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Inverted Pendulum Driven by Pneumatics*

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Teaching of the course mechatronics for undergraduate students at the Faculty of Mechanical Engineering and Naval Architecture on the University of Zagreb is described in the paper. Mechatronics has been taught through projects that students should complete during the semester. One of those projects, the inverted pendulum driven by pneumatics, is described in this paper.

INTRODUCTION

ALTHOUGH the 'mechatronic way' of analysis and design of systems has been presented in education and scientific research on the University of Zagreb for a long time, not until the last few years has mechatronics become popular and a well known term, both at the academic level and within the broader society. However, the real meaning of mechatronics, as an engineering methodology or even philosophy, usually is far removed from normal understanding. Of particular importance is to fully commit to it is for mechanical engineers, since mechatronics could be a career of the future for them, as has already been pointed out in [1]. Mechatronics as an elective course at the final semester of undergraduate studies at the Faculty of Mechanical Engineering and Naval Architecture on the University of Zagreb was introduced in 1999. Also, it has been an elective course at the fourth semester of postgraduate study of Robotics and Automation at the same Faculty for the past two years. Both courses have 30 hours of lectures and 15 hours of exercises ($2+1$ per week).

In making the concept of the new course Mechatronics, some ideas were being followed:

- throughout the course students should make a product that is obtained following mechatronics methods;
- students have to cooperate and make a project as a group work;
- respecting the limiting resources, the product should be affordable.

Until now several balancing mechanisms, all of them driven by pneumatics, have been selected as student projects in mechatronics. The first project/mechanism was an inverted pendulum that is described in this paper, than followed a kind of a swing, and a ball and beam mechanism (not completed so far). The reasons for the choice of aforesaid laboratory devices was in their acceptable complexity—modelling, control and hardware skills are necessary in order to complete the project, while at the same time they can be realised relatively simply and they are not expensive (considering the existing laboratory infrastructure). Notable is that the physics of balancing mechanisms is intuitively clear and their motion is very attractive as well. However, balancing mechanisms in control and mechatronic education are usual, and what distinguishes described mechanisms from common practice is their drive. They are driven by pneumatics. One reason for introducing pneumatics to drive balancing mechanisms is an already existing laboratory setup with different types of pneumatic proportional and on-off valves. For each mechanism it is just necessary to design and add some mechanical part including pneumatic actuator (linear or rotational cylinder), and add sensors, connect a cylinder and existing valve(s) with tubes and sensors, as well as to design and implement a control part for each mechanism, taking due note of mechanism parameters and control design goals. That can be economical because the elements that are usually the most expensive, the pneumatic valve and its controller, can be shared between several mechanisms. Moreover, pneumatics by itself is a very interesting task in modelling and control due to air compressibility and significant friction effects in cylinders, with a challenge to make models with pneumatic drives as part of common practice, although other drive possibilities were considered during the course. The extensive description of the laboratory setup and conducted research on pneumatics modelling and control is given in [2] and partly in [3].

TEACHING OF MECHATRONICS

Mechatronics consists of the synergistic use of mechanical, electronic and control systems and computers. It is not possible to learn thoroughly all about aforementioned disciplines in a short course. It is not even necessary since the students learn about them in many different courses during the study. It is necessary to learn the complete

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approach, though, emphasising synergy, integration through design and balance between theory and practice (implementation skill), as that is already stressed in [4]. Briefly, the course Mechatronics consists of the following topics:

- As an introduction students are asked to do their own research on the Internet and through articles about mechatronic definitions, ideas and products.
- Then the actual mechatronic laboratory model that students have to complete during the semester is considered.
- Design variations, parameters of moving parts, actuator and sensor possibilities, their characteristics and possible mounting places.
- Then control implications and implementation, affordability, and other relevant issues are thoroughly discussed.
- Derivation of a mathematical description of the model's dynamics follows, and that is a basis for analytical and simulation analysis. The mathematical model's significance is particularly emphasised.
- The modelling phase includes writing of a program for simulation of a nonlinear mathematical model. Some of subsystems or influences (like saturations, backlashes, or hysteresis) are included in the simulation program avoiding formal mathematical description.
- The linearisation of the model is made, so a brief analysis and a control design is done using linear model. The influence of variation of different design parameters (like geometry or mass variation, or possible sensor or actuator placement) is shown, observing positions of poles and zeroes and time responses.
- The controller design, considering model's design, actuator and sensor choice, and hardware possibilities, is an important topic.
- A final design of the model is accepted after close-loop simulation analysis.

