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An Investigation into Engineering Graduates' understanding of Probability Theory*

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This paper presents an investigation into the understanding of probability theory by recent graduates in engineering. The investigation included a specially set one-hour examination in probability theory taken by over 40 graduate engineers. The answers to the exam questions showed that engineering graduates could remember only a limited number of concepts from probability theory they had been taught as undergraduates. Since probability theory is generally taught as a special mathematics topic, this means that many probabilistic concepts are quickly forgotten by the time that students graduate. A solution to the problem is to incorporate probabilistic concepts into mainstream engineering subjects like design and solid mechanics and materials.

INTRODUCTION

ENGINEERING DESIGN and engineering management in industry involves dealing with uncertain data and uncertain events. This is particularly the case in the early phases of design and at the tendering stage [1, 2]. In order to cope with uncertainty, industry has been adopting more design and management methods that include probabilistic concepts [3–6]. It is therefore important that graduate engineers understand concepts in probability theory such as probability distributions and failure rate distributions. This paper presents an investigation into the understanding of probability theory by engineers who have recently graduated.

THE EXAMINATION

The investigation involved a one-hour examination with a group of recent graduate engineers and an analysis of the answers. The examination contained eight questions. In total, 41 graduate engineers with a wide range of experience took part in the examination. The graduates came from a wide range of institutions and the exam was carried out at Bristol University. A profile of the engineers' background experience is shown in Fig. 1. The graduates had an average of 1–2 years of industrial experience.

The graduates were not told anything about the subject of the exam so they could not carry out any study for the exam. In addition, they did not have access to books. Each of the graduates had taken an engineering course where probability theory was taught. They had encountered probability teaching typically in the second year as part of a maths unit.

This paper describes each question of the exam and discusses the answers given by the graduate engineers. For each question, brief observations are made about the understanding of the graduate engineers. At the end of the paper there is a general discussion and conclusion.

A standard deviation

The first question in the exam tested the graduate's understanding of a standard deviation. Standard deviations are often used in engineering to represent the confidence that is held with regard to a particular result set. In addition, standard deviations are used frequently by risk analysis and finite element analysis software packages to represent uncertainty.

The first question is shown in Fig. 2(a). The graduate engineers had to choose one of four answers, A, B, C or D. Figure 2(b) shows the range of answers given by the graduate engineers. The correct answer (C) is indicated in black.

QUESTION 1 What is the probability of a value falling within one standard deviation (σ), of a given sample mean? Select A, B, C or D.

As can be seen from the results in Fig. 2(b), only 39% of the graduate engineers gave the correct value for one standard deviation. When it is taken into account that there were only four options and a score of 25% would be expected if all graduates were guessing, 39% is not a good score.

Three standard deviations

The second question in the exam tested the understanding of the graduate engineers

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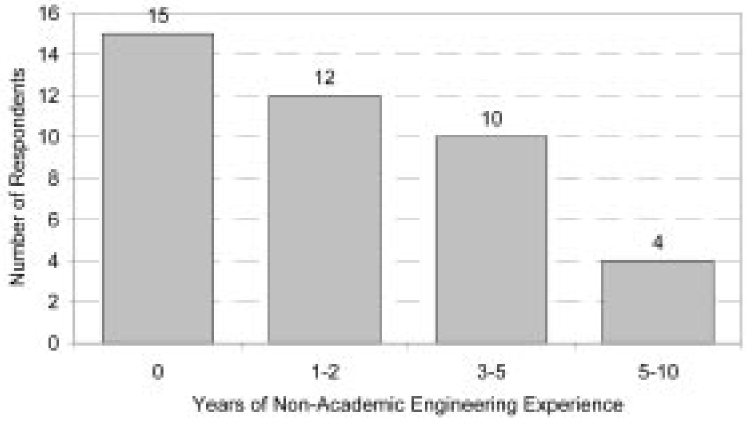
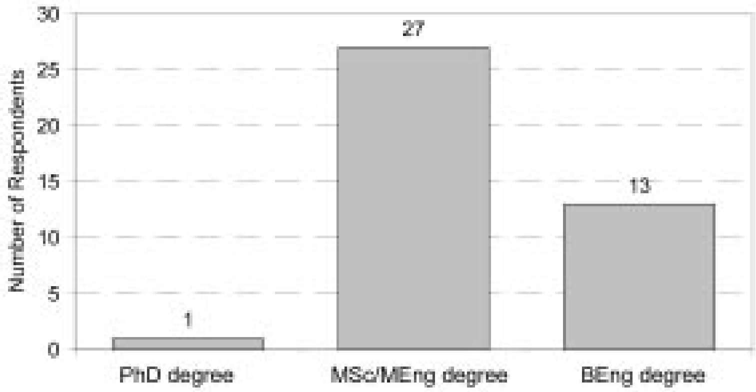
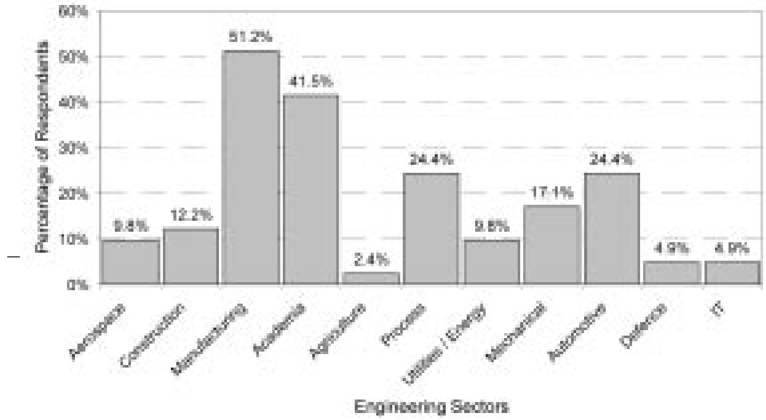


