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Applications of a Spreadsheet-based Wavelet Analysis Toolbox in Education*

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The relatively new Wavelet transform and Wavelet-based algorithms have found applications in virtually all engineering and many scientific disciplines. In this paper we point out their applications in enhancing education both from a teaching point of view and in carrying out laboratory-based experiments and projects. We take advantage of Excel's wide availability and familiarity to students in using it as a medium to bring the power of Wavelet analysis to the classroom and the lab. We give an overview of the implementation the Wavelet transform, inverse transform, and denoising algorithms into Excel, which was realized through a Dynamic Link Library (DLL) and Visual Basic for Applications (VBA) code. We present four examples that relate to education and research. We highlight the advantages of wavelets in detecting trends and events, removing noise, and achieving data compression.

INTRODUCTION

WAVELETS WERE DEVELOPED independently in the fields of mathematics, quantum physics, and electrical engineering but interactions between these fields during the past few years have led to an upsurge in the number of wavelet applications and in the amount of wavelet-related research, both at the applied and theoretical levels [1]. Just to mention a sample, wavelets have been applied in the areas of compression and fast transmission of medical images [2], video compression [3], image retrieval and digital libraries [4], digital communication [5–6], power electronics [7], antennas and electromagnetism [8], and even fluid dynamics [9]. Basically, any field that involves data processing and analysis can benefit from wavelets due to their properties that include the simultaneous localization of signal components in the time and frequency domains, which is not possible with Fourier analysis. Consequently, wavelets are increasingly entering classrooms and laboratories in the form of both theoretical and applied subjects. Our objective in this paper is to take advantage of the simplicity and familiarity of spreadsheets to instructors, students, and other computer users and encourage the exploration of wavelet analysis and its applications in educational subjects and research activities.

WAVELETS AND ALGORITHMS

Like the fast Fourier Transform (FFT), the discrete wavelet transform (DWT) is a fast and linear operation that operates on a data vector to transform it into a numerically different vector. Also like the FFT, the wavelet transform is invertible and in fact orthogonal. Both FFT and DWT can therefore be viewed as a rotation in function space, from the input space (time domain) to a different domain. For the FFT, the new domain has basis functions that are the sines and cosines. The wavelet basis functions on the other hand are quite localized in time and simultaneously like sines and cosines, are also localized in frequency (scale). This duality in localization is what makes the DWT a valuable tool when dealing with a large class of problems.

Wavelet transform coefficients

For a data vector $y_1, \ldots y_M$, the DWT computes another vector of N coefficients [10]:

$$\mathbf{w} = \{d_{j,k}, s_{J,m}\}, j = J, \ldots, \log_2 N, \quad k = 1, \ldots, 2^{j-1}, m = 1, \ldots\ 2^{J-1} \tag{1}$$

where J is such that $1 \leq J \leq \log_2 N$ and $N = 2^K$, the number of coefficients, is the smallest power of two that is greater than or equal to the number of samples in the signal, M. The parameter j is called the resolution level or scale while k is called the translation or shift index. The coefficients denoted by $s_{J,m}$, which make up the vector $\mathbf{s}_J$, represent the smooth behavior of the data at the coarsest level J while the coefficients $d_{j,k}$, which make up the vectors $\mathbf{d}_j$, represent progressively finer scale deviations from the smooth behavior. The coefficients $s_{J,m}$ and $d_{j,k}$ are known as the smooth and detail coefficients respectively. The vectors $\mathbf{d}_{\log_2 N}$, $\mathbf{d}_{(\log_2 N)-1}$, $\mathbf{d}_{(\log_2 N)-2}, \ldots, \mathbf{d}_J$ contain progressively coarser-level coefficients and having lengths $N/2$, $N/4, N/8, \ldots, N/2^{J-1}$ respectively.

