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Isotopic Engineering as a Conceptual Framework for Courses in Microelectronics and Quantum Informatics*

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Isotopic Engineering (IE) refers to use of the diversity of stable isotopes of chemical elements for a range of technological applications, including such areas as microelectronics, optoelectronics and quantum informatics. In a more extended version, it also includes technological applications of radioactive isotopes. The author argues that the incorporation of IE in university courses on electronics, informatics and general engineering provides a convenient and unifying tool for introductory exposure of students to a broad range of novel scientific and engineering concepts and ideas. Among them are quantum well structures and superlattices, random number generators, quantum computing, as well as ideas of chaos theory and nanoscale information storage. While most of these topics are scientifically quite involved and educationally challenging, their presentation in the context of isotopic examples allows for some reduction of the 'threshold of difficulty' for a typical engineering student. For the in-depth understanding of most of these topics the educational background requires only a base knowledge at the level of introductory courses in physics, chemistry and mathematics.

INTRODUCTION

MOST CHEMICAL elements exist in nature as mixtures of several stable isotopes. A number of major areas of science and technology takes advantage of this fact. Various effects stemming from isotopic diversity serve as key ingredients for a broad range of fundamental research directions as well as providing a basis for numerous applications. Some of them were discussed recently under the umbrella term of Isotopic Engineering (IE) [1–5]. While a broad variety of topics and numerous applications were suggested and discussed in this context [1–26], IE as such is still an incipient research and technological direction rather than a full-fledged area of a massive effort. At the same time, its potential for future electronic applications shows promise along several major lines.

The fact that about $\frac{3}{4}$ of all chemical elements are random blends of several stable isotopes is, of course, well known. The notion of isotopes is traditionally explained even in the most introductory courses of chemistry and physics. Many, seemingly distant, areas of science and technology are primarily based on this fact. For example, there are isotope separation technologies (several versions), nuclear industry, isotope labelling in biology, medical applications of radioactive isotopes, isotope geology [18], and isotope ecology [17]. In physics and chemistry isotope effects are important in such areas as chemical kinetics [13], diffusion studies, superconductivity (including recent developments in high temperature superconducting materials), structural phase transitions, impurity spectroscopy, etc. Such methodologies as Nuclear Magnetic Resonance (NMR) or Mössbauer spectroscopy make use of isotopic differences in nuclear magnetic moments.

It is quite curious, however, that in spite the common awareness about the existence of isotopes among scientists, engineers, and educated public in general, there is almost a complete lack of appreciation of the fact that isotopic diversity is a unique phenomenon in its own rights. One may wonder if it is possible to delineate a common thread (or threads) for various manifestations of isotopic diversity. Can we talk of isotopic diversity (or 'isotopicity', to be brief, [4]) as a kind of a singular phenomenon of Nature in its own right? If such a unifying view can be established, what are other possible implications that could be drawn from such an interpretation? And, furthermore, can such a unifying prospective lead to other gainful links between seemingly unrelated areas of research and applications?

The conceptual simplicity of IE and isotopicity makes it a convenient educational tool which allows to relate fundamental aspects with numerous engineering applications. One of the simplest examples, perhaps, is isotopic fibre optics. It takes advantage of the fact that crystalline structures made of different isotopes have slightly different refractive indices and hence provide a possibility for confinement of light [4, 6]. The present paper emphasizes the educational value of presenting

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isotopicity and IE from unifying positions. The author does not pretend to give here a final and exhaustive description of all possible implications of IE. Nevertheless, examples discussed below are aimed to stimulate a unifying vision of isotopicity in terms of its potential usefulness for discussing various facets of microelectronics and informatics. This approach can be tried at both undergraduate and graduate level courses on physical foundations of electronics and informatics. As to the personal experience of this author, some facets of IE were used in a variety of courses taught at the Department of Engineering Physics (electricity and magnetism, thermodynamics and solid-state electronics), although no special courses on IE were offered to this point. The offering of such a course could, in the opinion of this author, be implemented relatively straightforwardly upon the perceived curriculum need.

