

A Spreadsheet-Based Method for Thermal Radiation Calculations*

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The use of a spreadsheet program, e.g. Microsoft Excel, has been introduced as a platform for teaching engineering students to solve thermal radiation problems, particularly problems relating to multi-surface enclosures. The approach developed enables the network equations to be solved using Excel's Solver. The problem is formulated in a general way that permits the user to specify the required number of boundary conditions in the form of any valid combination of temperatures and heat flows. This includes boundary conditions where the net radiation to a surface matches external convection loss to a specified ambient temperature. Calculation of shape factors using user-defined functions written in VBA makes the method very flexible and permits use of redundant information to verify the consistency of the shape factors. The spreadsheet-based approach to radiation calculations is a major departure from the methods currently presented in heat transfer textbooks. The method has been found to be computationally robust and the approach has been well received by students.

NOMENCLATURE

A_i	area of surface i
E_{bi}	black-body emissive power of surface i
F_{ij}	shape factor (view factor) between surface i and surface j
J_i	radiosity of surface i
n	number of surfaces
q_i	external heat flow to surface i (positive if net flow is into the enclosure)
q_{ij}	net radiation exchange between surfaces i and j
R_i	resistance of surface i defined by equation (2)
R_{ij}	resistance between surfaces i and j defined by equation (3)
T_i	temperature of surface i
T_∞	ambient temperature outside the enclosure
U_i	overall heat transfer coefficient for external heat loss from surface i
ε_i	emissivity of surface i
σ	Stefan-Boltzmann constant

INTRODUCTION

THE APPROACH presented in standard heat transfer textbooks [1–3] to solving the equations generated in heat transfer calculations has changed little for over five decades. In the 1960s, numerical methods were introduced for conduction calculations. Students were encouraged to apply programming skills in programs such as Fortran or Basic to solve such problems, or else to use standard library routines to do the major computations.

The last decade has seen a major change in the way engineering students prefer to perform most calculations. Now, spreadsheets will be used whenever the students believe that this is an appropriate tool for solving the problem at hand. Unfortunately, there is still little recognition of spreadsheets in the literature available to students. There has been some attention, however, to the way of solving conduction problems using a spreadsheet [4–5]. One textbook [6] includes approaches to the spreadsheet solution of two-dimensional steady state and one-dimensional transient conduction problems. This appears to be the only comprehensive heat transfer textbook to date to acknowledge the value of spreadsheets in heat transfer education and their usefulness in carrying out heat transfer calculations.

The previous spreadsheet method for thermal radiation enclosure calculations [7] has been further developed and an alternative treatment of the calculation of the radiosity values evaluated. Over the last six years, the spreadsheet-based method has significantly improved the level of understanding reached by students and the complexity of problems which they can successfully solve given the same lecture and assignment time. This paper aims to show that a spreadsheet provides a very convenient and flexible means for solving heat transfer by radiation in multi-surface enclosures.

The radiation enclosure problem

The problem may be stated as follows: given an enclosure of n isothermal surfaces, with specified geometry and surface emissivities, calculate all unknown surface temperatures, T_i , and all

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unknown heat flows from outside the enclosure, q_i , and between surfaces, q_{ij} .

For the problem to be uniquely defined, a degrees-of-freedom analysis shows that it is necessary to specify n temperatures and/or heat flows. This is frequently assumed to mean that either T_i or q_i must be specified for each surface [1]. Although this is the most convenient problem specification, it is an unnecessary restriction imposed by the approach conventionally presented in textbooks [1–3] to structuring and solving the resulting equations. In fact, the only true restrictions are that at least one temperature must be specified for a unique solution, and that the quantities specified must be independent. One cannot, for example, independently specify the temperature of two blackbody surfaces and the net radiation heat exchange between them.

The example in [7] dealt with a problem in which the specified conditions were: the temperatures of surfaces 1 and 2, T_1 , and T_2 ; the net heat flow between surfaces 2 and 3, q_{23} ; and the external heat flows into surfaces 2 and 4, q_2 and q_4 . Note that this specifies two conditions relating to surface 2 and none relating to surface 5, yet the unique solution is readily found.