The wavelet transform coefficients of a data

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vector are computed hierarchically through convolution with the odd and even rows of a transformation matrix to produce the vectors $\mathbf{d}_j$ and $\mathbf{s}_j$ respectively. The elements of the mostly-sparse matrix are simple derivations from the wavelet filter coefficients [11]. For the first (lowest) resolution level, coefficients are produced from the data vector but for the remaining levels, it is the smooth coefficients of the previous level that are used to produce the coefficients. To reconstruct the signal, the wavelet coefficients are convolved in a similar manner by the rows of the inverse transformation matrix.

Wavelet properties

Unlike sines and cosines, which define a unique Fourier transform, there is not one single unique set of wavelet filters. Actually, there are infinitely many possible sets. In general, the different sets make different trade-offs between how compactly they are localized in space and how smooth they are. (There are also further fine distinctions.) A special family of wavelets that was discovered by Ingrid Daubechies [12] and consists of four classes is described below:

1. The Haar wavelet is a square wave and has compact support. It is the only compact orthogonal wavelet, which is symmetric. However and unlike the other wavelets, the Haar wavelet is not continuous.
2. The Daublets (or Daubechies) were the first type of continuous orthogonal wavelets with compact support.
3. The Symmlets also have compact support. While the Daublets are quite asymmetric, the Symlets were constructed to be as nearly symmetric (least asymmetric) as possible.
4. The Coiflets were constructed to be nearly symmetric and also have additional properties thought to be desirable—vanishing moments, which are useful in compression applications.

Signal denoising

Wavelet-based denoising is based on the Waveshrink algorithm that was developed by Donoho and his colleagues [13–15]. The algorithm works by shrinking the wavelet coefficients of a data vector, that typically includes additive noise, and then inverse-transforming them to obtain a denoised approximation of the true underlying data. There are several shrinkage methods that basically differ in the way the threshold is computed and to which coefficients it is applied. The details of these methods are beyond the scope of this paper but a somewhat detailed treatment can be found in [5].

IMPLEMENTATION IN EXCEL

The wavelet transform, inverse transform, and denoising code is quite involved (for an example implementation of the transform and its inverse, see [16]) and hence, its implementation using elementary Excel functions is highly unmanageable and impractical. A more straightforward approach that benefits from publicly available open source code and works with Excel is much more attractive. The three algorithms were implemented as a dynamic link library (DLL) that was written in C++. The DLL interacts with Excel as a Common Object Model (COM) object. This latter is a Microsoft technology that represents a mechanism allowing programs to communicate. The DLL becomes accessible to Excel after being added to Window's Registry and proper communication with it is made possible by way of adding its type library to Excel through the menu of its Visual Basic Editor. This allows for correctly marshalling (converting to a 'neutral' data type) Excel's data so they are correctly received by the DLL code. The VBA code of Excel calls the functions of the DLL through interfaces, which are specified in an Interface Definition Language (IDL) file. Each interface includes the set of data types and the set of functions to be executed from Excel. Interfaces specify the function prototypes for remote functions and for many aspects of their behavior from the point of view of interface users.

As seen in Fig. 1, the DLL exposes three interfaces through which the Excel forms send and receive data. Three forms are provided, which allow users to run the wavelet transform, inverse wavelet transform, and the wavelet-based de-noising algorithm.

User Interface

To allow the user to control the actions of the wavelet transform, inverse transform, and denoising algorithm, three user interfaces were designed using VBA. These are shown in Fig. 2. In particular, the wavelet transform form allows for a great flexibility in picking the desired wavelet filter, the number of applicable vanishing moments, the number of resolution levels, and the format of the presentation of computed wavelet coefficients on the Excel sheet. Two options were implemented. The first one allows for isolating coefficients per resolution level while the second arranges the coefficients into groups that progressively contain more detail coefficients. The first option can be used to reconstruct the data from coefficients belonging to a single resolution level while the second one allows for adding more details to the reconstruction process and hence, acting as a low pass filter with a controllable cutoff frequency.