STABLE ISOTOPICITY

When we hear the word 'isotope' our most common reaction is to think of radioactivity. Use of radioactive isotopes in nuclear energy technology, in medicine and in several other areas of human activity (e.g., carbon-14 dating) is so well known, that in the public perception it tends to obfuscate even the more basic fact of Nature: that the majority of chemical elements are mixtures of several stable isotopes.

Chemical elements have various numbers of naturally occurring stable isotopes. If Z is the place number of an element in the Periodical Table, the range of stable isotopes runs from hydrogen ($Z = 1$) to bismuth ($Z = 83$). Some elements have only one stable isotope, while their immediate neighbours (next Z) may have several. The two most numerous groups are mono-isotopic and di-isotopic elements. They are almost equal in numbers (20 and 22 elements, respectively) and together encompass about half of all stable elements. It is a some kind of curiosity of Nature that 2 elements 'inside' the stable range of Z do not have stable isotopes at all, namely, technetium ($Z = 43$) and promethium ($Z = 61$). While the author is not aware of any universally accepted explanation for the existence of these two 'gaps', it is interesting to note that both the above values of Z (43, 61) are prime numbers. Taking into account the frequently emphasized role of prime numbers in various natural phenomena (see, for example, [27]), one may refer (metaphorically, perhaps) to some peculiar numerological game that Nature plays in this case.

Groups of elements with 3, 4, 5, and 6 stable isotopes are almost equal in numbers. The 'champion' of polyisotopicity is tin ($Z = 50$) which has 10 stable isotopes. Altogether, there are 283 stable isotopes for 81 stable elements—an average of about 3.5 stable isotopes per element.

Apart from some isotopic-sensitive studies (as, for instance, in the above mentioned NMR method), common chemistry in most cases ignores isotopic differences in defining molecular specificity. Chemical properties of, say, benzene (C_6H_6) are, by and large, the same when the C_6-ring contains only ^{12}C atoms, or only ^{13}C atoms, or any combination of these two isotopes. However, due to their mass differences isotopes are classically distinguishable particles. This leads to interesting statistical implications and in terms of educational efficiency allows for a relatively elementary and vivid discussion of such complex issues as Gibbs' paradox of identicality in statistical physics and the issue of quantum identity of particles [28].

ISOTOPIC RANDOMNESS

Classical distinguishability of isotopes can be described within the concept of isotopic freedom which is applicable to any molecular or crystalline structure which involves at least one polyisotopic element. In order to illustrate this concept let us consider, for example, an 'ideal' diamond crystal. Since natural carbon is 99% to 1% mixture of stable isotopes ^{12}C and ^{13}C, the crystal lattice of diamonds, even in the absence of 'common' defects (vacancies, impurities, dislocations, etc.), still possesses an inherent disorder due to random locations of two different isotopes at regular lattice sites. As classically distinguishable particles, these isotopes can form a tremendous number of positionally different combinations. Strictly speaking, such isotopic randomness violates the translational invariance of crystal. It is also obvious that no two microcrystals of the same size and shape are fully equivalent—each of them has its own pattern of isotope distribution ('isotopic individuality').

It is important to stress that the above mentioned arbitrariness of a particular microscopic isotopic arrangement in any segment of a crystal lattice is realized within the framework of rigid constrains of chemically-specific structure. One may say that isotopic freedom, with its enormous number of possible spacial combinations, can be exercised 'inside' a fully deterministic (in terms of chemical bondings) realm of a given lattice structure. Thus, isotopicity provides an interesting dichotomy between categories of determinicity and freedom at the level of atomic structures. This can be seen as a complimentary aspect to the 'usual' quantum-mechanical indeterminacy based on a probabilistic nature of the quantum dynamics [24]. However, isotopic fluctuations differ from ordinary compositional or impurity fluctuations in their 'built-in' character, that is, they occur within the chemically-ordered structures.