The work is done in the workshop of the Faculty. Experimentations are certainly the most exciting part. Usually additional changes of some parameters are necessary, in order to improve behaviour of the laboratory model. Final discussion and conclusions are mostly about new ideas, or possible changes and improvements of the system that are surveyed during the experiments. However, that is left to students for their individual projects at the final part of their studying.

DESCRIPTION OF AN INVERTED PENDULUM

The photography of the laboratory model of an inverted pendulum is given in Fig. 1, and the schematic diagram of the model is given in Fig. 2. The actuator is a rodless cylinder by SMC, type CDY1S15H-500 with stroke 500 mm and diameter 15 mm. The linear motion of the slider (the piston of the cylinder) is controlled by the proportional directional 5/3 valve, FESTO type MPYE-5 1/8 HF-010B. The linear position of the slider is measured by the linear potentiometer FESTO MLO-POT-500-TLF, and the angle of the pendulum is measured by rotational servopotentiometer by Spectrol. The electronic reference voltage cards (with ref-chips) for potentiometers reduced the measured signals in the needed domain. Pressure transducers are SMC ISE4-01-26. The controller is implemented on the PC computer via PCL-812PG

Fig. 1. Photograph of inverted pendulum.

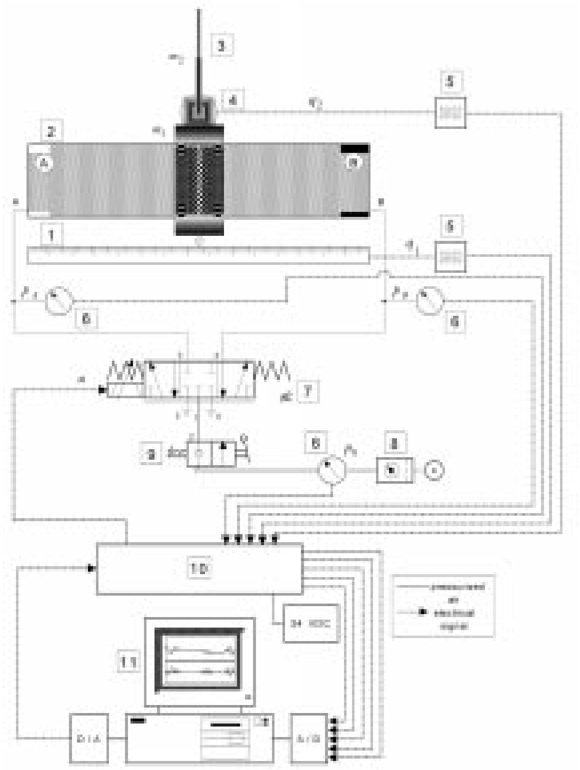

Fig. 2. Schematic diagram of inverted pendulum. 1: Linear potentiometer; 2: Rodless cylinder; 3: Inverted pendulum; 4: Rotational servopotentiometer; 5: Electronic reference card; 6: Pressure transducer; 7: Proportional valve; 8: Filter with pressure regulator; 9: Air supply valve; 10: Electronic interface; 11: PC computer.

acquisition card with 12-bit A/D and D/A converter. The working pressure is set up to 6 bar. The mass of the pendulum is 0.06 kg, and its length is 400 mm (the gravity centre length is 180 mm), but it is telescopic (car radio antenna) and it can be easily extended or shortened, clearly showing influence of the parameter changes on behaviour of the system. The slider mass is 1.5 kg. The equipment consists of some more components, like a pair of proportional pressure valves SMC VY1A00-M5 and a pair of simple on-off 3/2 valves SMC EVT307-5D0-01F (in Fig. 1 marked by numbers 12 and 13, respectively). Their application (combining some other control methods, like for example pulse width modulation control method) can be also considered as a cheaper alternative of the proportional directional valve employment. That is, however, more closely investigated in individual (postgraduate) student projects.