The inverse transform form inherits all the options from the coefficients but the denoising form allows for selecting one of the available shrinkage methods. Additionally, the user is able to select the specific wavelet to be used for the computation of the coefficients which get subjected to shrinkage. The choice of the wavelet greatly

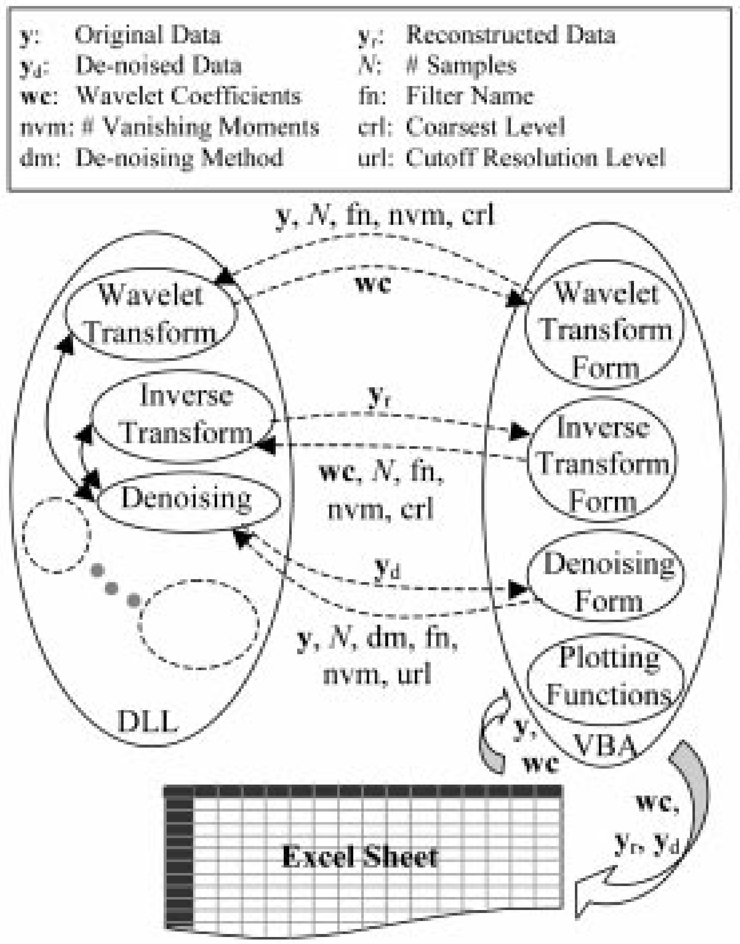

Fig. 1. Interaction between the Excel forms and the DLL's implementation. The dotted circles in the DLL represent support math and signal processing functions that are used by the wavelet transform, inverse transform, and de-noising functions.