In the example in this paper, either the temperature or external heat flow is specified for each surface, as is conventional. However the external flow for one surface is defined in terms of the external convection loss, as often occurs in practice, and so depends on the temperature of that surface. The method is designed to be robust in reaching the solution, whatever conditions are specified, while requiring a minimum of problem-specific setup.

The steps in solving a radiation enclosure problem can be generalized as:

1. Specify the problem geometry.
2. Select the surfaces to be treated as isothermal and specify the surface emissivities.
3. Calculate the surface resistances (Equation (2) below) and the resistances between surfaces (Equation (3) below).
4. Specify n temperatures or heat flows to satisfy the n degrees of freedom.
5. Obtain a set of equations to solve for the remaining temperatures and heat flows. This will require solution for the radiosities of all surfaces. The radiosity is the total radiation leaving a surface, emitted plus reflected, and is treated as a potential in the same way as the black body emissive power.
6. Express the equations in a form suitable for the solution method to be used.
7. Solve the equations for the radiosities, then the unknown temperatures and heat flows.

The well-known equations relating to greybody radiation enclosures are summarized below [1–3].

Black body emissive power, E_{bi} :

$$E_{bi} = \sigma T_i^4 \quad (1)$$

Surface resistance, R_i :

$$R_i = \frac{1 - \varepsilon_i}{A_i \varepsilon_i} \quad (2)$$

Resistance between surfaces i and j , R_{ij} :

$$R_{ij} = \frac{1}{A_i F_{ij}} = \frac{1}{A_j F_{ji}} \quad (3)$$

The net heat flow, q_i , from the surroundings in through surface i :

$$q_i = \frac{E_{bi} - J_i}{R_i} = \sum_{j=1}^n q_{ij} \quad (4)$$

The net radiation exchange, q_{ij} , between surfaces i and j :

$$q_{ij} = \frac{J_i - J_j}{R_{ij}} = -q_{ji} \quad (5)$$

It is usual [3] to use Equation (4), after rearranging to Equation (6), for each surface i if the surface temperature T_i is specified, and hence E_{bi} is known. Otherwise it is used in the form of Equation (7) if the net heat flow, q_i , is known. This provides a set of n equations which can be rearranged to a set of n linear equations for the unknown radiosities.

$$\frac{E_{bi} - J_i}{R_i} = \sum_{j=1}^n \frac{J_i - J_j}{R_{ij}} \quad (6)$$

$$q_i = \sum_{j=1}^n \frac{J_i - J_j}{R_{ij}} \quad (7)$$

This formulation of the solution requires that either T_i or q_i be specified for each surface i . It is, however, a simple matter to substitute Equation (5) for (6) or (7) if an inter-surface heat flow q_{ij} is given instead.

Where the external heat flow is governed by external heat exchange with the surroundings, Equation (8) can be used, where the overall heat transfer coefficient, U , incorporates conduction resistances in the enclosure wall and the external convection resistance:

$$q_i = -U_i A_i (T_i - T_\infty) \quad (8)$$

The method usually recommended for solving the linear equations for the radiosities is by a matrix inversion technique or by an iterative method such as Gauss-Siedel [1–3]. Little indication of the accuracy to be expected is given apart from statements such as ‘the method might give a large truncation error for some ill-conditioned matrices, but will always give a solution if the matrix is non-singular’ [1].

Many textbooks now contain computer programs to allow students to solve problems, including radiation enclosure problems. Mills [1] has a shape factor calculation program for a few

standard situations. These shape factors must then be transferred to a second program to generate the solution. The temperature or net heat flow must be specified for each surface. Holman's program [2] is even more restrictive. It solves only for a rectangular cube with all surface temperatures specified, although it does demonstrate well the progression through the shape factor and subsequent calculations.

The method of structuring the spreadsheet to solve for a radiation enclosure calculation is demonstrated for an example with five surfaces. This is a recent assignment problem from the author's course to Year 3 chemical engineering students.