(a) Graduates' years of industrial experience



(b) Graduates' engineering qualifications (or equivalent)

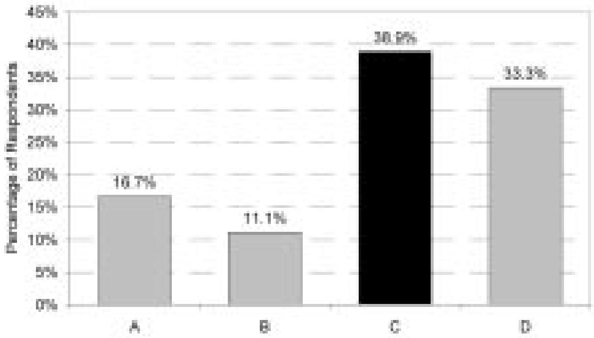


(c) Graduates' experience in engineering sectors

Fig. 1. Experience of the graduate engineers in the study.

A	B	C	D
58.26 %	63.26 %	68.26 %	73.26 %

(a) Question 1



3. Distributions of answers to Question 1

Fig. 2. Question on one standard deviation.

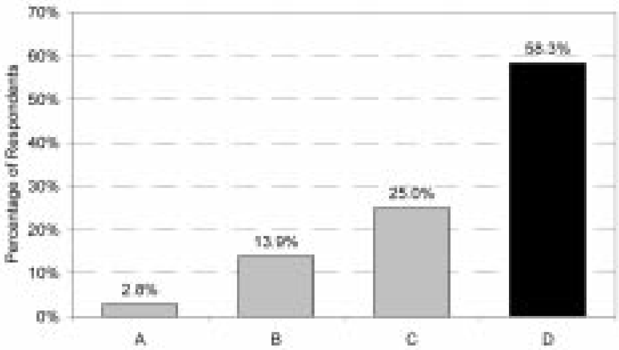
concerning three standard deviations (3σ). This question is interesting as engineers often use three standard deviations to signify high confidence in a value. Question 2 is shown in Fig. 3(a).

QUESTION 2 What is the probability of a value falling within ± three standard deviations of a given sample distribution (also known as 3σ)?

Figure 3(b) shows that 58% of the engineers gave the right answer to Question 2 which is significantly higher than for Question 1 where only 39% of graduates answered correctly. From this it is clear that graduate engineers are more aware of the value of three standard deviations than one standard deviation. The fact that three standard deviations is sometimes used in engineering design and analysis clearly helps engineers to remember the value of three sigma.

A	B	C	D
93.73 %	95.73 %	97.73 %	99.73 %

(a) Question 2



(b) Distribution of answers to Question 2

Fig. 3. Question on three sigma.

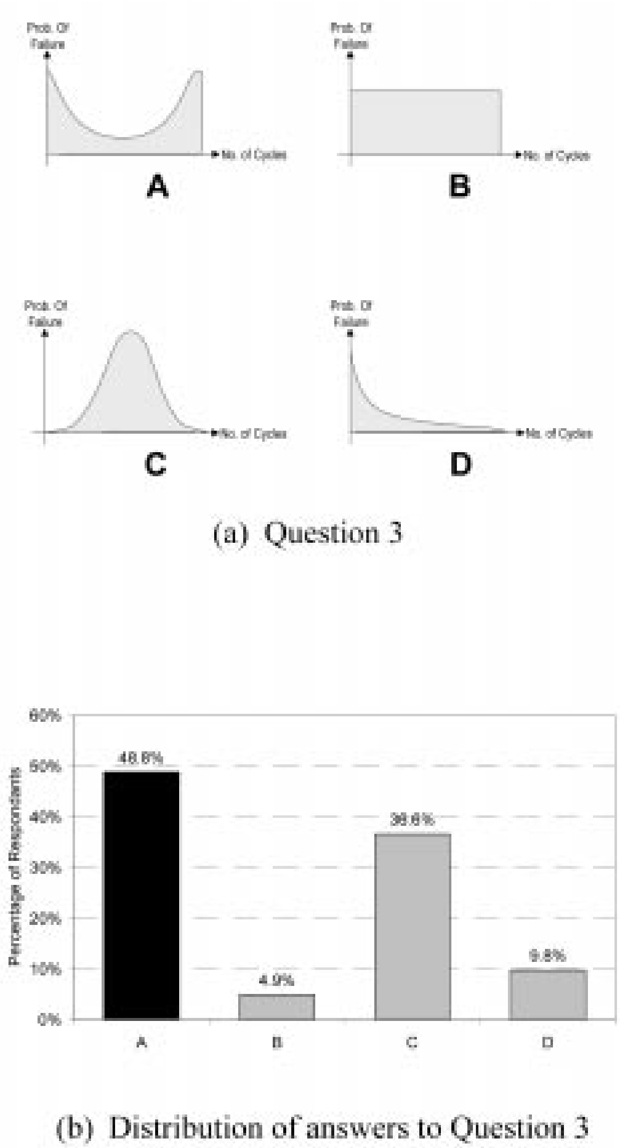


Fig. 4. Question on failure rate distributions.