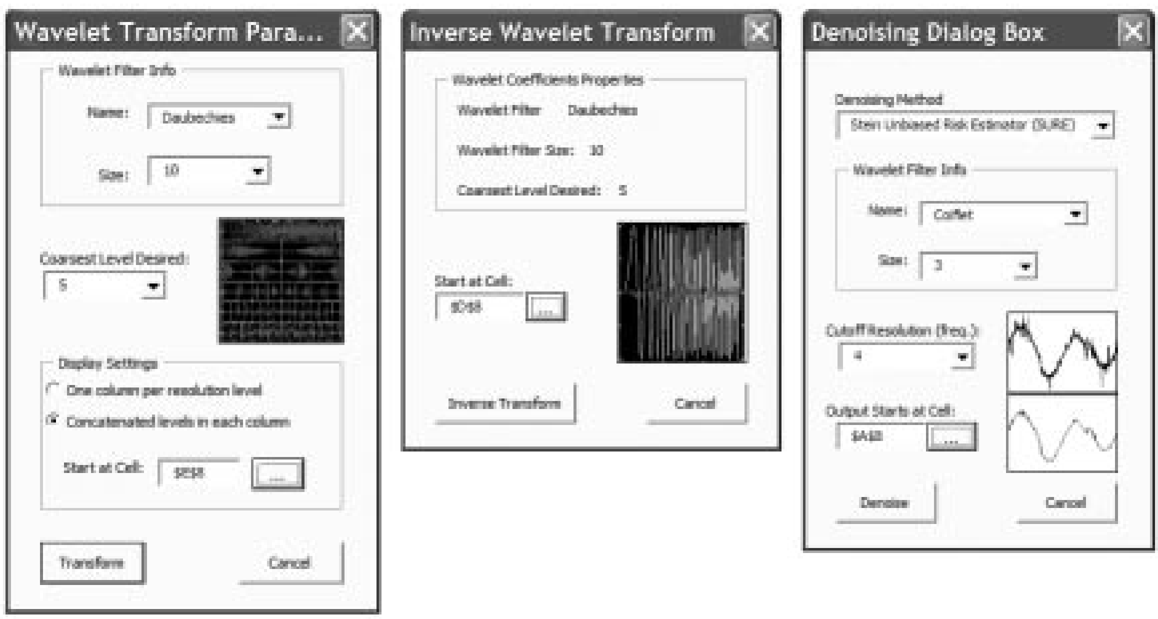

Fig. 2. User Interfaces for interacting with the wavelet DLL. The left window allows the user to control the actions of the wavelet transform, the middle one allows for performing the inverse transform, and the right one allows for controlling the denoising algorithm.

impacts the results as certain wavelets are able to highlight particular features within the data more than others. As an example, the Haar wavelet is known for making discontinuities more pronounced in the wavelet coefficients when compared to other wavelets.

Comparison with Matlab's Wavelet Toolbox

The obvious package to compare the developed toolbox to is MATLAB's Wavelet Toolbox. In this paper, it is not claimed that the developed toolbox has advantages over MATLAB's toolbox in terms of computational capabilities and functionalities.