Unless crystals are composed exclusively of monoisotopic elements, they have a 'built-in' isotopic diversity. Most materials which are important for microelectronics have isotopic diversity. Exceptions to this are relatively rare (e.g., AlAs, where both Al and As have only one stable isotope each). Most chemical compositions forming natural or synthetic crystals have isotopic diversity. Consequently, each next sample of the chemically-identical crystal (e.g., each microregion of a silicon chip) has its own individual pattern of microscopic isotope distribution ('personal isotopic signature'). Nevertheless, isotopicity generally has relatively little appreciation, even in the domain of phonon physics and acoustic devices where the mass-dependent isotope effects could be quite noticeable. Since vibrational time scales are directly mass-sensitive (usually depending on mass M as 1/SQR(M), isotope shifts of phonon frequencies should be seen as first-order effects [1, 22].

Probably, the best known effect of isotopic randomness is the modification of lattice heat conductivity in a form of additional isotope scattering [6–8]. The latter means that local mass variations due to different atomic masses of isotopes produce additional scattering of phonons. This reduces heat conductivity of isotopically-random crystals in comparison with a monoisotopic (isotopically purified) crystal of the same chemical identity. Difference in heat conductivity between isotopically pure and isotopically blended crystals could reach a factor of 2 to 3. Less pronounced, but still observable, are variations in electrical conductivity in metals and semiconductors due to differences in scattering cross-sections of charge carriers on different isotopes. The additional electronic scattering factor can be interpreted as a Rayleigh-type scattering on isotopic fluctuations whose size is comparable with an electronic wavelength.

In favourable cases (e.g., narrow electron bands, strong electron-phonon coupling) isotopic disorder can induce Anderson localization of electrons and/or holes in semiconductors [1, 19]. Anderson localization is purely quantum phenomenon when random fluctuations of electrostatic potential lead to a formation of discrete (bound) quantum states. This effect is closely related to semiconductor-insulator transitions which are used for fast electronic switches (Mott-Anderson transitions). *Using isotopic randomness as an example, allows for an easy introduction of undergraduate students to the understanding of this electronically-important (but conceptually challenging) physical effect.*

Similarly, isotopic disorder could be a possible reason for the presence of phonon-assisted hopping conductivity in some electronically important materials [1]. Hopping conductivity means a change of mode of charge propagation through a semiconductor from a diffusion-like continuous motion to discontinuous jumps of electrons or holes from one localized state to another. Because of the variety of isotopic surroundings of the otherwise identical lattice sites, isotopic diversity can lead to a conversion of these sites to trapping levels for electrons or holes. Thus, a whole range of quantum concepts (discrete and continuum states, localization, conductor-insulator transitions, etc) can be illustrated on the basis of the isotopicity model.

A special avenue of IE is the possibility of constructing 'physical' random number generators [10, 11]. Typically, generation of random numbers in computers is based on mathematical procedures of truncation of various transcendental functions. Random numbers which are produced this way are, in fact, pseudo-random: due to a deterministic character of computer codes the strings of random numbers is repeated every time the same seed numbers are used. Mixture of stable isotopes, provided they can be probed at the atomic level [10], is free from this limitation. Counting different isotopes of the same chemical element (say, carbon) as digital '0' and '1' a non-repeatable genuinely random binary string can be generated. *This example of ISENG can, therefore, be used to expose students to subtle aspects of randomness and pseudo-randomness.*

An alternative version of isotopic random number generators uses the physical randomness in the decay of individual radioactive isotopes. This randomness is a direct manifestation of the fundamental laws of quantum physics. *Thus, using it as an educational example helps students to relate fundamentals of quantum physics (which some students may see as quite remote from practice) to a straightforward and important technical application of random number generation.*

INFORMATIONAL AND MEMORY ASPECTS OF ISOTOPICITY

It is a common assumption that the distribution of different stable isotopes of the same chemical element over the corresponding lattice sites in crystalline structures is always perfectly random. This assumption is equivalent to a statement of the lack of any noticeable positional correlation in the distribution of isotopes. There are some general reasons to doubt the universal validity of this assumption. On the contrary, one can expect that there is (at least in some crystal-forming situations) a non-vanishing positional correlation for different isotopes which may turn up in atomic-scale analysis [15, 18, 23].