EXAMPLE PROBLEM

The enclosure is a cylinder treated as five isothermal surfaces (Fig. 1). Surface 1 is a disk of radius 0.2 m, from which 10 kW is radiated into the enclosure. Surface 2 is the remainder of that end of the cylinder, being an annular disk with outer radius 0.3 m and inner radius 0.2 m. The rate of heat loss through this surface is governed by Equation (8), with the overall heat transfer coefficient being $10 \text{ W m}^{-2} \text{ K}^{-1}$. Surface 3 is the curved side wall of the 0.5-m long cylinder, and is assumed to be perfectly insulated externally. Surface 4, also perfectly insulated externally, is a 0.15-m radius disk in the far end of the cylinder. 'Surface' 5 is an annular opening between surface 4 and the end of surface 3 that permits loss of radiant energy to the surroundings at 300 K. The surroundings are assumed to be large, so that 'surface' 5 can be treated as a blackbody, also at 300 K.

The specified conditions are thus temperature T_5 , heat flows q_1 , q_3 , q_4 , (explicitly) and q_2 (implicitly). The specified emissivity values can be read from Fig. 2. Because the cylinder ends each contain two co-planar surfaces, it follows that $F_{12} = 0$ and $F_{45} = 0$.

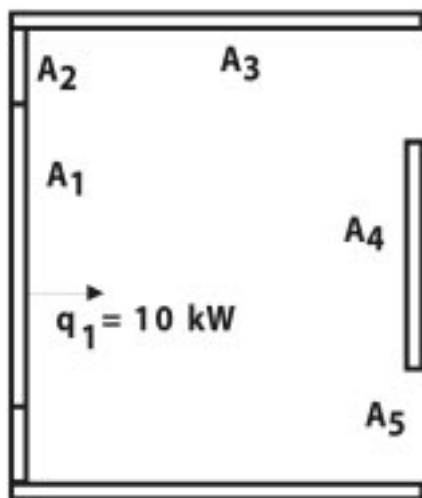


Fig. 1. The cylindrical enclosure with five isothermal surfaces.

Spreadsheet layout

The spreadsheet, shown in Fig. 2, contains the following major blocks:

1. The Stefan-Boltzmann constant and the geometric variables. These values in C4 and C6:C10 are in named cells with the names in column B.
2. The external overall heat transfer coefficient for surface 2 is entered in C11. For this method of specifying the external heat flow, the ambient temperature is also required, but is available in this example as T_5 .
3. Surface-related values are in rows 16 to 20. Areas are calculated from the geometry, while emissivity values must be supplied. Temperatures are entered for all surfaces in D16:D20. If temperatures are not specified for a particular surface, an initial estimate is made. Black-body emissive power is calculated in E16:E20 using Equation (1). Initial guesses for the surface radiosity values are entered in F16:F17. All radiosities will be between the highest and lowest black-body potentials. Where it is known that the radiosity and black-body potential for a surface will be the same, the radiosity is best set equal to the black body potential for that surface, and is not included among the optimization variables (Solver changing cells). In the example, this is done for surfaces 3 and 4 (because the external heat flow is zero) and for surface 5 (which is treated as a black body).
4. The calculation of the shape factors in rows 24:28 is described separately later.
5. Resistances are calculated in rows 32 to 36. Inter-surface resistances are evaluated in B32:F36 using Equation (3). Surface resistances using Equation (2) are in G32:G36. The layout of the spreadsheet is such that the cell formulae for Equations (2) and (3) need to be entered only in one cell before being copied to the remaining cells. Because some shape factors in this example are zero, the resistances between these pairs of surfaces are infinite. These resistances have been assigned an arbitrary high value of 10^{20} to prevent 'divide by zero' errors.
6. Heat flows are calculated in rows 43 to 47 using equations (5) for the inter-surface flows in B43:F47 and (4) for the external flows in G43:G47. For example, $D43 = (\$F16 - D\$38) / D32$ and $G43 = (E16 - F16) / G32$. The sum of heat flows at each node, calculated in H43:H47, should be zero according to Kirchhoff's law. For example $H43 = G43 - \text{SUM}(B43:F43)$. These provide an essential check on the correctness of the solution, but also play a vital part in arriving at the solution. As was the case for the resistances, Equations (5) and (4) need to be entered into only one cell each before being copied to the other cells. To make this possible, the radiosities are also displayed in B38:F38 so that they are available as a row as well as in a column in F16:F20.