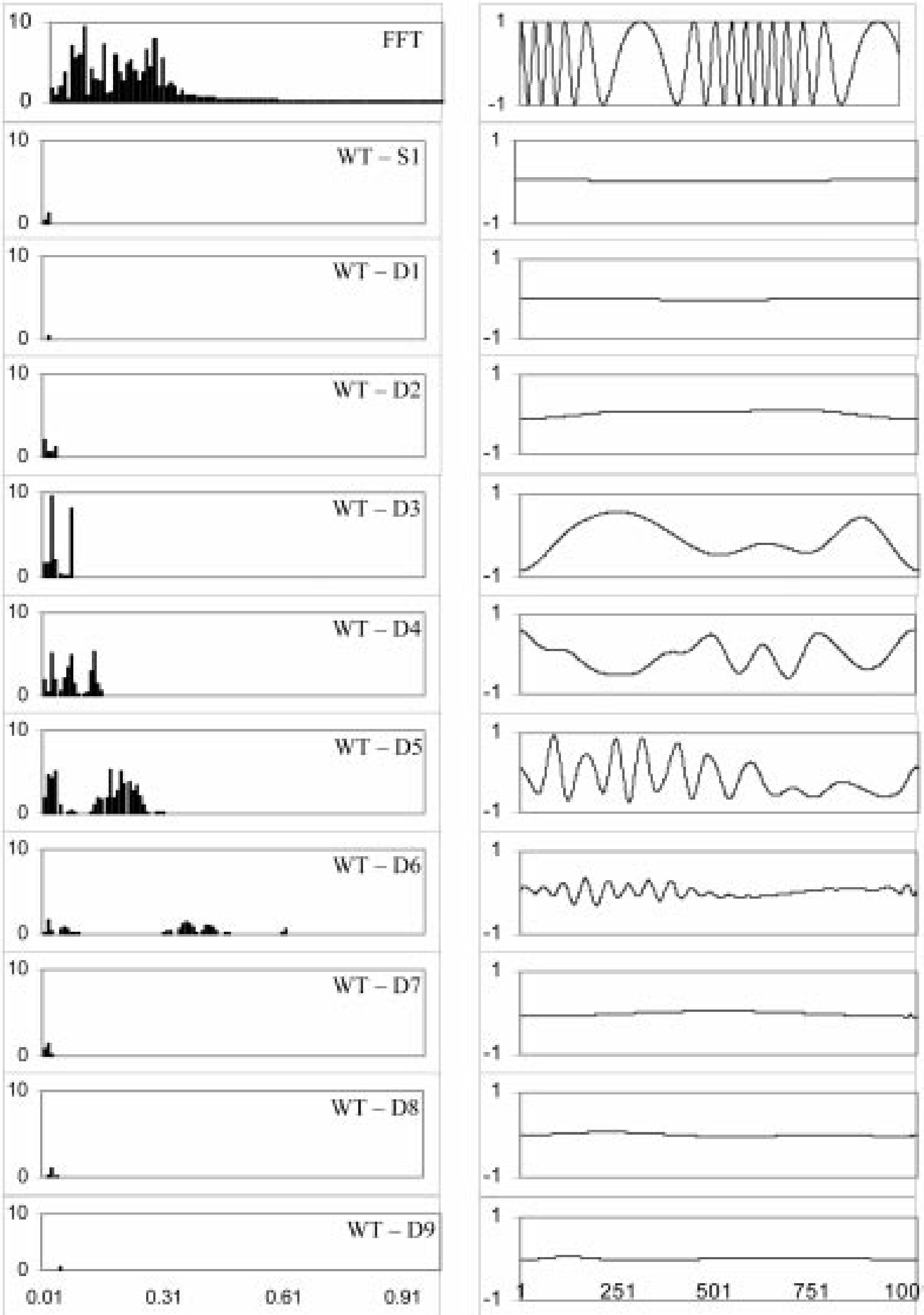

Fig. 3. Multiresolution analysis of an FM communication waveform. The top two plots show the FFT transform (left) and the FM waveform (right) while the remaining plots show the wavelet transform at each resolution level and the corresponding inverse transform.

The developed toolbox however can be used wherever Excel is present and inherits from it its flexible data viewing capabilities, ubiquitous accessibility, and interface simplicity. We also note that the code of the developed toolbox, including the C++ code that compiles into the DLL and the VBA code of the Excel forms, is available from the author for educational purposes, at no cost. This allows students and instructors to make changes, add functionality, or customize the toolbox to perform special functions.

APPLICATIONS IN EDUCATION

In this section, we present four applications in education and research. The first two are in the areas of analog communications and antenna theory and explore the capability of multiresolution analysis. The other applications are in the areas of feature detection and signal denoising, which are topics that relate to many research fields and are encountered in several graduate courses.

Example in Analog Communications

Figure 3 shows several plots that belong to an FM signal. The top left plot shows the FFT of the signal while the top right one shows its inverse transform, which produces an exact replica of the original waveform. Starting from the second row from the top and going down, we show the amplitude of the wavelet coefficients of each resolution level and the corresponding inverse transform. We start with the smoothest coefficients at the lowest level and end up with the detail-most coefficients at the highest level (the signal was represented with 1024 samples). The smooth coefficients render the trends in the data while the detail ones highlight events, sudden changes, and noise. The figure shows that there were insignificant details beyond level 6, which leads to a very significant discovery: The entire signal can be represented with $1024-(512+256+128)=128$ samples with negligent losses in terms of reconstructed signal quality. This property of the wavelet transform can positively impact the bandwidth utilization if the wavelet coefficients are transmitted across the communication channel in place of the signal samples themselves. Such properties can be further explored by students using laboratory experiments not only in the area of analog communications but in the fields of digital communications and computer networking as well.