Failure rate distributions

Failure rate distributions are an important concept in design. To test graduates' understanding of failure rate distributions, a question was given as shown in Fig. 4(a).

QUESTION 3 A new mechanical product has been manufactured. During product testing it was observed that the product failed over the course of its lifecycle in a manner typical of many mechanical devices. Please tick the option that most accurately models the typical probability of failure of a mechanical product over its lifecycle.

Figure 4(b) shows that 49% of graduates gave the right answer and identified the 'bath tub' reliability model as the most likely probability failure model for a new product. It is interesting to note that a significant proportion of engineers (~37%) stated that the near normally distributed probability model was the most likely representation of product failure. The normal distribution is obviously unsuitable because it would involve products not failing early or late over the product lifetime. The reason why so many graduates were drawn into this wrong answer may be because they had a vague recollection of an up and down curve but then failed to choose the right curve. This could be an example of where incomplete knowledge leads to a worse answer than just a guess.

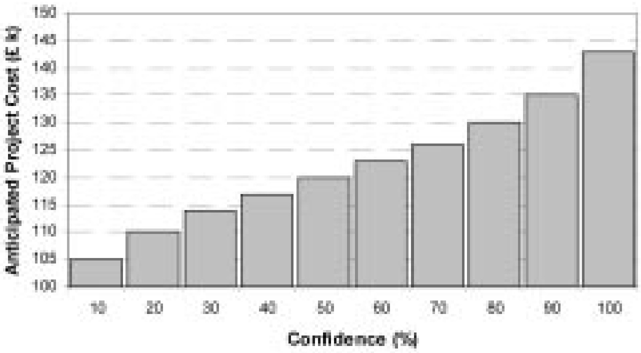
Confidence levels

Confidence levels are often used in probabilistic modelling to present outputs to engineers. Figure 5(a) shows the question set to test the graduates understanding of confidence levels.

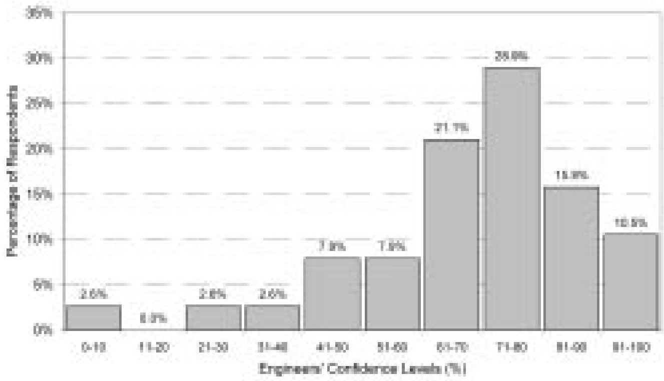
QUESTION 4 Assume you are in the position to bid for an engineering contract on behalf of your company. Company policy is to aim for a 2%

Confidence in Project Cost

Confidence (%)	10	20	30	40	50	60	70	80	90	100
Project Cost (£ k)	105	110	114	117	120	123	126	130	135	143



(a) Question 4



(b) Distribution of answers to Question 4

Fig. 5. Question on confidence levels.

profit on every engineering contract your department undertakes. Your fellow engineers provide you with the information below, stating their confidence in the anticipated cost to your department of completing the project. You are aware that there will be other competing bids but you are unsure how many or at what level they will bid. At what bid level would you be confident of extracting a profit from the project but also remaining competitive?

In order to achieve a profit it is clearly best to choose a price which has a confidence level of greater than 50% which corresponds to a price of £120,000. About 16% of the graduates gave a price less than £120,000 which represents a clearly inappropriate answer. To ensure a level of safety in the quotation a value greater than 50% confidence should be proposed. A reasonable minimum level of confidence would be 60%. 76.4% of the engineers gave an estimate greater than this value although several graduates opted for 90–100% confidence which is over-cautious since the question mentions the existence of competition.

Single summation of probability distributions

Since engineering products and systems contain many individual parts, it is important for engineers to understand what happens when probability distributions are added. Question 5 involved the addition of two identical probability distributions as shown in Fig. 6(a).

QUESTION 5 An assembly has two identical components. Each component has a mass of between 1 kg to 2 kg. The probable distribution of mass is graphically demonstrated in the diagram below. As