The mathematical model of the inverted pendulum dynamics is developed by the students. Usually the Lagrangian mechanics is employed in order to get the model, but approaches using other methods are welcomed, too. The nonlinear equations of motion are given in (A.1) and (A.2) in the Appendix. The dynamics of pneumatics is simplified, and given as a first-order lag system represented with a differential equation (1), where the input is voltage signal u (0–10 V) on the solenoid of the pneumatic valve, and the output is a pressure difference Δp between two chambers of the rodless pneumatic cylinder:

$$\tau \Delta \dot{p} + \Delta p = Ku \tag{1}$$

where τ is a time constant (0.12 s), and K is a gain (0.11 MPa/V), and they were experimentally determined from transient responses. An extensive, physical approach to modelling a pneumatic valve and cylinder is given in [3]. It includes modelling of a proportional solenoid dynamic and thermodynamic changes in the pneumatic system. That is a complex approach, which gives a highly nonlinear differential equation as a result, and the parameter's determination is more demanding. Hence, the physical approach is sooner avoided in the course of mechatronics, and its simplified, linear and experimentally determined alternative, given in Equation (1), is accepted. An inclusion of pneumatics dynamics in the model given by (A.1) and (A.2) is straightforward, using the equation for the force F that moves the piston (the slider of the pendulum):

$$F = \Delta p A \tag{2}$$

where A is area of the piston of the cylinder ($0.00018\ \mathrm{m}^2$).

Pneumatics dynamics (that includes solenoid dynamics, too) increased the number of system's states by one; the total number of the states are five. The state space description of a linearised system is given in Equations (A.3) and (A.4) in the Appendix. A friction of the slider is included in a simulation model using the reset integrator algorithm given in [5]. A coefficient of viscous friction and a coefficient of stick effect were also determined experimentally.

The control scheme using state controller is given in Fig. 3. The students implemented the controller on a PC. Alternative controllers and filter implementations are considered, too, and that is discussed in the following section. A list of symbols is given in the Appendix.

EXPERIMENTAL RESULTS AND DISCUSSION

A goal in the project completion certainly was to make the laboratory model successful, so in this case the pendulum had to stay vertical as necessary. There were different possibilities to realise the inverted pendulum regarding design parameters, actuator, and choice of sensors or control method. They were considered in the course, some of them using numerical simulations or linear analysis. The modest timetable anticipated for the course Mechatronics $(2+1)$ and the available equipment put limitations on possible solutions. Still, further research is left to students for their individual projects.

Regarding controller design, the students first

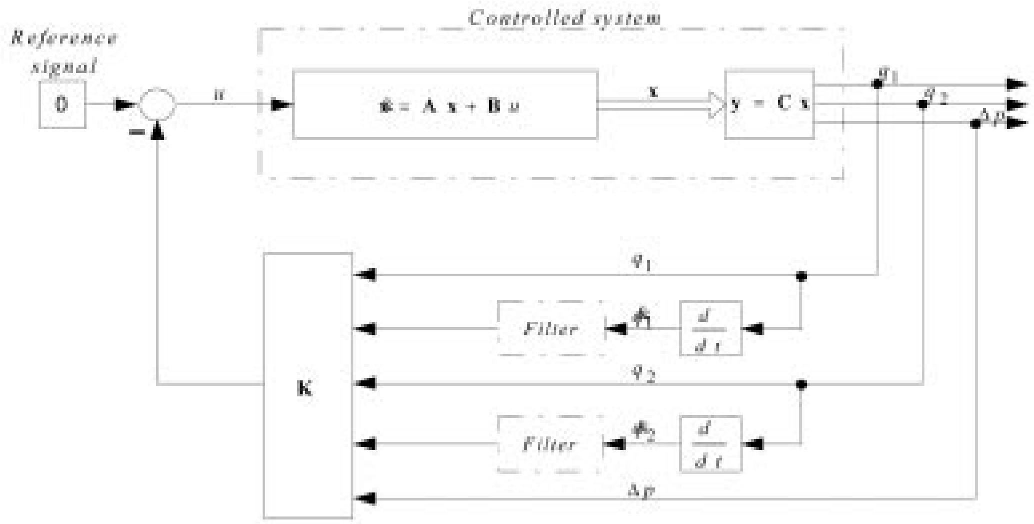

Fig. 3. Control scheme.

tried to apply a simple proportional-plus-derivative (PD) compensator. It had been successful in hold the pendulum upright, while the slider had been moving to infinity. Then the state-variable feedback controller was applied, and properly designed it was successful in the task of control of the inverted pendulum. The controller gain (given in matrix **K** from Fig. 3) was obtained by using linear quadratic regulator (LQR) optimal design. The weighting matrix **R** was kept equal to identity matrix, while initial values for weighting matrix **Q** was equal to **C′C.** They were changed after analysis of closed loop poles and simulations. Finally, the applied controller gave experimental responses of the slider position and the pendulum angle showed in Fig. 4. The disturbances marked in Fig. 4 were just impulses given to the pendulum by flicking it with fingertips. It can be seen that the pendulum was completely calmed after circa 5 seconds.