Likewise, isotopes can potentially store digital information at an atomic scale [2]. Isotopically different regions within crystalline matrices can be assigned informational 'zero' or 'one'. Such informational loads can be carried by monoisotopic micro islands or even by single atoms within a bulk crystalline or thin film structure. This amounts to a very high density of information storage of up to 10E20 to 10E23 bits per cubic cm. Such possibility is based on the fact that isotopic information storage (unlike most other information storage techniques) allows an information bit be carried by a single atom. We note in this respect that the information content of the Library of Congress (estimate: 10E8 volumes of 1000 pages each, each with generous allowance of 10E5 bits per page—the latter to account for the digitized photographs) is 'only' about 10E16 bits. In principle, with isotopic information storage, all the information accumulated by the whole civilization (about 10E21 bits) can be stored in a volume of less than one cubic centimetre (!).

As such atomic-scale isotopic information storage forms (a somewhat specific) subdivision of nanotechnology. Because of its extremely high information packing density it opens the potential possibility of creating a universal library of 'all books'. The latter is something which was foreseen in a speculative vein by several earlier authors, such as, for example, a well-known essay '*The Library of Babel*' (1941) by Jorge Luis Borges (1899–1986). At this time, miniaturization of spatial scale of information storage (and speed of information processing) still follow well-known Moore's law (doubling every 18 months). Storage at the single atomic level (e.g., by specific isotopes) gives, therefore, a natural stop-point for Moore's law (at least as far as atomic physics is concerned). It should be noted, however, that because of enormously high packing density which isotopic information storage could potentially provide, an eventual atomic-scale limit to the Moore's law (even if it will be reached), will, for all practical purposes, be of little or no of real significance for the future information systems.

Further to that, in isotopically mixed lattices there could be a number of similarly structured isotopic microcomplexes. This might lead to low-frequency vibrational resonance effects and account for memory storage phenomena. For example, holographic-type memory effects in quartz crystals may be related to complexes involving minority isotopes of oxygen and/or silicon (e.g., ^{17}O and ^{29}Si isotopes) in a manner describable as a formation of isotopic neural networks, similar to neural networks in spin glasses [4, 26].

ISOTOPIC DIVERSITY AS A METAPHOR OF CHAOLOGY

Most compounds and crystals are, in principle, available in different isotopic versions. Their physical characteristics are slightly different. This fact provides a basis for representing isotopic variations as points in a multidimensional configuration space. Each axis corresponds to a given physical parameter. These points form 'isotopic paths' resembling sequences of points which could be stroboscopically sampled along the phase trajectories of strange attractors in chaotic dynamics [20, 29, 30].

Quite often isotopic replacements produce noticeable variations of physical parameters, e.g., temperatures of structural phase transitions of different isotopic versions of the same crystal can differ by as much as several degrees centigrade—a clearly 'macroscopic' variation even in terms of crude measurement standards [9].

The multiplicity of isotopic combinations and physical parameters available for measurements leads to a wide range of possible topological representations of data. The physical properties of isotopically different crystal lattices are not exactly the same. Therefore, the relative isotopic variations of different physical parameters can be represented by corresponding isotopic points in multidimensional configurational spaces [20]. The set of all isotopic points in a chosen space forms an isotopic cluster. The line connecting all isotopic points of a given cluster one-by-one, according to a gradually increasing value of some physical 'label', forms an isotopic trajectory.

As an example, consider crystalline SnTe which has as many as 80 isotopic versions (10 isotopes for Sn and 8 for Te, respectively). Let us assume that some set of physical parameters is experimentally known (measured) for all $N = 80$ isotopic versions of this crystal. Let D be a number of measured physical parameters for a given run of measurements. This number is, of course, the same as the dimensionality of a given configurational space. Each of the axis of this parametric space represents a given physical parameter. For example, if $D = 3$ one can take as physical parameters the speed of sound, the melting temperature, and the refractive index of the corresponding isotopically pure crystal (in this example, 1 out of 80 possible versions).