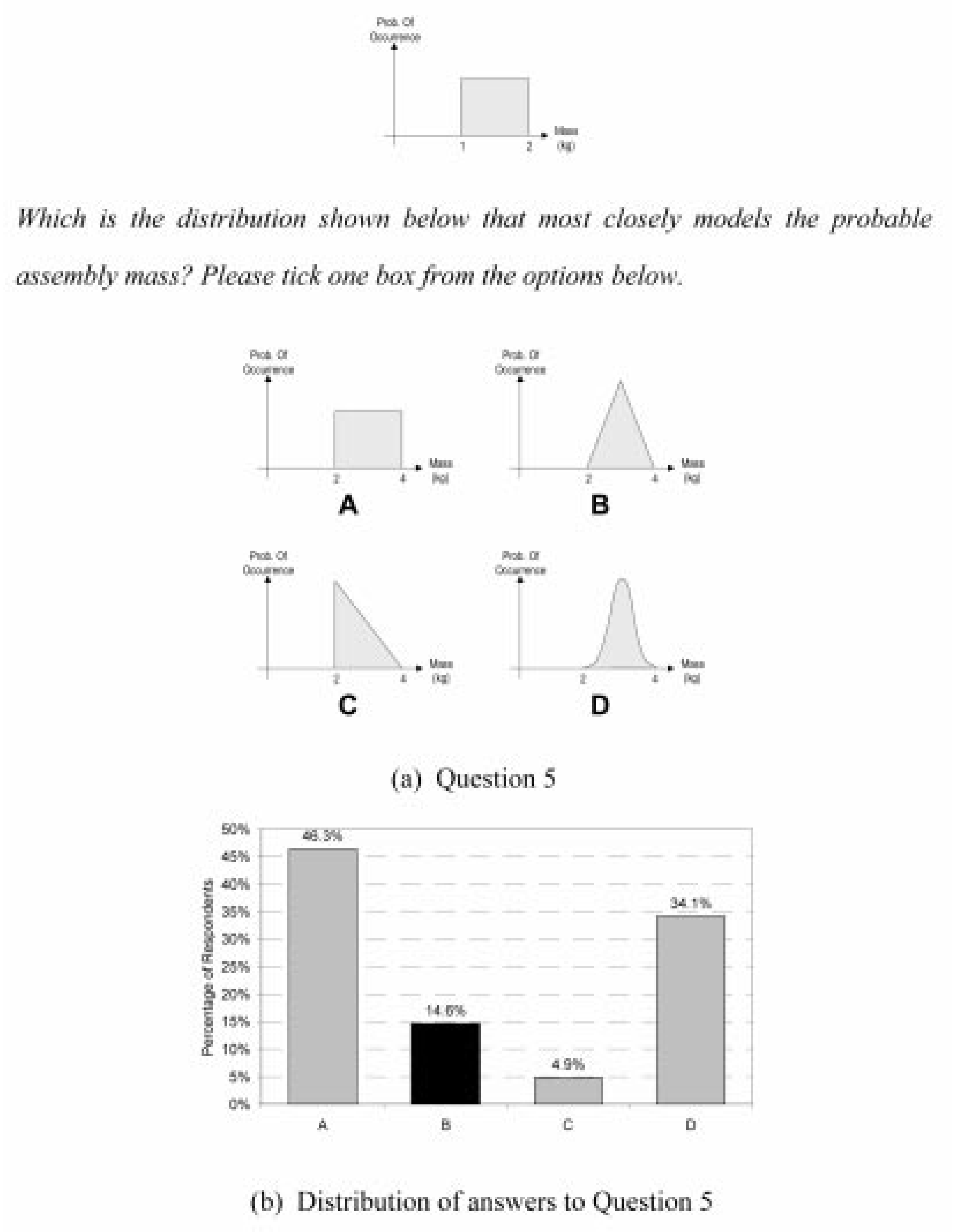


Fig. 6. Question on summation of two probability distributions.

shown there is an equal probability of the component mass being any value between 1 kg and 2kg.

Figure 6(b) shows that only 15% of graduates gave the correct answer which is very low. In fact, if the graduates had simply guessed the right answer, the group would have got close to 25% and performed a lot better. The graduates clearly assumed that if two individual components had a certain weight distribution then the combination of the two components must have the same distribution.

Some simple reasoning could have shown that Option A was incorrect. For example, the only way to get 2 kg is if both components are at the bottom end of the scale of weight, that is 1 kg. However, there are many ways to get 3 kg including $2 + 1$, $1 + 2$, $1.5 + 1.5$ and so on. From this it is clear that there is much more chance of getting 3 kg than 2 kg. Therefore, the distribution cannot be flat as shown in Option A. The poor performance of graduates in Question 5 shows that graduates