A certain change of design parameters, like an extension or a shortening of the pendulum, obviously influenced behaviour of the model. That can be seen in Fig. 5, where the pendulum's length is almost doubled. The pendulum could not calm completely, but it did not fall. (It succeeded to recover angles more than $\pm 10^{\circ}$ from equilibrium.) However, that could be compensated for by controller redesign and then almost identical behaviour as in Fig. 4 were obtained. For very large pendulum extensions (above 1 m), strong vibrations occurred, and it could not be stabilised. In that case it might be very interesting to include some knowledge from active vibration damping or flexible robot control research.

Some other alternatives were considered as well. The linear potentiometer was changed with a rotational one connected to the slider by wire (not marked in Fig. 1). That was much cheaper while the results were almost equal. Also, implementation of filters (see Fig. 3) were anticipated and discussed. Since the results were successful,

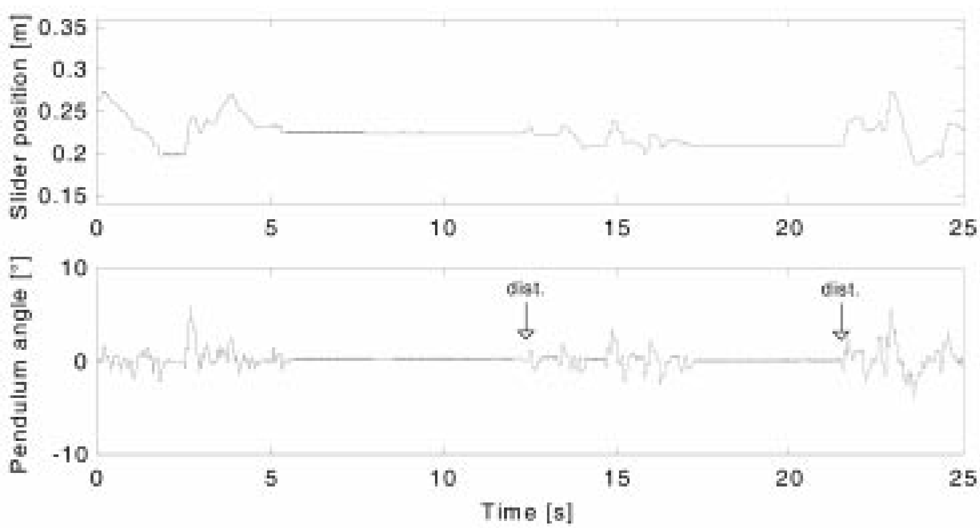

Fig. 4. Slider position and pendulum angle.