Example in antenna design

Figure 4 show polar plots of the array factor of a 10-element antenna array. The top left plot corresponds to the entire data while the remaining plots represent the array factor for given resolution levels, reconstructed from corresponding groups of wavelet coefficients. Basically, each plot represents the response of the antenna array to a particular range of frequencies, as determined by

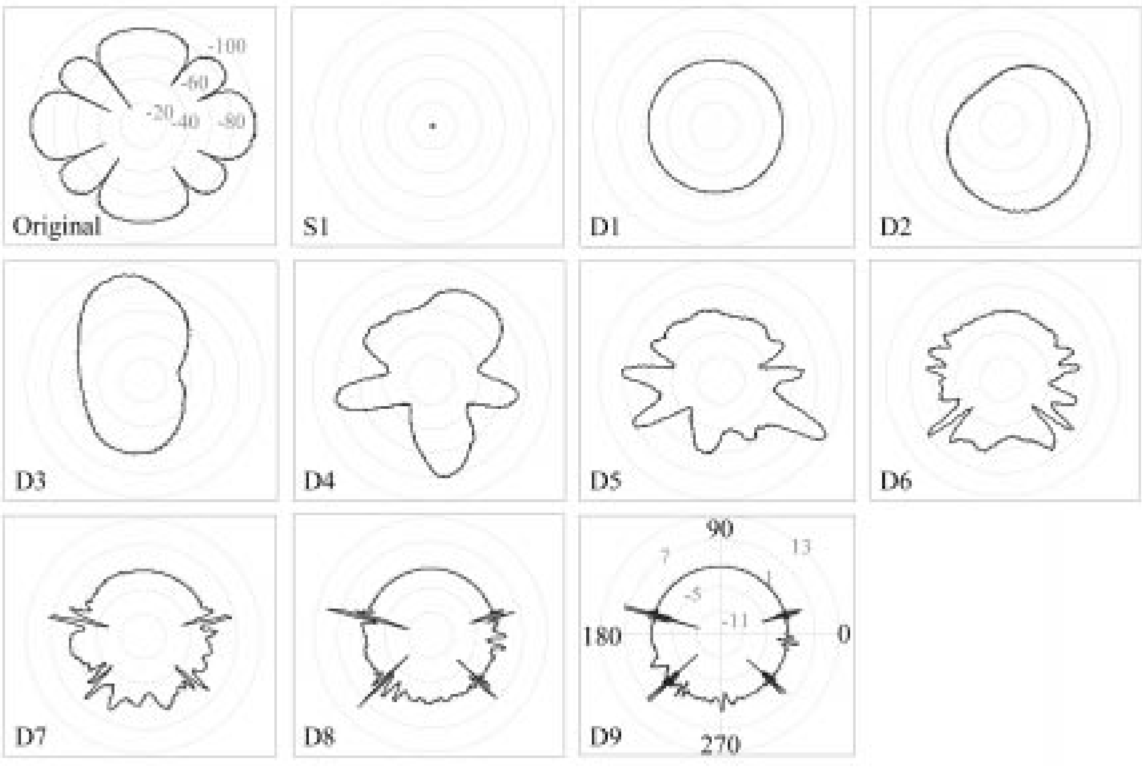

Fig. 4. Polar plots of the Array Factor of a 10-element antenna array. The top left plot shows the array factor for the original data while the remaining ones show the inverse transform of the wavelet coefficients at each resolution. The scale used for the plot of the original data is 0 to 100 while the one used for all the remaining ones is -17 to 13.

the corresponding level. Educators, students, and researchers can use these findings to explore the behavior of antenna systems under certain conditions, e.g., combinations of phase angle and frequency ranges. The plots also reveal trends and sharp fluctuations that could only be isolated to various degrees by using the power of wavelets.

Example in feature detection

One of the benefits of wavelet analysis is the ability to detect and locate hidden trends and events within the data. Basically, events represent sharp and sudden fluctuations in the data that may not be obvious when the data series is visually inspected. Figure 5 shows two examples of waveforms that include obvious jumps in the data and illustrate the ability of the wavelet transform to identify those jumps through the magnitude and the positions of the transform coefficients. The first plot is about a sine wave with increased and decreased amplitudes during certain intervals. The wavelet coefficients clearly indicate the relative value of each jump and its position. This property is clearly demonstrated in the example of the second plot.

