theta_1 = 36.87^\circ \pm 0.18^\circ \\ \theta_2 = 11.53^\circ \pm 0.18^\circ \\ \theta_3 = 19.40^\circ \pm 0.18^\circ \\ \theta_4 = 47.16^\circ \pm 0.18^\circ \end{array}$

The value of R_B will be the weight of the pan together with the masses on it. The horizontal B_x and vertical B_y components of the reaction R_B at support *B* can be easily obtained using the value of θ_4 initially calculated.

The experimental measurements at the supports *A* and *B* were:

$$R_A = 2.67 \pm 0.05 \text{ N}$$

 $R_B = 3.18 \pm 0.05 \text{ N}$

Determination of the tensions in the segments of the cable

Once the values of R_A and R_B are known, the tensions in the segments of the cable may be easily calculated in a similar way to that described under Analytical resolution, taking as the initial data the values of the loads applied, the angles calculated and the reactions measured at the supports. In order to calculate the tension T_4 , we consider point B in Fig. 5 and apply the equilibrium equation $\Sigma F = 0$. We then consider point P_3 in the same figure, and so on.

The calculated values of the tensions were:

$$T_1 = 2.67 \pm 0.05 \text{ N}$$
$$T_2 = 2.18 \pm 0.05 \text{ N}$$
$$T_3 = 2.26 \pm 0.05 \text{ N}$$
$$T_4 = 3.14 \pm 0.05 \text{ N}$$

GRAPHICAL ANALYSIS

Culmann pointed out the importance of graphical methods for the analysis of structures in engineering [7]. Although the construction of the funicular polygon and forces polygon was known in Varignon's time (18th century) [8], it was Culmann who performed a systematic introduction to the use of graphical methods in the resolution of static problems [9], in particular, in the analysis of several types of structures. He was, in fact, the first to publish a book on graphical statics, in which he included many original graphical solutions [10].

In the case of the cable we are analyzing, the funicular polygon $AP_1P_2P_3B$ of the cable in equilibrium can be obtained from the experimental study (Fig. 2). It is also possible to determine graphically the reactions $R_A = (A_x, A_y)$ and $R_B = (B_x, B_y)$ at supports A and B, the resultant R and the position of its line of action (the central axis, r, of the system of co-planar and parallel external forces applied), and also the tensions T_1 , T_2 , T_3 and T_4 in the segments of the cable. In this way, information about the equilibrium of the cable may be obtained from the experimental measurements at the supports A and B, and at points P_1 , P_2 and P_3 .

Resultant R and its line of action

To find the single-force resultant R of the system of parallel forces W_1 , W_2 and W_3 acting on the cable, the forces polygon is obtained from the funicular polygon [2]. To do this, we draw the funicular polygon with scaled relative distances, together with the scaled applied forces W_1 , W_2 and W_3 , their lines of action passing through points P_1 , P_2 and P_3 of the cable respectively (see Fig. 6). Through a point M, an equipollent force to W_1 , MN, is traced. From point N an equipollent force to W_2 , NP, and from P an equipollent force to W_3 , PQ, are traced. The vector MQ, with its origin at point M and end at point Q, will be the resultant R of the system of forces applied. As the forces have



Fig. 4. Experimental determination of the reactions at the supports.



Fig. 5. Relation between the reactions at the supports and tensions in the cable.

been drawn using a scaling factor, the modulus of the resultant R may be obtained by simply measuring the distance MQ.

In order to find the line of action r of the resultant R and, consequently, the position of the central axis of the system of forces, we trace a line parallel to the segment AP_1 of the cable from the point M; a line parallel to the segment P_1P_2 of the cable from the point N; a line parallel to the segment P_2P_3 from the point P, and from the point Q a line parallel to the segment P_3B (see Fig. 6). All

these lines will intersect at the same point O, known as the 'pole' [2]. With the aid of vectors OM, ON, OP and OQ, which have as their origin the point O, it may be easily determined that:

$$W_1 = ON - OM$$
$$W_2 = OP - ON$$
$$W_3 = OQ - OP$$

From the funicular polygon (Fig. 6), it may be easily verified that the force W_1 is equivalent to the



Fig. 6. Graphical determination of the resultant, its line of action and the reactions at the supports.

concurrent forces ON and -OM in the directions of AP_1 and P_1P_2 ; the force W_2 is equivalent to the concurrent forces OP and -ON in the directions P_1P_2 and P_2P_3 ; and the force W_3 is equivalent to the concurrent forces OQ and -OP in the directions P_2P_3 and P_3B .

In segment P_1P_2 the forces ON and -ON are equal and opposite and so cancel each other out. The same occurs in segment P_2P_3 with forces OP and -OP. However, force -OM in segment AP_1 and force OQ in segment P_3B do not cancel each other out:

$$R = W_1 + W_2 + W_3 = OQ - OM = MQ$$

These two forces, -OM and OQ, are concurrent and they are equivalent to the resultant R passing through the point E (see Fig. 6). This point is the intersection of the extensions of segments AP_1 and P_3B . The straight line r parallel to the resultant R, traced through the point E, is the central axis of the system of forces (line of action of the resultant) [2]. Because the system is composed of parallel forces, the resultant is the algebraic sum of the three loads applied. It therefore corresponds to an applied mass of 400 g and so the resultant modulus is R = 3.92 N.