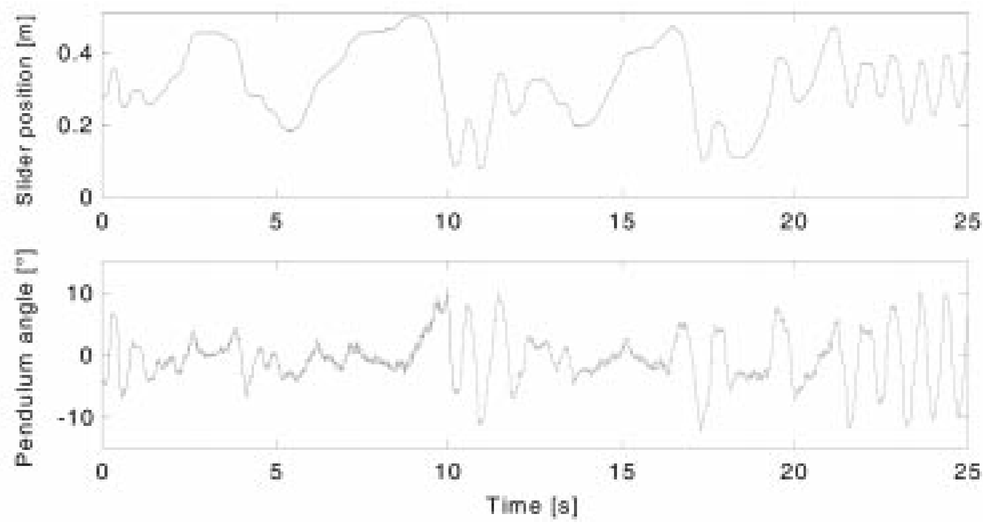

Fig. 5. Slider position and pendulum angle for doubled pendulum length.

they were omitted, although the noise in derivative of the signals was observed. Changes of air pressure (originally set up to 6 bar) significantly influenced the parameters of the drive, given in Equation (1). Influences of slower or faster drive were easily noticed on the response of the system, and their compensation with controller redesign or mechanical parameter changes (mass, pendulum length) were analysed.

CONCLUSIONS

Many disciplines that form part of mechatronics and their composite subjects can be taught with projects like the pneumatically driven inverted pendulum described in the paper. It requires good knowledge in modelling, simulation, control and hardware implementation, yet it is affordable and relatively simple. Influences of design parameter variation (although just mass or pendulum length can be changed) on the behaviour of the system can be observed and compensated for by different control parameters or methods. Different possible sensors, outputs, actuators, and their influences on the system can be considered on this laboratory model. The particular importance of mathematical models of system dynamics is emphasised for the inverted pendulum, since the controller design is model-based. In addition, behaviour of the system and many of its variations are analysed using a mathematical model and computer simulation. The pneumatics dynamics is included in the model using simple mathematical description based on experimental determination.

The presented laboratory model is open-ended, and that is in its favour. Many other possible alternatives that can be applied on this inverted pendulum, like application of cheaper valves or some other sensors combined with other control methods, remained for further student projects and their education.

Similar student projects, either group or individual, that will result with different laboratory models are anticipated in the future for mechatronic education.

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APPENDIX

Nonlinear mathematical model of the dynamics of the inverted pendulum is given by:

$$m_1\ddot{q}_1 + b\dot{q}_1 + m_2\ddot{q}_1 + m_2L\dot{q}_2^2\sin q_2 - m_2L\ddot{q}_2\cos q_2 = F \tag{A.1}$$

$$m_2L^2\ddot{q}_2 - m_2L\ddot{q}_1\cos q_2 - m_2Lg\sin q_2 = 0 \tag{A.2}$$

The state space description of a linearised system, and including pneumatics dynamics from Equation (1) is given in Equations (A.3) and (A.4). The linearisation is obtained near the equilibrium point, assuming $\cos q_2 = 1$; $\sin q_2 = q_2$; $\dot{q}_2 = 0$.

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \\ \dot{x}_5 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & \dfrac{-b}{m_1} & \dfrac{m_2g}{m_1} & 0 & \dfrac{A}{m_1} \\ 0 & 0 & 0 & 1 & 0 \\ 0 & \dfrac{-b}{m_1L} & \dfrac{g(m_1+m_2)}{m_1L} & 0 & \dfrac{A}{m_1L} \\ 0 & 0 & 0 & 0 & \dfrac{-1}{\tau} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ \dfrac{K}{\tau} \end{bmatrix} u \tag{A.3}$$

$$y = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} + [0]u \tag{A.4}$$

System states are:

$$x_1 = q_1 \qquad x_2 = \dot{q}_1 \qquad x_3 = q_2 \qquad x_4 = \dot{q}_2 \qquad x_5 = \Delta p.$$

List of symbols:

A = area of the piston, 0.00018 m^2
b = coefficient of viscous friction, 65 Ns/m
F = applied force [N], given in eq. (2)
g = gravity acceleration, 9.81 $\mathrm{m/s}^2$
K = pneumatic gain, 0.11 MPa/V
L = length of pendulum's gravity center, 0.18 m
m_1 = mass of the slider, 1.5 kg
m_2 = mass of the pendulum, 0.06 kg
Δp = pressure difference between two chambers [Pa]
q_1 = slider position [m]
q_2 = pendulum angle [rad]
u = input voltage applied on the valve [V]
τ = pneumatic time constant, 0.12 s

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