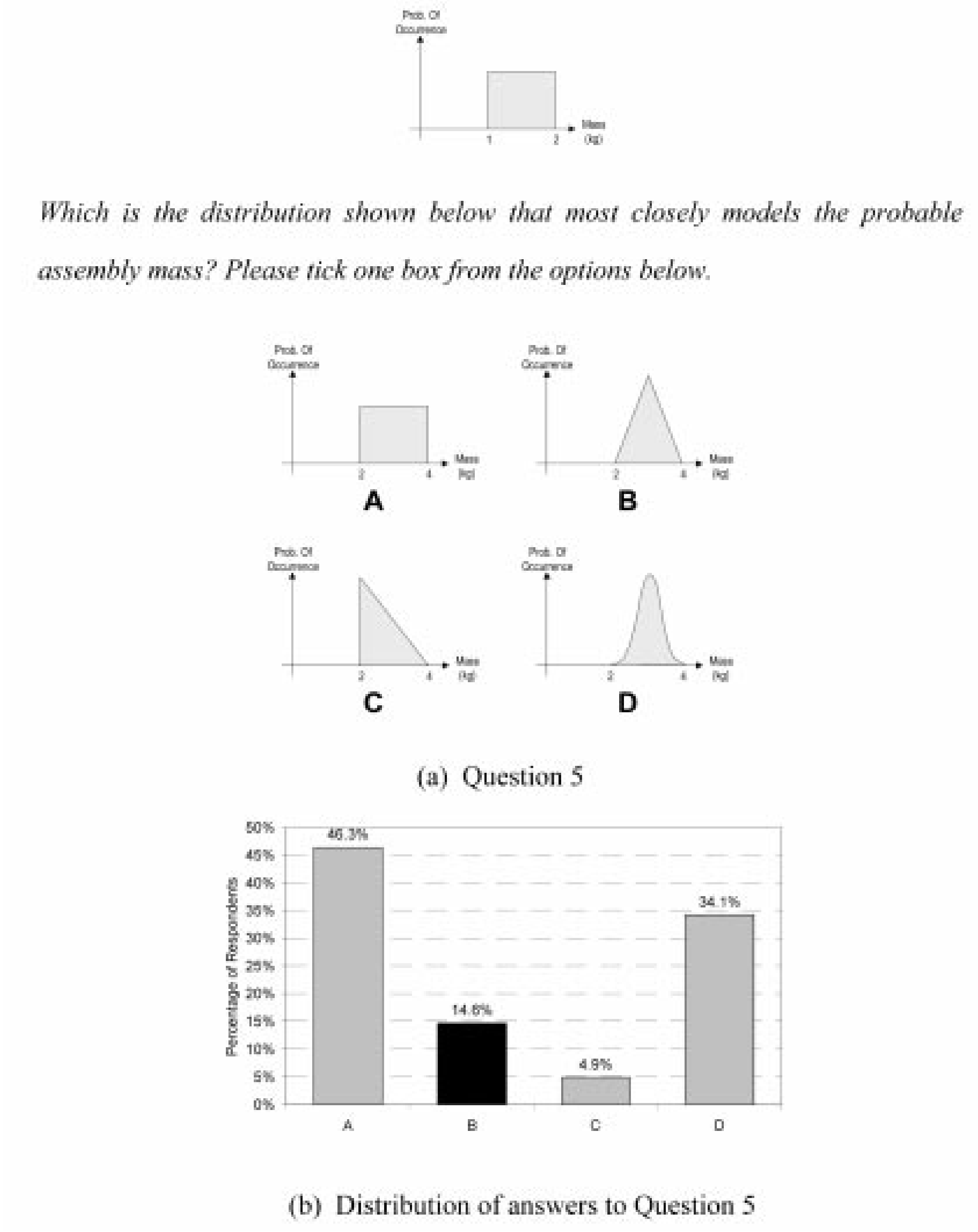


Fig. 7. Question on summation of multiple probability distributions.

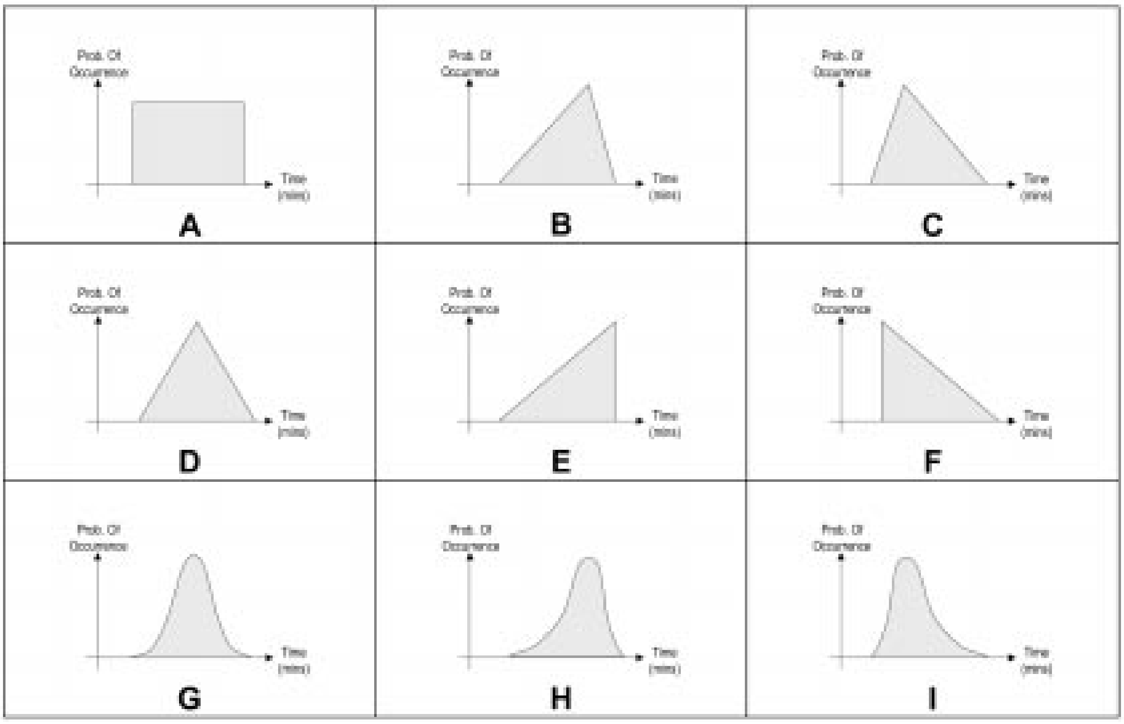
do not have a good intuitive understanding of probability theory.