	A	B	C	D	E	F	G	H
1	Radiation enclosure example, five surfaces							
2	P J Jordan	9 July 2003						
3	Constants							
4	S-B const.	sigma	5.67E-08	W m ⁻² K ⁻⁴				
5	Geometry and external heat transfer data							
6	Length	L	0.5 m			Colour key		
7	Radius1	rad1	0.2 m				Specified value	
8	Radius2	rad2	0.3 m				Solver changing cell	
9	Radius3	rad3	0.15 m				Solver target cell	
10	Radius4	rad4	0.3 m					
11	H T coeff	U	10.0	W m ⁻² K ⁻¹				
12								
13	Surface Information							
14	Surface i	Areas	Emissivity	Temp T _i	E _{net}	Radiosity		
15		(m ²)	(-)	(K)	(W m ⁻²)	(W m ⁻²)		
16	1	0.126	0.8	1233.32	1.31E+05	1.11E+05		
17	2	0.157	0.6	807.22	2.41E+04	2.75E+04		
18	3	0.942	0.7	901.47	3.74E+04	3.74E+04		
19	4	0.071	0.6	947.75	4.57E+04	4.57E+04		
20	5	0.212	1.0	300.00	4.59E+02	4.59E+02		
21								
22	Shape factors F_{ij}							
23	i\j	1	2	3	4	5	Check	
24	1	0.0000	0.0000	0.7569	0.0727	0.1704	1.0000	
25	2	0.0000	0.0000	0.7996	0.0554	0.1451	1.0000	
26	3	0.1009	0.1333	0.5316	0.0561	0.1781	1.0000	
27	4	0.1292	0.1230	0.7478	0.0000	0.0000	1.0000	
28	5	0.1010	0.1075	0.7916	0.0000	0.0000	1.0000	
29								
30	Resistances	Inter-surface, R _{ij} (m ⁻²)					Surface	
31	i\j	1	2	3	4	5	R _i	
32	1		1.00E+20	10.51	109.50	46.70	1.989	
33	2	1.00E+20		7.96	115.00	43.88	4.244	
34	3	10.51	7.96		1.89E+01	5.96	0.455	
35	4	109.50	115.00	1.89E+01		1.00E+20	9.431	
36	5	46.70	43.88	5.96	1.00E+20		0.000	
37								
38	Radiosities	1.11E+05	2.75E+04	3.74E+04	4.57E+04	4.59E+02		
39							Radiosities copied from above to facilitate heat flow calculations	
40	Calculated heat flows							
41		Inter-surface, q _{ij} (W)					Surface	Node heat balance
42	i\j	1	2	3	4	5	q _i	residual (W)
43	1		0	7025	599	2373	9998.2	1.8
44	2	0		-1254	-159	615	-797.4	0.7
45	3	-7025	1254		-439	6208	0.0	0.7
46	4	-599	159	439		0	0.0	0.8
47	5	-2373	-615	-6208	0		-9196.8	0.0
48		Sum of external heat flows (W)					4.0	
49		Sum of squares of node heat balance residuals					4.91E+00	
50	Specified heat flows							
51		Inter-surface, q _{ij} (W)					Surface	
52	i\j	1	2	3	4	5	q _i	
53	1						10000.0	
54	2						-796.7	
55	3						0.0	
56	4						0.0	
57	5						0.0	
58		Sum of squares of differences between specified and calculated heat flows					3.70E+00	
59		Solver target cell: total sum of squares.					8.60E+00	

Fig. 2. Spreadsheet for the example radiation enclosure after convergence using Solver.