If all such parameters are measured at the precisely specified and fixed conditions, there is a plethora of other choices of measurable physical parameters. In addition, D itself can be any positive integer. The set of all N points representing each of N participating isotopic combinations (isotopic cluster) forms a 'swarm' in a corresponding configurational space. It is important to note that the same set of N isotopic points can lead to quite different shapes and appearances of these swarms if different combinations of physical parameters are used for plotting. *Due to the wealth of accumulated data on various isotopic effects this opens an opportunity for numerous tutorial projects and assignments when students can use multidimensional plotting programs to attain a conceptual grasp of parametric spaces and gain a deeper insight into underlying physical effects.*

The next step is to extract some physical information from a set of measurements for a complete isotopic 'collection' of a given compound or a crystal. For a particular experimental run one can look for all types of meaningful correlations between various subsets of physical parameters and labels designating various isotopic combinations as some functions of atomic numbers of participating isotopes.

ISOTOPIC COMBINATORICS AND ISOTOPIC STRANGE ATTRACTORS

In a polyelemental compound, isotopic replacements for each element can be done independently of all other elements. Therefore, for each individual compound, isotopicity can be seen as a discrete (stepwise) multidimensional variable having the dimensionality equal to the total number of polyisotopic elements in a given compound (F). For the above example of SnTe the so-defined isotopic dimensionality, $F = 2$. There is, of course, no special limitation on a number of different physical characteristics (D) which can, in principle, be measured for all isotopically distinguishable compositions. This leads to a rich variety of possible isotopic spaces and isotopic trajectories which can be studied for compounds with isotopic variability.

For $F = 1$ isotopic points can be connected according to consecutive values of atomic number of stable isotopes of a given element. For substances with isotopic dimensionality $F > 1$, the order in which N points are to be connected is less obvious and may not be unique. One can use, for example, the following simple recipe of consecutive numbering of all N isotopic points. List all polyisotopic elements of a given compound in order of their atomic number Z. Within each Z count all stable isotopes according to the increase of their atomic weight A. The shape, length and the topology of isotopic trajectory will, of course, depend on the enumerating rule used in each given case.

One route of extracting non-trivial physical information from isotopic trajectories is to apply the same protocols as commonly used in the theory of chaos. In a frequently quoted experiment with the leaky faucet ([29], p. 262) consecutive time moments of the falling drips exhibit a subtle structure having informational interpretation in terms of strange attractors. On the other hand, there are indications that isotopic effects may lead to non-linear shifts of physical parameters with atomic mass. In this case a natural way to analyse isotopic points is to calculate the correlation sum according to the method of Grassberger and Procaccia [30]. Power approximation for the correlation sum gives an estimate of power (n) which, in analogy of the theory of chaos, can be called the dimensionality of an isotopic attractor.

Another route of informational assessment of isotopic attractors (which could be especially useful for cases when the number of collected isotopic points is small) can be based on exploiting more specific connections between chaology and information theory (e.g., by relating isotopic points to informational Shannon's entropy). Due to a limited number of isotopic points and (generally) high level of experimental uncertainty, methods based on Bayesian version of probability theory and the Maximum Entropy principle of Edwin Jaynes [31] can be of special relevance in this context.

There could be numerous ramifications of the above method when it is applied to cases with various degrees of isotopic diversity, varying relative magnitude of isotopic effects for different compounds, attainable precision of measurements, etc. The best candidates for such studies are likely to be found among materials with unusually strong isotopic effects [9]. It is quite possible that variously chosen subsets of the whole 'isotopic collection' of a given compound or a crystal will reveal different resulting values of the dimensionality of isotopic attractor (n) or a different degree of quality of fitting to a power dependence.

Even for cases with relatively small degrees of isotopic diversity (say, for $N = 10$) the number of possible subsets could be quite significant to allow the extraction of reliable conclusions about the degree the robustness of the dimensionality of isotopic attractor. It should be remembered, however, that isotopic swarms with a finite number N of points can only tentatively imitate the behaviour of the 'genuine' strange attractors of chaology for which N tends to infinity. Nonetheless, prime concepts of chaology can be introduced on vivid and chemically lucid examples of isotopically-different compounds.