Summation of multiple probability distributions

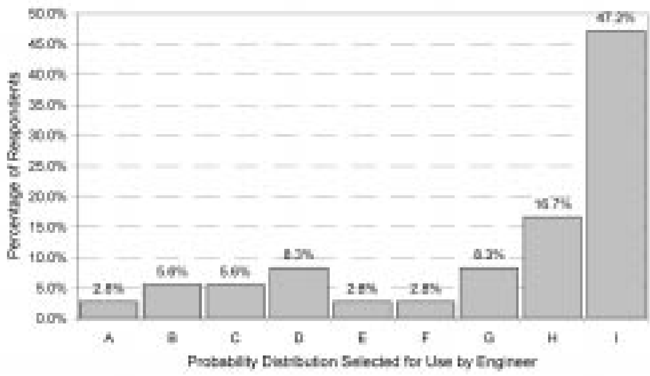
Question 6 involved the summation of a large number of probability distributions. The same individual distribution as Question 5 was used. Fig. 7(a) describes Question 6.

QUESTION 6 An assembly is constructed from 1000 identical components. Using the same component mass and distribution stated in Question 5 (i.e. minimum mass is 1 kg and maximum mass is 2 kg). Which is the distribution shown below that most closely models the probable assembly mass? Please tick one box from the options below.

Figure 7(b) shows that 63% of the graduates gave the right answer. This represents a very much higher number of right answers compared to Question 5 where only 15% gave the right answer. This is surprising because Question 6 is arguable a more difficult question than Question 5.



(a) Question 7



(b) Distribution of answers to Question 7

Fig. 8. Question on appropriate probability distributions.

The results may suggest that engineers understand that normal distributions will often be formed when large quantities of data are involved. However, the result may simply be because graduates like choosing the normal distribution. It is interesting that in Question 5, a normal distribution was chosen by 34% of graduates and in Question 3, a normal distribution was chosen by 36% even though it was not the right answer in either case. This indicates that engineers seem to assume that the normal distribution often applies in probability questions.

Selection of appropriate probability distributions

Question 7 had the aim of assessing graduates ability to select an appropriate probability distribution. Question 7 is presented in Fig. 8(a).

Event Description	Did you include the possibility of this event occurring in your original list	What do you consider to be the probability of this event occurring? (e.g. 25%, >50%, <10%, NA)	What do you consider would be the effect on the time to carry out the task if the event actually occurs? (e.g. +5 mins, ∞, >30mins, NA)
Jack cannot be set on ground under car therefore car must be pushed to an alternative position where the jack can be set.	Yes ☐ No ☐	%	mins
No spare tyre is on the car	Yes ☐ No ☐	%	mins
No wheel wrench is on the car	Yes ☐ No ☐	%	mins
A wheel nut is corroded and will be difficult to remove	Yes ☐ No ☐	%	mins
A wheel nut is badly corroded and cannot be removed	Yes ☐ No ☐	%	mins
A wheel nut is tightened up too tightly and cannot be removed	Yes ☐ No ☐	%	mins
A wheel nut is tightened up too tightly and will be difficult to remove	Yes ☐ No ☐	%	mins
Spare tyre not fully inflated and time is required to inflate tyre	Yes ☐ No ☐	%	mins
Deficient background light hinders the time to do the task	Yes ☐ No ☐	%	mins
Jack takes time to find	Yes ☐ No ☐	%	mins
Jack collapses and needs to be reset	Yes ☐ No ☐	%	mins
Jack is unusual and it takes time to familiarise yourself with it	Yes ☐ No ☐	%	mins
Wheel wrench takes time to find	Yes ☐ No ☐	%	mins
Spare wheel is awkward to detach from its retaining mechanism	Yes ☐ No ☐	%	mins

Fig. 9. Question on identifying unusual problems.

QUESTION 7 Assume that you are driving a hire car, unaccompanied, late at night in darkness through isolated countryside roads. The hire car is not one that you are familiar with. The car develops a puncture and you are forced to pull over in order to change the punctured tyre. You possess no forms of communication and no help is therefore immediately or seemingly likely to be available. Please select a distribution below that models most accurately the time estimate for changing the tyre. Leave blank if you do not understand probability distributions.

As can be seen in Fig. 8(b), the majority of engineers selected the probability distribution designated I (~47%). This distribution is of the type commonly termed a 'Beta' distribution. The selection by the majority of a distribution type with left leaning skew was in line with the practices of probabilistic modelling 'experts'. It makes sense when modelling the time required to complete a task to have a most likely value closer to the minimum than the maximum value. This is due to the fact that it is difficult to complete a task far in advance of the expected completion date but highly possible that the task will take significantly longer than the expected date.

Interestingly, a number of engineers, 13.9%, did not choose a probability distribution. As can be seen in the question, the engineers were told to offer no selection if they did not understand probability distributions.

Unusual problem events

A key problem in engineering design and management is that designers sometimes overlook unusual problem events that can have a significant effect on a product or process [7–9]. The eighth question investigated the graduate's ability to identify unusual problem events for a simple task. The task involved changing a tyre on a car in a remote place. The question is shown in Fig. 9. The question had two parts. In the first part, the graduate was asked to make a list of problem events for the task of changing a tyre on a car. In the second part, the graduates were shown a list

Table 1. Scheme for the qualitative measure of event probability and impact

	Qualitative Measure				
	Very Low (VL)	Low (L)	Moderate (M)	High (H)	Very High (VH)
Event Probability, EP (%)	$EP \leq 10$	$10 \leq EP \leq 20$	$20 \leq EP \leq 30$	$30 \leq EP \leq 40$	$EP > 40$
Event Impact, EI (minutes)	$EI \leq 5$	$5 \leq EI \leq 10$	$10 \leq EI \leq 20$	$20 \leq EI \leq 30$	$EI > 30$

of unusual problem events and asked to identify which events they had not thought of and how significant were the events that they had not considered.