The graphical study was carried out by hand using a sheet. The drawings were done with the aid of two setsquares and a graduated rule. The distances were represented using a scale of 4 cm to 1 cm. For the sake of simplicity, when drawing the forces we considered their value expressed in grams instead of Newtons, taking a scale of 1 cm for each 40 g. Once the different reactions and tensions are obtained graphically, the centimeters are converted into grams, then transformed into kilograms and finally multiplied by $g = 9.8 \text{ m/s}^2$ in order to obtain the result in Newtons. As the smallest divisions on the rule used are of 1 mm, with the above scales the sensitivity of the distances measured on paper will be of 0.4 cm and that of the masses 4 g, which results in a sensitivity of 0.04 N for the measurements of the forces. Obviously, the sensitivity can be increased by using a larger sheet of paper and reducing the scale. Figure 7 represents a diagram of what was obtained graphically on paper.

Vertical components of the reactions at supports

In order to find the vertical components of the reactions, we are going to equilibrate the system of vertical forces W_1 , W_2 and W_3 by means of two forces A_y and B_y , which are also vertical and consequently parallel to the resultant, that must pass through points *A* and *B*. To do this, a parallel line to the segment *AB* is traced passing through the point *O* (see Fig. 6). This line intersects the resultant R at point *S* yielding two forces SM and QS which correspond to the vertical reactions at the supports A_y and B_y , respectively [2]. Because the forces were drawn using a scale, it is possible to measure the values of A_y and B_y using a rule. From Fig. 7, using the above scale and multiplying by $g = 9.8 \text{ m/s}^2$, the following values were obtained:

$$A_y = 1.61 \pm 0.04 \text{ N}$$

 $B_y = 2.31 \pm 0.04 \text{ N}$

Equivalent of the system of forces

Since the funicular polygon was drawn using a scale for distances, it is possible to measure the distance x_A between the vertical line containing the support A and the line of action r of the resultant, as can be seen in Fig. 6. The value obtained, taking into account the scale for distances, was $x_A = 70.8 \pm 0.4$ cm. Next, in the experimental setup, the three vertical loads W_1 , W_2 and W_3 were substituted by the resultant R = 3.92 N set at a distance x_A , so the experimental funicular polygon seen in Fig. 7 was obtained.



Fig. 7. Photograph of the cable under the action of the resultant of the system.



Fig. 8. Graphical determination of the tensions in the different segments of the cable.

Tensions in the segments of the cable

In order to find the tension in the different segments of the cable graphically, we again use the funicular polygon. The forces we have at the moment are the applied loads W_1 , W_2 and W_3 , and the vertical components A_v and B_v of the reactions at supports A and B, and all of them are drawn, using the appropriate scaling factor, on the funicular polygon. In the beginning, for instance, at support A (see Fig. 8), it is easy to find the value of the reaction R_A at point A as its horizontal component A_x , by simply extending the segment P_1A .

The modulus of R_A will be the same as that of the tension T_1 . Once the tension T_1 is known, and using W_1 , we can obtain graphically the tension T_2 at point P_1 , and so on. It is easy to see that the horizontal components of all the tensions in the segments are the same and it can be easily shown that the following relation holds: $A_x = B_x$. Figure 8 shows the diagram of the results of the tensions obtained graphically. In this figure, the scale defined in section 4.1 was used for the distances and for the forces (loads, reactions and tensions), and the final results for the horizontal components of the reactions at the supports are:

$$A_x = 2.16 \pm 0.04 \text{ N}$$

 $B_x = 2.16 \pm 0.04 \text{ N}$

and for the tensions in the different segments of the cable:

 $T_1 = 2.70 \pm 0.04 \text{ N}$ $T_2 = 2.20 \pm 0.04 \text{ N}$ $T_3 = 2.31 \pm 0.04 \text{ N}$ $T_4 = 3.16 \pm 0.04 \text{ N}$

ANALYTICAL RESOLUTION

It is possible to study the cable in equilibrium by solving the problem analytically, starting from a series of experimental measurements. To do this, we use the equilibrium equations:

$$\Sigma F = 0 \tag{5}$$

$$\Sigma \mathbf{M}_P = 0 \tag{6}$$



Fig. 9. Diagram of the cable analyzed.



Fig. 10. Free solid diagrams for the support points A and B and the points P_1 , P_2 and P_3 where the external forces are applied.