7. Any specified heat flows are entered in B53:G57. The remaining cells in this range are left blank. The cell formula for q_2 is: $G54=-C11*B17*(D17-D20)$.
8. To this stage, all heat flows will be correctly calculated if the correct temperatures and radiosities have been entered. This will be indicated by having zero values for all node sums in H43:H47. The sum of all external heat flows should also be zero and is displayed in G48 as a check. Two measures of the correctness of the solution are provided in cells H49 and G58. H49 contains the sum of the squares of the node sums of heat flows. This is calculated using $H49=SUMSQ(H43:H47)$ and will be zero for an exact solution. The agreement between the calculated and specified heat flows is calculated in G58. This is most conveniently done with the array formula $\{=SUM(IF(B53:G57="","",B43:G47-B53:G57)^2)\}$. This sums the squares of the differences between the specified values in B53:G57 and the calculated heat flows in B43:G47 for those heat flows for which a value has been specified. Again the sum of squares of differences will be zero for an exact solution. Note that it is necessary to use Control/Shift/Enter instead of just Enter after entering or modifying an array formula.
9. The solution is reached from the initial guessed values for temperatures and radiosities by using the built-in optimisation package Solver in Microsoft Excel. This needs a single target cell as the objective function to optimize. Cell G59, which sums the two sums of squares of differences in H49 and G58, provides the target cell for Solver.

Solution of the equations using Excel's Solver

The equations for the radiation enclosure are solved using Excel's Solver. The target cell to be minimized is the global sum of squares calculated

in cell G59 in the example (Fig. 2). The changing cells are the radiosity values in F16:F17 as well as the unknown surface temperatures: in this example those for surfaces 1 to 4 in D16:D19. No constraints are needed. The window for Solver for this example is shown in Fig. 3.

The Solver convergence parameter should be reduced to a smaller value such as 10^{-8} . Otherwise no changes need be made to the default settings. A typical solution will take less than five seconds and require fewer than 100 iterations by Solver. After solving, the accuracy can be readily seen from the heat flow residuals at each node in H43:H47 and the sum of squares between specified and calculated heat flows in G59. If all surface temperatures are specified, the errors are extremely small. In Fig. 2, the largest error is associated with the flows at surface 1 (1.8 W or about 0.02%). This is smaller than the errors that would be caused by uncertainties in the emissivities in a real problem.

Shape factor calculations

In describing the solution method, the calculation of shape factors between surfaces was deferred. Calculation of the shape factors, used in calculation of the resistance between each pair of surfaces, is the most time-consuming part of solving most radiation enclosure problems. Use of Excel for performing these calculations has certain advantages. Firstly, they can be evaluated on the same worksheet as the remaining calculations, bypassing the requirement to transcribe or enter them from a separate calculation medium. Secondly, there are shape factor equations available for many more geometries than are given in the standard textbooks [8]. These can be readily programmed as functions in Visual Basic for Applications (VBA) which is present within any recent version of Excel. The VBA functions can then be used from within Excel as user-defined functions in the same way that Excel's built-in functions are accessed. Programming the required



Fig. 3. The Excel Solver window for the example.

user-defined functions is treated as an integral part of radiation assignments in the author's course.

The shape factors in the example are evaluated in rows 24:28 in Fig. 2. All shape factors are calculated by using the VBA functions or applying the basic equations of shape-factor algebra such as the reciprocity rule. For example, F_{15} is obtained using the cell formula: =UnequalDisks(rad1, rad4,L)-UnequalDisks(rad1,rad3,L). This cell formula uses the fact that $F_{15} = F_{1(4+5)} - F_{14}$. UnequalDisks is the name of the function for the shape factor from a disk to a parallel coaxial disk of unequal radius [8, case C35]. The other shape factor equations supplied to the class were Howell case C45 [8] for an annulus to a parallel coaxial annulus, and Howell case C75 [8] for the shape factor between a disk in a cylinder base and the inside of a right circular cylinder. The code for this last case is given in the Appendix.

The fact that the shape factors from any surface to the other surfaces, 1 to 5, must sum to unity has not been used in calculating any shape factors. This is then used as a check in column G.

Alternative treatment of the radiosity calculation

In the above procedure, those surface radiosities known to equal the surface blackbody potentials were excluded from the set of Solver changing cells. An alternative is to use only the unknown temperatures as the Solver changing cells and to rearrange Equation (4) to Equation (9) to solve for the remaining radiosities:

$$J_i = E_{bi} - q_i R_i \quad (9)$$

Since Equation (4) is also used to solve for q_i , this creates circular references. As long as iteration is enabled, Solver will usually converge to the correct temperature solution. The advantage of this approach is that the sum of squares of the node heat balances is reduced almost to zero. However, depending on the order in which the spreadsheet equations are set up, it is possible for the calculation to get trapped in an incorrect solution. The extra effort required to recover from this occasional problem means that this alternative method is not recommended, despite the slight improvement in accuracy.