QUESTION 8

1. *Write down problem events that can have an effect on the time taken to repair a puncture.*
2. *In the list shown, identify unusual problem events which you did not consider. For these events write your estimate for the probability of this event occurring and also your estimate of how much time it would add to the task time.*

On average 45.8% of problem events were identified and 54.2% were not identified. Therefore, the majority of the problem events were not identified. This is concerning because estimates will be inherently over optimistic if they do not take into account possible problem events. In order to identify the significance of the events that were not identified by the graduates, the problem events were grouped into the quantitative groupings shown in Table 1.

For each of the events not considered by each of the graduates, a qualitative measure was given of either: VL, L, M, H or VH in accordance with what the graduate had stated in their answer and in accordance with the scheme in Table 1. Having classified each entry, a 'probability versus impact' matrix was produced, as shown in Fig. 10. The value in each box shows what percentage of the unconsidered events fell into that particular category. Therefore, the different percentages in all boxes add up to 100%. The matrix is useful because it shows what kind of problem events graduates did not think of.

The results, shown in Fig. 10, show that it is the low probability events that are typically neglected by the graduate engineers. Figure 10 also illustrates that it is the 'very low' and 'very high' impact events that are ignored by the engineers. Examples of low probability and high impact events are:

- no spare tyre in car;
- no wheel wrench in car;
- wheel nut badly corroded and cannot be removed.

There are several reasons why engineering graduates do not consider low probability events. One reason is that the events are inherently unusual and it takes creative thinking to identify them. A second reason is that the graduates are not experienced in real life design and management. A third reason is that traditional questions in engineering education do not require such creative thinking. A fourth reason is that it may be assumed that low probability events are not significant.

DISCUSSION

The answers given in the exam show that graduates have a limited understanding of terms and concepts used in probability theory. Question 2

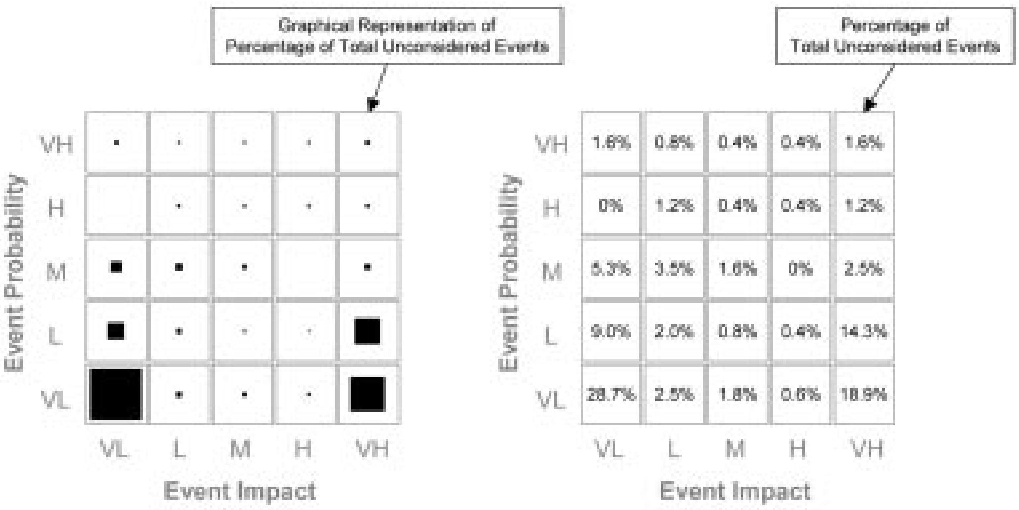


Fig. 10. Probability versus impact of unconsidered events.

showed that many graduates were aware of the three-sigma term. However, in general the answers showed a lack of true understanding. There was a trend for the graduates to simply select a normal distribution whether this was right or not. There were some poor answers. In Question 5 it was surprising that only 15% of graduates gave right answers for the summing of two simple probability distributions, especially since guessing would have achieved around 25% for the group. This is an example of where a little knowledge can make a situation worse.

The lack of proper understanding is not surprising because probability theory is traditionally taught as a minor topic within a maths unit and it is not integrated into the mainstream engineering science subjects. Traditional engineering subjects contain teaching material that is based on deterministic methods and data rather than probabilistic methods. In particular, the vast majority of exam questions contain deterministic questions where there is unique data and unique answers. In reality, practical design problems do not have such certain parameters.

A typical example of a deterministic question can be found in structural mechanics. A question in structural mechanics will generally have unique values for the material properties and well-defined boundary conditions. Such a question contains unique solutions and unique answers. When an engineering undergraduate has experienced three or four years of deterministic questions in all the different engineering science subjects, it is not surprising that they get the impression that engineering is a deterministic subject.

It is worthwhile considering incorporating probability theory and uncertainty into traditional subjects like structural mechanics. If an engineering undergraduate is exposed to questions in mainstream subjects with probability theory there will two advantages. One is that they will develop more expertise in probability theory. The second is that they will develop an awareness of the presence and source of uncertainty in real-life engineering problems.