In both plots, the shown coefficients belong to the highest detail resolution level. Since this level contains $N/2$ coefficients (N is the number of samples that make up the waveform), we had to interleave the coefficients with $N/2$ zeros to align the coefficients with the samples of the signals. Once this is done, the analyst is able to pinpoint the exact positions within the waveform using the positions of wavelet coefficients that have amplitudes above a certain threshold.

Example in denoising

Figure 6 demonstrates the capabilities of the non-linear denoising algorithm. The two shown plots illustrate the effectiveness of the algorithm when used with continuous as well as non-continuous waveforms. Using the cutoff level (see the screen dump in the far-right of Fig. 2), which is the level above which shrinkage is applied, the user can control the degree of denoising. An aggressive level can cause loss of signal information while a relaxed one does not remove all the noise. Second, it is important and as mentioned before, to select the appropriate wavelet filter. In the lower plot for example, the discontinuous Haar wavelet was used to denoise the square-like noisy waveform. By comparing this denoised waveform of this figure to the bottom-left plot of Fig. 5, a close approximation of the original data was obtained from a severely noise-corrupted data.

CONCLUSION

In this paper we focused on the applications of an Excel-based implementation of the wavelet transform, inverse wavelet transform, and the wavelet-based denoising algorithms in education. We presented the user interfaces of the developed toolbox, which were implemented using VBA to allow users to perform different types of data analyses. We have shown through two applications the potentials of wavelets in uncovering trends and details within the data using multiresolution analysis. We presented two other applications that demonstrate the role of wavelets in localizing