TEACHING APPROACH

Discussion of how to use spreadsheets to solve problems, where appropriate, is now an integral part of the author's heat transfer courses. The underlying theory is first taught using conventional 'chalk and talk' methods. Methods of structuring the calculations for solution on a spreadsheet are then discussed, supported by demonstration of the creation of a spreadsheet and solution of a problem using a data projector in the lecture room. Issues discussed include whether or not to use named cells for particular

variables, and such tips as repeating the radiosities in a row as well as the column to permit easier copying of formulae. In the lecture demonstration, using coloured cell backgrounds is a considerable aid in clarifying the role of various cells. For example, one colour is used for the entry of geometric data, another for specified temperatures and heat flows, and another to highlight Solver adjustable and target cells. During construction of the spreadsheet, it is also instructive to use one colour for cells into which a computational formula must be typed, and another colour for cells into which the formulae are copied.

An assignment problem, such as the example problem in this paper, provides an exercise in which the students must apply all aspects of the theory taught. Requests for the demonstration spreadsheet to be made available electronically are declined, as this would potentially degrade the assignment into an exercise of putting the numbers into a prepared program. Students are permitted to work individually or in pairs, and this seems to eliminate the temptation to plagiarise others' spreadsheets. The mean time taken for the assignment is 7 hours (range 4 to 10 hours), including the programming of the necessary VBA functions for the shape factors.

Students are expected to demonstrate how they have verified the correctness of their VBA functions and their spreadsheet solutions. Emphasis is also placed on the value of good documentation of spreadsheets. This is reinforced by allocating 30% of the assignment marks to the quality of the documentation. Assignments are handed in only in hardcopy form, and the documentation is expected to permit verification of answers and, if desired, complete construction of the spreadsheet from the hard copy and documentation submitted. The expected documentation on the worksheet includes a statement of the aims and assumptions, the equations used, sufficient sample cell formulae to permit checking that the equations have been correctly coded, and instructions on manual operations needed such as the use of Solver. Much of this has been omitted from Fig. 2 in the interests of space. The documentation of spreadsheets is an aspect neglected in textbooks on spreadsheet use, even those aimed at engineers and scientists. Approaches have been described by Morison and Jordan [9], but there is scope for considerably more discussion and consensus on the subject.

STUDENTS' RESPONSE

The students have responded positively to being taught how to apply spreadsheets to solve a range of problems. By the time they reach this course in the third year of their degrees, spreadsheets are their preferred computation tool and they are keen to extend their repertoire of spreadsheeting techniques. They are by then competent in the use of

Solver, but have not usually encountered array formulae before.

Typically two thirds of the class will opt to solve the problem with a partner. The remaining third produce an individual solution. The mean time taken for the assignment does not vary significantly between those doing an individual effort and those working as a team.

Feedback from recent graduates indicates an appreciation of the emphasis on documentation of spreadsheets in this and other courses in the author's department.

DISCUSSION

The pre-packaged programs that accompany some texts enable students to solve certain radiation problems. However they have some drawbacks. The main skill developed is that of entering numbers into menus, rather than gaining understanding of the underlying structure of the problem. Students cannot see the underlying code in the program to understand what it is doing.

Provision of pre-packaged programs is a response to students' lack of time to program the calculations themselves and to an increasing lack of fundamental programming skills in traditional programming languages. However students are now developing considerable skills in applying spreadsheets to a wide range of calculations. Ideas developed in solving one problem can often be transferred to other apparently unrelated problems. The approach used here encourages and takes advantage of this.