ISOTOPIC TRAJECTORIES AND 'DATA MINING'

There is a priori no known 'law' which requires that mutual dependances of physical parameters represented by isotopic trajectories should be smooth or even monotonous functions. For instance, in heteroatomic crystal lattices, especially those with strong ionicity of the lattice, the effect of atomic weights on features of phonon spectra can be quite peculiar. The resulting isotopic trajectory can, generally speaking, be a smooth featureless curve or, contrary to that, it may have some peculiar features, like cusps and other singularities.

As a possible tutorial example, we can put forward the following indirect argument in favour of a likely expectancy of non-trivial possibilities of isotopic mappings. Taking the above example of SnTe, let us compare the following 3 of its 80 isotopic combinations: $^{124}Sn^{130}Te$, $^{124}Sn^{120}Te$, and $^{124}Sn^{124}Te$. In the first sample Sn is lighter than Te, in the second Sn is heavier than Te, and in the third they have the same weight. Because SnTe is a partially ionic material, it is highly unlikely that the vibrational properties of all three samples are so close that their differences can not be detected by the available methods of vibrational spectroscopy. On the contrary, it is more likely that the third, equal-mass ('resonance') combination will show some distinct features in phonon spectra in comparison with two other combinations with unequal masses of Sn and Te. Such a contrast may be due to a different positioning (symmetric for the third and asymmetric for the first and second combinations) of the centre of mass of a unit cell within the unit cell itself.

The development of an isotopic mapping methodology can lead to even more tricky results. For instance, the situation could turn out to be especially interesting if labelling of isotopic points according to a yet another physical parameter will result in a different isotopic trajectory for the same N isotopic points. Suppose we choose the fourth parameter, e.g., the hardness or magnetic permeability and connect isotopic points according to the consecutive values of this new parameter. This may result in the same numbering of points and it would be a kind of a trivial case. However, it may also happen that a new labelling, according to consecutive values of a new parameter, will result in a different order of connecting same N isotopic points. Wealth of non-trivial physical information can be obtained along this route of experimentation and conceptualization.

We will not analyse all other topological options for isotopic clusters and isotopic trajectories in the present paper any further. Any, more or less comprehensive, analysis of these options should probably wait before any systematic experimental data will be collected for isotopic versions of chemically identical crystals. However, a few additional and more concrete experimental routes can be indicated:

1. Impurity spectroscopy for the case when identical impurity is placed in isotopically different host crystals. This is an ‘inverse case’ to more traditional studies of isotopic effects for impurity centres which mostly look for isotopic shifts of isotopically different impurities in isotopically fixed matrix. In the suggested case isotopic points can be collected by measuring isotopic spectral shifts (e.g., zero-phonon lines whose positions can be measured with a high precision) of isotopically-same impurity in several isotopically different matrices. For polyisotopic impurity both the impurity centre and the host lattice can be tried in various isotopic combinations.
2. Isotopically engineered structures (e.g., isotopic superlattices grown with alternating layers of different isotopes) can be of a double benefit for collecting isotopic points. Firstly, they open a possibility for various ‘junction’ measurements, e.g., similar to the known measurements of band-gap offsets for variously doped semiconductor homojunctions. Secondly, they could often lead to noticeable ‘savings’ in terms of a number of required isotopic combinations for a given number of isotopic points. For example, 5 isotopes of a metal (e.g., Zn) together with 5 isotopes of a semiconductor (e.g., Ge) produce 25 isotopically distinguished Schottky barriers. More refined and sensitive experiments can be performed on isotopically varying Josephson junctions near their superconductivity transition. Another example of structural IE is isotopic fibre optics mentioned above [4, 6].
3. Ultrahigh pressures (e.g., in experiments with a diamond anvil cell) can be quite useful in a number of ways. One is that they tend to reveal new structural phases for crystalline materials. Experiments with isotopically different samples could likely be optimal when pressures and temperatures are kept near the boundaries of such transitions as it increases sensitivity to applied parameters and could lead to a ‘magnification’ of isotopic effects. Ultrahigh pressure experiments can be especially instructive for isotopically structured samples [see item (2)] as conditions can be found when spatially different parts of the sample experience transitions at slightly different pressures and/or temperatures.
4. Isotopically pure versus isotopically mixed materials give another experimental leverage—a simple example will be to study the comparative properties of pure ^{29}Si crystal and 50–50 % mixture of ^{28}Si and ^{30}Si. Both crystals have the same averaged weight of a unit cell and yet clearly are not the same. The questions which can be offered in this respect aim to determine whether these (isotopically different) crystals have exactly identical lattice constants, melting temperature, etc. For heteroatomic combinations the combining of all possible isotopic subsets in pairs, triplets, etc., can, in principle, generate an almost unlimited collection of additional isotopic points.