ANALYTICAL SOLUTION From a free-body diagram for the cable (Figure 9): $+ \sum M_g = W_1(a-x_1) + W_2(a-x_1-x_2) + W_3(a-x_1-x_2-x_3) - A_ya = 0 \quad \Rightarrow \quad A_y = 1.607 N \uparrow 10^{-1} N \downarrow 0^{-1} N \downarrow 0^{-$ + $\sum M_A = -W_1x_1 - W_2(x_1 + x_2) - W_3(x_1 + x_2 + x_3) + B_p a = 0 \implies B_p = 2.313 N^{+}$ The horizontal components of the reactions at supports A and B can be easily obtained into account the geometrical relations: $A_x = \frac{A_y}{\tan \theta_1} = A_y \frac{x_1}{y_1} = 2.143 N \quad \leftarrow \qquad \qquad B_x = \frac{B_y}{\tan \theta_x} = B_y \frac{a - x_1 - x_2 - x_3}{y_3} = 2.145 N \quad \rightarrow \quad$ The reactions R_A and R_B at supports A and B are: $R_{d} = \sqrt{A_{x}^{2} + A_{y}^{2}} = 2.679 N$ $R_{B} = \sqrt{B_{x}^{2} + B_{y}^{2}} = 3.155 N$ Now the tensions in the four segments of the cable may be easily obtaineds. First we note that the horizontal components of all of the tensions are the same: $A_x = T_1 \cos \theta_1 = T_2 \cos \theta_2 = T_1 \cos \theta_1 = T_4 \cos \theta_4 = B_x$ In order to calculate T1 we fix on a point A of Figure 10: $A_y = T_1 \sin \theta_1 \implies T_1 = \frac{A_y}{\sin \theta_1} = 2.678 N$ Next we consider point P1 in Figure 10: $T_1 \sin \theta_1 = W_1 + T_2 \sin \theta_2 \quad \Rightarrow \quad T_2 = \frac{T_1 \sin \theta_1 - W_1}{\sin \theta_2} = 2.155 N$ And we carry on successively calculating the tensions T3 and T4: $T_1 = 2.244 N$ $T_4 = 3.153 N$ The resultant R consists of asingle vertical force computed as: $R = W_1 + W_2 + W_3 = 3.920 N \downarrow$ To obtain the line of action of the resultant R we take into account (Figure 6):

 $+ \sum M_A = -Rx_A + B_y a = 0 \implies x_A = \frac{B_y a}{R} = 70.8 \text{ cm}$ and from Figure 7: $L_A^2 - x_A^2 = (L - L_A)^2 - (a - x_A)^2 \implies 2LL_A = L^2 - a^2 + 2ax_A$ so finally: $L_A = \frac{L^2 - a^2 + 2ax_A}{2L} = 79.3 \text{ cm} \qquad L_B = L - L_A = 60.7 \text{ cm}$

Fig. 11. Summary of the analytical solution of the problem in steps in the manner in which a student might write a homework problem.

where P denotes the point with respect to which the moments are calculated. The starting data will be the experimentally measured horizontal distances x_1 , x_2 and x_3 and the vertical distances y_1 , y_2 and y_3 (Table 1), which allow us to obtain the angles θ_1 , θ_2 , θ_3 and θ_4 formed by the different segments of the cable with the horizontal line. Firstly, we are going to obtain the vertical components of the reactions at supports A and B. Figure 9 shows a free-body diagram of the cable, while Fig. 10 shows free-body diagrams for support points A and B and for points P_1 , P_2 and P_{3} . In Fig. 11 we have summarized the analytical solution of the problem in steps in the manner in which a student might write a homework problem.

To conclude, it may be mentioned that the analytical resolution of the system, taking L_1 , L_2 , L_3 , L_4 , a, W_1 , W_2 and W_3 as the data, poses a more complex problem. In this case we have an extremely difficult set of equations to solve. The equations obtained are very difficult to solve because of the non-algebraic, trigonometric functions that appear [4]. The solution is, therefore, very difficult if the calculus is done manually. Therefore, in order to solve the problem of the cable using this formulation, the use of a computer is recommended.

CONCLUSIONS

The laboratory project described in this paper provides students with a better understanding of the basic concepts in engineering mechanics: statics. The use of a simple cable, on which a series of weights were hung, has allowed the experimental study of a cable under the action of a system of punctual forces. The problem has been analyzed by three different methods: experimental, graphical and analytical. In this way, the students acquired an ample perspective of the problem analyzed. We have shown that there is good agreement between experimental, graphical and analytical results. The laboratory project may be integrated into an introductory engineering mechanics course by considering both laboratory sessions as formal lectures. Students can verify findings of the experiments by hand and this reinforces the importance of the physical fundamentals of the problem. In the three different approaches to the problem there are important concepts of statics such as force, moment of a force, reaction at a support, resultant of the system of forces and tension. It is evident that the experiments could be generalized to a situation in which the points of support are not at the same height. The method of measuring points directly on the cable with different weights hung on it, as shown in this paper, can be used to explore other cases of equilibrium. For example, the same scheme can be applied to study the equilibrium of a cable under the action of its own weight and to measure the catenary [11]. Finally, it is important to point out that this is a simple, inexpensive, easyto-assemble experiment that enables us to experimentally study the statics of cables by means of a series of simple measurements such as lengths and masses.

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