Quantitative Modelling of an Air Conditioning Problem*

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To solve a saturation problem involving the moisturizing effect of an air conditioning unit, a practical model has been developed. In particular, this model demonstrates that the humidity feedback control in a printing office was not effective under certain ambient disturbances. Rather than measuring the relative humidity within the building, the air conditioning's redesigned control system uses the absolute humidity of the ambient atmosphere for a corrective feedforward action. This paper is based on a related case study for students in the Technical Business Studies programme.

INTRODUCTION

SHORTLY AFTER the installation of a new air conditioning at the printing works L. Jzn. Coster (Haarlem, The Netherlands) during the mid 1990s, an annoying effect occasionally disturbed the printing process. Specifically, during the spring and the autumn seasons, the humidity control section saturated, thereby causing an interruption of the air conditioning.

After several resets, performed by the technician of the printing works, the air conditioning stopped completely, resulting in a considerable loss of production in the printing works. The control technician had to carry out a hard reset. The cause of the breakdown was found to be the result of a sudden drop in the ambient relative humidity, a phenomenon that occurs frequently in the spring and autumn seasons at the seaside location in Holland.

Eventually the control technician, skilled in installation, calibration and trouble shooting, lowered the capacity of the moisturizing section of the air conditioning unit. He also reconfigured the feedback control of the relative humidity within the building into a feedforward control of the absolute humidity of the air in the building. After this adjustment the air conditioning unit functioned faultlessly.

However, apart from the instrumentation scheme of the installation, no model for calculation of disturbance effects was available. On the basis of the accessible information, a contextual model was constructed to demonstrate the effect of this phenomenon on the operation of the air conditioning.

Unfortunately, the author was not given permission to visit the printing works premises. For this reason, a fictitious name for the printing works has been chosen. Moreover, two decisive parameters of the air conditioning remained unknown: the value of the controlled supply air and the ventilation frequency. However, these values are assumed in the sequel in such a way that, together with all the approximations used to make the case study manageable to the students, the problem mentioned above clearly shows up.

DIDACTICAL CONCEPT AND ASSESSMENT

The Hogeschool van Amsterdam, a university of professional education, has chosen *Learning to Learn* as one of its three superordinate goals. This concept is i.a. based on the assumptions that '. . . the teachers and staff members, in their role of coach, expert and role model, facilitate the learning process by process-oriented instructions . . . [and] . . . the current challenges in the real life professional environment are the starting point of the teaching and learning process . . .'.

To meet these requirements in the programme Technical Business Studies, the emphasis has increasingly been placed upon cases which are both real and practical, multidisciplinary with inputs across several branches of engineering, and possibly open-ended with a range of options available. This air conditioning problem seemed appropriate to serve as a starting point for such a case study.

To stress the real life situation, students have to refer to and to decipher the relevant product information sheets describing the equipment of the air conditioning present in the printing works. Apart from the installation scheme, the following sheets were offered to the students: the universal controller RWF61.100, the controller input signal conditioners AZW61, the signal splitter SEZ61.5 and the absolute humidity transmitter

QFM61.1 (all from Landis & Staefa AG, Switzerland), the rotary heat exchanger (Econovent AB, Sweden), the Condairmatic TS 760 (Condair Ltd, Switzerland), the relative humidity safety device HBC 10 (Sauter AG, Switzerland).

When preparing this case study, in addition to a description of the situation the stance has been adopted to offer a carefully structured series of assignments serving as milestones for the students. The course was concentrated in one week, with a student total learning time of 40 hours, and had the form of a workshop. During this week, the teacher acted as a consultant giving advice as requested. In this sense, there has been a feedforward control of the student's learning process.

In teams of (maximum) five, students were challenged to devise a model for the calculation of the relevant quantities of the air conditioning system, both in the original and in the revised situation. In some respects, the case study allowed the possibility for each group member to undertake a specific task.

Each group was required to write a report. The report was assessed according to the following aspects: formulation of the model, investigation of the milestones and identification of the problem occurring in the printing works. Because of the limited student total learning time, the students were not required to make an oral presentation before the class.

Problems the students encountered while working on the case study may be summarized as follows:

- the formulation of the equation of state of an ideal gas;
- the description of a mixture of dry air and unsaturated water vapour;
- the calculation of the molar mass of dry air starting from the air composition in volume percentages or mass percentages;
- the notions of absolute and relative humidity;
- the calculation of the settings of the three SEZs and the Condair units;
- studying the literature to obtain a model for the thermodynamical behaviour of the air in the printing works;
- the calculation of the bias of the controller in the feedback control situation, the original situation in the printing works;
- the saturation of the controller in the sense that the output signal remains maximal;
- the identification of the problem with the safety device in the stationary state of the air conditioning system;
- the calculation of the time development of the absolute humidity in the original and the revised situation.

It is important to be acquainted with the technological level of the students. Students had only a rudimentary knowledge of calculus, physics and control theory, and had no experience in using simulation software tools. The case study was the subject of an optional module Workshop Technology during the final year curriculum of the institution. The case was part of this curriculum for three consecutive years, starting in the academic year 1997/1998. After finishing the study programme successfully, the students received a Bachelor's degree of Engineering.

To judge the merits of the Workshop Technology module, it is also instructive to know the grade system employed at the institution. The grade system ranges from 1.0 to 10.0. The Workshop Technology module belonged to a cluster of several other technology modules. To obtain the credits for this cluster, the student was required to attain an average mark greater than or equal to 5.5, with the lowest mark allowed in the cluster being a 4.0 . The raw grade result over three years is summarized in the following table.