Additionally, isotopes could be combined in variable proportions which is even further increases potential information which can be extracted from ‘hidden’ correlations between physical parameters (data mining).

This, theoretically speaking, produces an infinite number of possible isotopic points and leads to continuous isotopic trajectories—the latter could even better mimic the essentials of strange attractors of chaology. However, in case of isotopic mixing, statistical aspects, such as isotopic fluctuations, should be taken into account.

QUANTUM INFORMATICS AND QUANTUM COMPUTING

Apart from the above mentioned isotopic random number generators [10, 11], isotopes have recently found their way into a novel area of quantum computing.

The rapidly unfolding area of quantum informatics and quantum computing is an unquestionable growth industry at the frontier of physics and electronics. While basic principles of quantum computing are becoming generally known [32–35], technological applications are still mostly in infancy stage. The applicational potential of quantum computing and quantum informatics, however, is truly enormous. Instead of handling fix bits (zeros and ones) quantum computers work with superpositions of bits mixed in various proportions (qubits). The expected advent of a reliable quantum computer (which many experts believe is likely to happen within a decade) will make the problem of the factorization of arbitrary long integers solvable by relatively simple means. The most important advantage of quantum computers is their ability to probe multiple possibilities simultaneously. This could be invaluable in such applications as extremely fast data mining of large-scale databases. Some authors go as far as ascribing enormous potential of quantum computers to their ability to borrow computing power from parallel universes [35]. In spite of speculativeness of such ideas, almost everyone finds them fascinating and mind-boggling. Thus, the very mentioning of them to aspiring students can significantly boost their interest in quantum informatics and strengthen their learning process.

Most of the modern cryptology (such as security of bank transactions) is based on the practical impossibility to factor long numbers (over a few hundred digits). For example, to find the prime factors of (arbitrarily chosen) 500-digit integers is a practically unsolvable problem, even with the best modern computers. Such popular schemes as RSA algorithm [31, 32] are fundamentally based on the fact of this factorizational difficulty. Quantum computers will likely eliminate physical basis for such cryptology schemes which, in turn, may require fundamental social re-evaluation of such issues as secrecy and privacy.

At the time of writing, there are several research lines attempting to implement quantum computing in practice. Almost all of them use nuclear spin states of specific isotopes in crystalline matrices [34]. In practical implementation on the basis of solid state structures, quantum computing is isotopically selective. Thus, quantum computing naturally falls into the domain of IE. Specifically, because of the quantum identity of the same isotopes (as opposed to the quantum distinguishability of different isotopes of the same element), isotopicity provides a natural playground for the establishing of quantum entanglement among large clusters of atoms. The latter (sustained quantum entanglement) is one of key requirements needed for a functional quantum computer [32].

DISCUSSION: ISOTOPICITY AS PRIME FACT OF NATURE

We have reviewed here several, quite different, aspects of isotopic diversity of chemical elements. On top of chemical diversity, isotopicity provides an additional 'built-in' degree of freedom in condensed matter structures (crystalline or not). This extra degree of freedom has a distinct informational bearing [2, 4]. In a kind of general philosophic modality isotopicity can be seen as an independent outfit for a general trend in Nature for a pattern-forming activity [4, 15, 16]. One may possibly say that various examples of spontaneous pattern formation are concrete manifestations of inherent 'quest for intelligent patterns' deeply rooted in Nature [25, 26].