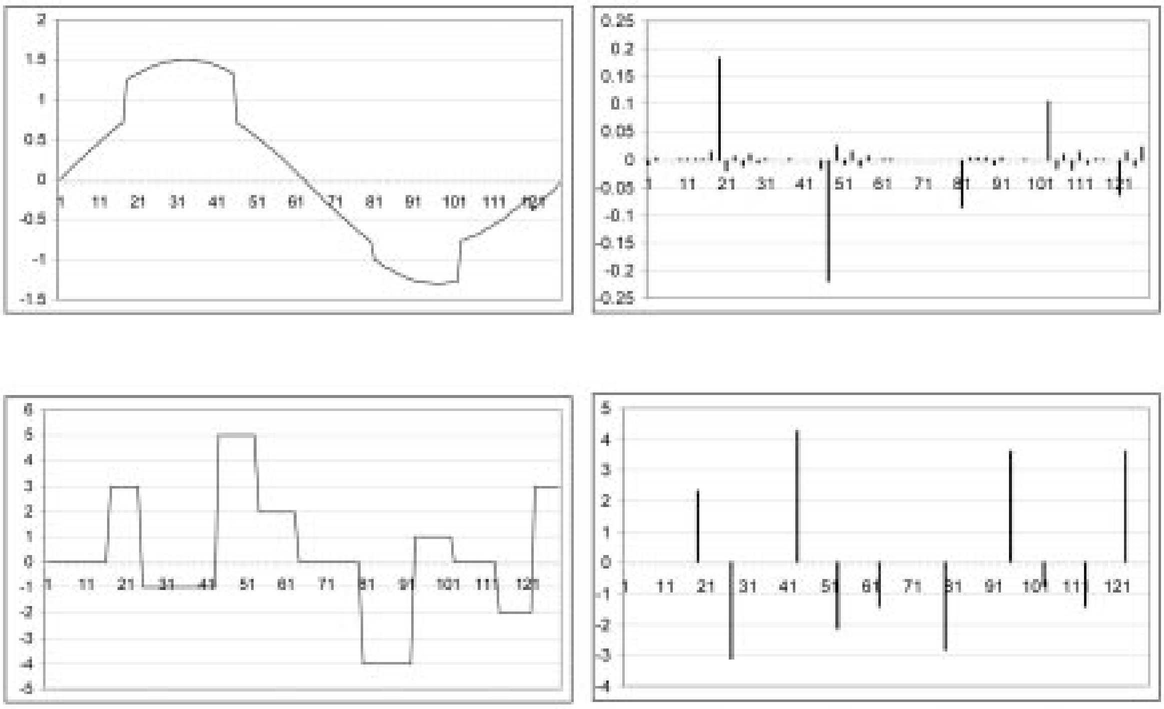

Fig. 5. Feature detection and localization examples. The figure shows the plot of the waveform data (left) and the corresponding detail coefficients (right). The magnitude, polarity, and location of each coefficient clearly describes a particular event in the waveform.

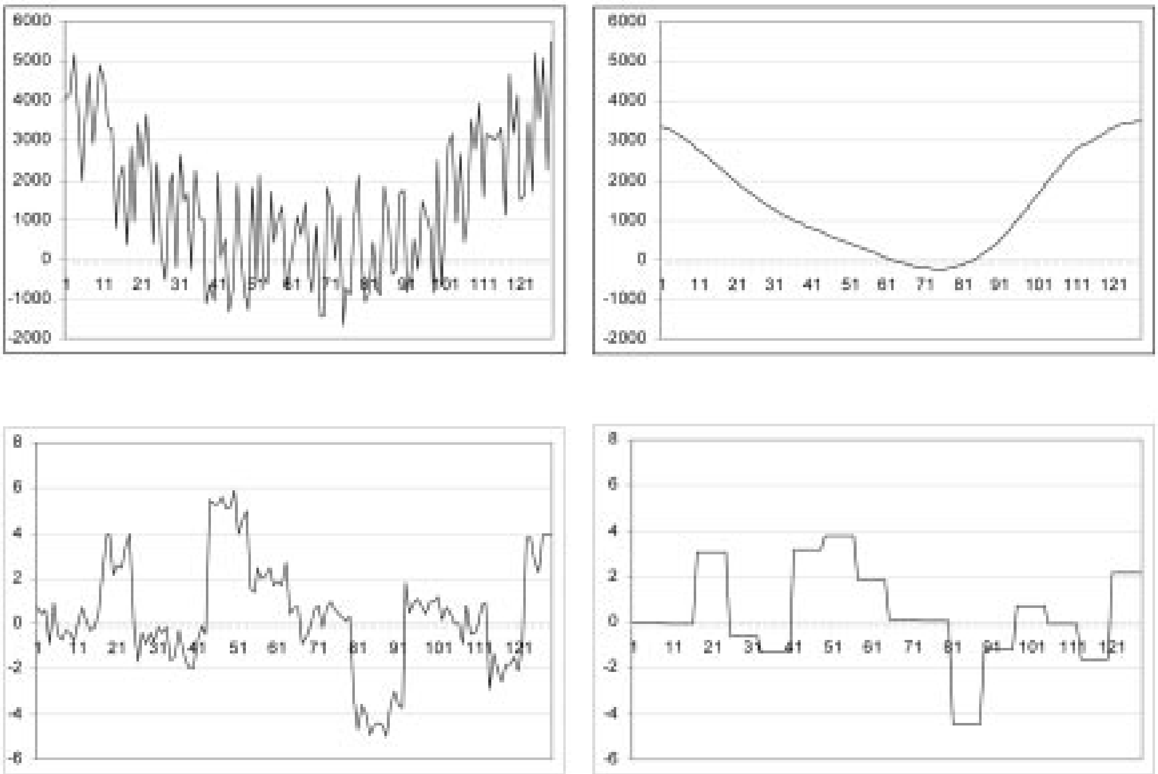

Fig. 6. Signal denoising examples that demonstrate the effectiveness of the denoising algorithm in removing noise both from continuous and discontinuous waveforms.

features and in effectively removing noise from data. The popularity and wide use of Excel among students and instructors represents a motivating factor for using such a tool for all types of data and signal analyses within classroom and lab settings.

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