APPLICATION OF STATISTICAL CONCEPTS INTO MAINSTREAM SUBJECTS

Following the research studies presented in this paper, statistical methods have been integrated into some of the mainstream subjects in the department of mechanical engineering. For example, a 10-credit fourth year unit 'Product Design' gives application of statistical methods in product design. The syllabus includes:

- Probabilistic design and variance analysis.
- Probability density functions.
- Coefficient of variation.
- Process capability indices.
- Common and special cause variability.
- Parts-per-million failure and statistical process control.

A full explanation of these terms is given in reference [10]. An example of an exam question from the Product Design unit (2003–2004) is given in the Appendix. The question is essentially a solid mechanics question. However, the question includes uncertain data for all of the design parameters. The question is presented in a way which is very different from a typical solid mechanics question that would have discrete and unique design parameters.

The question achieves three important goals. Firstly it illustrates that real-life problems have a large amount of uncertainty. Secondly, it provides a means for testing the application of probability theory. Thirdly, it shows how probability theory can be applied to a practical engineering problem. Student feedback from the Product Design has shown that students appreciate the way that statistics is integrated with engineering problems. The fact that students are taught statistical principles in the second year and then apply them in the subsequent years of their course will undoubtedly improve their long term appreciation and understanding of statistical principles.

CONCLUSIONS

A special exam has been conducted to test the understanding of probability theory of recent graduates in engineering. The following specific conclusions were drawn from the answers given to the exam questions:

- The majority of the graduate engineers examined do have some understanding of standard deviations.
- Many graduates had difficulties in identifying appropriate probability distributions.
- The majority of graduates could not summate probability distributions.
- Many of the graduate engineers had difficulty in identifying problem events with a low probability of occurrence.

The examination showed that recent graduates could remember only a few concepts from the probability theory they had been taught as undergraduates. Since probability theory is taught as a single topic within a maths unit early in the degree programme, this means that it has been largely forgotten by the time that students graduate. An effective solution to this problem could be to incorporate probability concepts into mainstream subjects like structural mechanics. This solution would enable undergraduates to develop confidence in probability theory and to develop an awareness of the presence and source of uncertainty in real-life engineering problems.

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APPENDIX

Q1. A shrink-fit is a semi-permanent assembly system that can resist the relative movement or transmit torque between two components through the creation of high radial pressures at the interface of its constituent parts. The pressure established between the inside diameter of a part such as a hub and the outside diameter of a shaft is created through interference in dimensions. Figure A1 shows the arrangement of a typical shrink-fit assembly with notation of the key parameters.

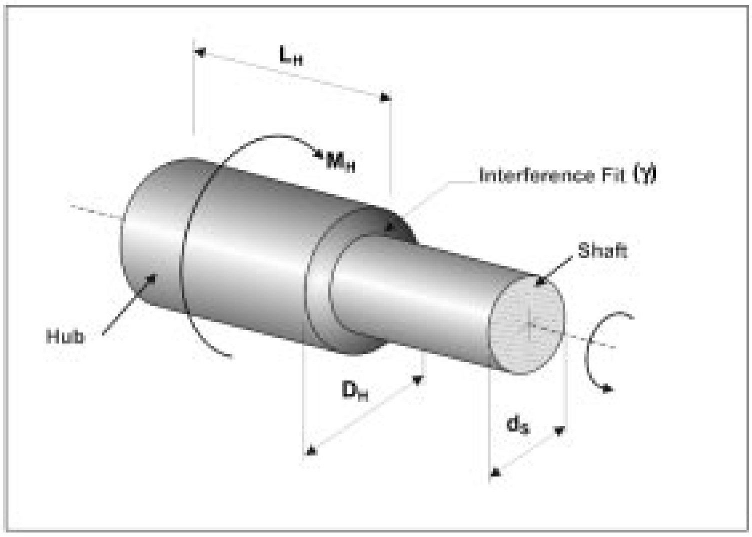

Fig. A1. Shrink-fit Assembly and Notation

The holding torque, M_H, is the torque that is needed to cause a slipping of the shaft through the whole length of a hub and is given by:

$$M_H = \frac{\pi}{4} \cdot f \cdot L_H \cdot E \cdot \gamma \cdot d_s \cdot \left[1 - \left(\frac{d_S}{D_H}\right)^2\right]$$

where;

f = coefficient of friction between shaft and hub
d_S = shaft diameter
L_H = length of hub
E = modulus of elasticity
γ = interference between the shaft and the hub
D_H = hub outside diameter.

Experimental testing and measurement of a small sample of each parameter yielded the following statistical information described by the normal distribution:

$f \sim N\ (0.148, 0.03)$
$d_S \sim N\ (22.083, 0.011)$ *mm*
$L_H \sim N\ (15.191, 0.057)$ *mm*
$E \sim N\ (218.15, 7.73)$ *GPa*
$\gamma \sim$ N (0.027, 0.006) mm
$D_H \sim$ N (59.914, 0.043) mm

a) Find the mean, μ, and standard deviation, σ, of the holding torque for the design variables given. (8 marks)
b) Construct a Pareto chart showing the relative contribution of each design variable to the holding torque variability. What factors could affect the two main variables in terms of maintaining their current level of variability in manufacture/assembly? (8 marks)
c) Briefly discuss the use of the variance equation and Sensitivity Analysis (SA) information in a typical iterative design activity. (4 marks)