It is known that Nature has found ways to utilize biologically almost all chemical elements from the Periodical Table. Many so called 'microelements' have distinct biological function. In spite that some of these functions may still not be fully understood, our body's need for a variety of mineral supplements is generally well recognized. Not only major life-related elements (C, O, N, H) are important for biological processes but also such trace elements as, say, vanadium or molybdenum have their own biological functions as well. A whole range of chemical elements (often several dozens) are typically used in poly-vitamin supplement pills. Likewise, the same can be expected for isotopic diversity of elements. Informational and pattern-forming capabilities of isotopic diversity are so multisided and universal that it is highly unlikely that Nature could completely overlook the use of this additional option in its inner fabric.

EDUCATIONAL POTENTIAL OF ISOTOPIC ENGINEERING

As a concept, isotopic diversity of chemical elements (isotopicity) is quite lucid and transparent. Its fundamental essence can be reasonably well understood and appreciated by a person with just a base knowledge of chemistry and physics. This makes it a convenient and attractive tool for the presentation of a broad range of topics of novel science and technology as was briefly outlined above. The advantage of a unification of a variety of topics under one conceptual umbrella is obvious. It allows for the economy of time and educational resources. Furthermore, an appeal to such a conceptually transparent phenomenon as isotopicity makes it suitable for an introductory exposure to a number of 'hard' topics for the mixed (interdisciplinary) student audience. Some educational experiments in this vein were undertaken by the present author in his teaching of a course on the Discovery and Innovation for the McMaster Theme School on Science, Technology and Public Policy (1998–2000) [36].

It is important to note that in the context of the present article IE (and isotopicity) should not be seen as an entirely new separate discipline. It is rather a venue along which a variety of engineering and technological topics can be presented and integrated. In this way, IE displays itself as primarily a unifying and synthetic educational tool which adds to a synergy in a coverage of a broad range of methods and ideas.

EPILOGUE

Exposing students to IE and the isotopicity concept, especially at the undergraduate level, can serve a two-fold objective. One one hand, the very phenomenon of isotopicity is one of the prime facts of Nature which branches into numerous applications in science and technology. Thus, a more detailed familiarity with isotopicity has a distinct engineering value in itself. On the other hand, by providing a rich illustrative tapestry for various effects and applications, it assists to some degree in simplifying and making more accessible the treatment of such complex topics as atomic scale information storage, quantum computing, quantum entanglement and qubits, physical random number generators, or some aspects of bioengineering.

In addition to technical merits, the exposure to the ideas of isotopic diversity and related areas such as quantum computing, contributes to a broadening of general scope and in-depth comprehension which is normally expected from the modern engineer. For example, one of the founders of quantum computing, David Deutsch, is a strong proponent of one of the most radical and interesting interpretations of quantum physics known as Many-World interpretation [35]. This brings into serious consideration such mind-boggling ideas as an issue of the physical existence of parallel universes. It is the experience of the present author (23 years of teaching in engineering) that many students, even those whose specialties seem to be remote from physics (such as, e.g., civil engineers) are often fascinated by these frontier ideas. It is obvious that the intellectual excitement provided by such ideas is almost invariably serves as a strong contributing factor into the quality of engineering education.

While in this paper we covered only IE based on stable isotopes, it should be mentioned that extending isotopicity to include radioactive isotopes allows for even greater leverage of presenting engineering related topics. The nuclear industry alone comprises a separate area of engineering. Likewise, nuclear fusion, neutron channelling, nuclear activation analysis and several other technologically important areas are all, in fact, subareas of IE in a broader sense.

Because of the multiplicity of conceptual facets and applications, IE and isotopicity have a distinct interdisciplinary flavour. In practical terms, however, it may be a dubious blessing. While superficially, interdisciplinary approaches are often acclaimed as creative and fruitful, this praise is often hardly more than a lip service. Quite often, the reality of the research community, its 'Realpolitik' and its accepted practice of peer review are de-facto aimed at the protection of narrow sectarian professional interests and turn out to be overtly conservative and generally remains in opposition to interdisciplinary efforts [36, 37]. In this vein isotopicity provides an alternative route for expanding horizons in interdisciplinary engineering education.

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