The spreadsheet-based approach to teaching radiation enclosure calculations has been employed for the past six years, building on the earlier introduction of spreadsheet methods for steady-state and transient conduction calculations. It has proved to be an effective means for conveying the essential relationships in the problem. It has enabled students to tackle and solve non-trivial problems from first principles in a way that enhances their problem-solving skills. The use of VBA as the language of instruction for computer programming has aided the use of this approach. The students learn VBA in the year prior to the thermal radiation topic. Because they are able to use VBA to enhance their spreadsheet calculations, they use it sufficiently often that writing code for user-defined functions is a straightforward part of the process.

The treatment of the external flow for surface 2 in the example deals with a common situation, where the external heat loss depends on the surface temperature, which is not discussed at all by the standard texts [1–3].

The solution method demonstrated in the example provides a very flexible way of specifying and solving radiation problems. For example, to

change from specifying a surface temperature to specifying a heat flow, it is simply a matter of:

- entering the specified heat flow, or calculation using Equation (8), in the 'specified heat flows' section of the spreadsheet;
- entering an initial guess for the temperature that is now unknown;
- solving the problem again with Solver, after adding the new unknown temperature to the set of changing cells.

The main parts of the spreadsheet that contain problem-specific formulae are the area and shape factor calculations. The remainder of the spreadsheet can be used as a template to apply directly to other problems. This suggests that there is merit in doing the area and shape factor calculations on a separate worksheet so that there is a geometry-specific part and a generic part of the solution on separate worksheets. However the author's preference is to have all calculations on a single worksheet, when this is not excessively large. The template developed for a problem with, say five surfaces, can be easily extended to more surfaces by inserting rows and columns as appropriate and then copying the necessary resistance and heat flow formulae into the extra cells.

The solution method has proved to be very robust. If the sum of squares of deviations cannot be reduced to a reasonable value by Solver (say less than 10^2), then this has invariably been found to be the result of inconsistent specification of the set of temperatures and heat flows. When all temperatures are specified, convergence to the solution is extremely rapid and the sum of squares of deviations is trivial. In the example in [7], one surface was not involved in any of the specified quantities. In such a case, the accuracy is not quite so good, but the errors in nodal heat balances will typically be well below 1%. The information displayed on the spreadsheet makes it very easy to see how well all equations are satisfied, compared with matrix-inversion methods where there is generally no information readily available about the accuracy of the result.

Occasionally Solver will reach a solution with one of the calculated temperatures being negative. This will happen for surface 4 in the example from many initial guesses. Because the temperatures, apart from T_2 , are used only in calculating the blackbody potentials, the sign of any other temperature can simply be changed to positive. The use of constraints or other Solver options to prevent negative temperature results can hinder Solver's progress to the solution.

The approach used in this paper can be equally applied to any network problem where there is flow through branches of the network determined by the relationship:

$$flow = \frac{potential\ difference}{resistance} \quad (10)$$

CONCLUDING REMARKS

Using a spreadsheet provides a convenient and flexible tool to solve radiation enclosure problems. It builds on the considerable skills students already possess in spreadsheet computation and

provides an opportunity to reinforce the necessity for good practices in documenting spreadsheets. The change to this approach to teaching calculations has been stimulating and satisfying, and the students have responded very positively.

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APPENDIX

Sample VBA Code for shape factor calculation

Function DiskToCylinder(dR1, dR2, dH) As Double

'Calculates radiation shape factors for a disk in a cylinder base or top to inside of a right circular cylinder
'using the equation for Case C75 in Howell (1982).

'dR1 = radius of disk

'dR2 = radius of cylinder

'dH = length of the cylinder

'Written by P J Jordan, 25 March 2002

Dim dRR As Double, dHH As Double

dHH = dH/dR1

dRR = dR2/dR1

DiskToCylinder = 0.5 * (1 - dRR ^ 2 - dHH ^ 2 + ((1 + dRR ^ 2 + dHH ^ 2) ^ 2 - 4 * dRR ^ 2) ^ 0.5)

End Function

Patrick Jordan is a senior lecturer in the Department of Chemical and Process Engineering, University of Canterbury where he has taught for thirty years. He uses spreadsheets extensively in research and teaching, particularly in the teaching of heat transfer. He also teaches spreadsheeting, optimisation, computational fluid dynamics and process modelling and dynamics. In conjunction with Ken Morison, he has run several continuing professional development courses for engineers and scientists in Excel and in VBA.