Mark intervals	Number of students
4.0—4.4	9
4.5—4.9	33
5.0—5.4	40
5.5—5.9	56
6.0—6.4	36
6.5—6.9	17
9.0—9.4	6

From the year 2000, the Workshop Technology module has further been developed into a Project Technology module carrying more credits requiring both oral and written reports.

THE BUILDING AND THE AIR CONDITIONING SYSTEM

This section will focus on those components of the air conditioning system which were present in the initial situation and in the revised situation. The controllers and the sensor elements will be considered in the appropriate sections.

The building consists of five floors. Each floor is 34 m long by 34 m wide and has a height of 4 m.

Generally, an air conditioning system fulfils three main functions: moisten the air within a building at a desired level, maintain the temperature at a desired value, and refresh the air within the building.

This paper concentrates mainly upon the humidity aspect of the air conditioning system at the printing works. Figure 1 shows the piping and instrumentation diagram (P&ID) of the humidity aspect system.

The printing office has one central air conditioning unit. Moreover, each floor has its own support air conditioning unit. The hydration capacity of the supporting air conditioning units is negligible with respect to the moistening capacity of the central air conditioning. The support air conditioning units, therefore, have in this case a limited function: to deliver an air inflow, heated or cooled
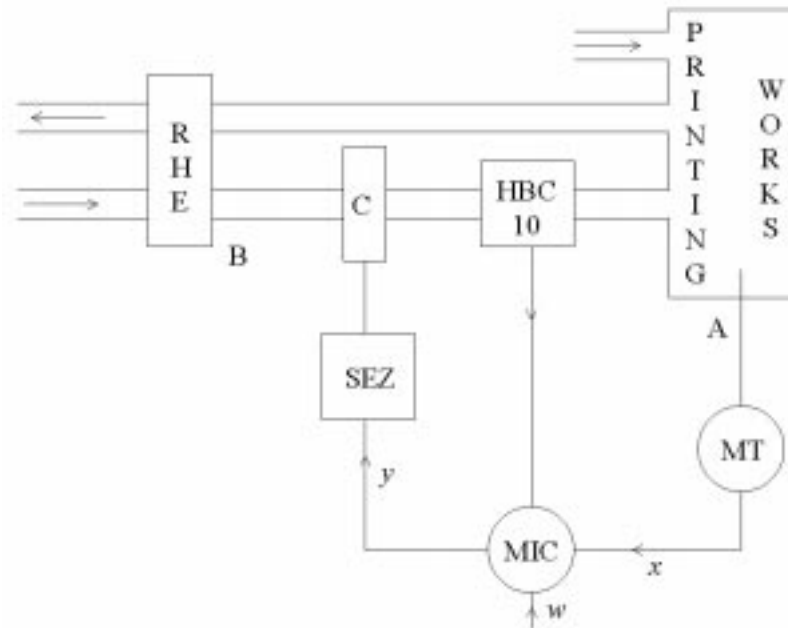
Fig. 1. P&ID of the moisture aspect system of the printing work's air conditioning.

according to the desired temperature of the air within the building.

The temperature of the ambient atmosphere in the spring and autumn seasons will be assumed to be a constant 15°C.

The essential part of the air conditioning system is positioned in a compartment on the roof of the building. This compartment contains a rotary heat exchanger (RHE in Fig. 1) that transfers heat (and moisture) from the exhaust air to the supply air as it rotates. The ambient supply air flows through one half of the heat exchanger, and the exhaust air flows in the opposite direction through the other half. As the supply air leaves the heat exchanger, it has a constant temperature of 20°C.

The dry part of the supply air has a mass flow rate $\Phi_{in,dry} = 3.06 \, \text{kg s}^{-1}$. The dry part of the exhaust air has a mass flow rate $\Phi_{out,dry} = 13.9 \, \text{kg s}^{-1}$.

The mass flow of the exhaust air includes the air of the five supporting floor air conditioning units, as well as the ambient air coming in through the doors, the windows and chinks. As alluded to in the introduction, the exact values of the mass flow rates were not available. With the above values, the ventilation frequency amounts to 1.7/hour. This value is apparently too low as in the literature ventilation frequencies of 4.0/hour and higher are accounted for. The value 1.7/hour can be considered as a good average value because the inventory and the distribution centre of the printing works are also included.

The control elements

'Control elements' is a generic name for the components between the output signal, y, of the controller and the input signal applied to the air in the printing works.

In the system under consideration, the control elements consist of two components: a transducer depicted as the function block SEZ in Fig. 1, and a control system actuator depicted as the function block C.

In the supply channel close behind the heat exchanger there are four Condair outlets C. The function of a Condair unit is to moisturize the incoming ambient air with steam. This process occurs without any appreciable temperature change of the incoming air. In the supply channel, an isothermal mixing of supply air (from the heat exchanger) and vapour (from the Condair units) occurs. This mixture is called 'wet air'.

The four Condair units, shown as one block C in the P&ID and called a control system actuator, are connected in parallel but operate in a cascading manner. To achieve this cascading operation, a transducer SEZ (in fact three SEZs) splits the controller output signal into four standard signals. This splitting technique is subject of an additional problem for the students and is further elaborated later on. The graph of the transfer function of the transducer and the actuator, considered as one module, is shown Fig. 2. For subsequent calculations, this graph is simplified by a straight line through the points $(2 \, \text{V}, 3 \, \text{kg h}^{-1})$ and $(10 \, \text{V}, 120 \, \text{kg h}^{-1})$.

$$\Phi_{vapour}(y) = \begin{cases} 0 \, \text{kg s}^{-1} \text{ for } 0 \, \text{V} \leq y \leq 2.0 \, \text{V} \\ 0.4063 * 10^{-2}y - 0.7292 * 10^{-2} \\ \qquad \text{for } 2.0 \, \text{V} < y \leq 10.0 \, \text{V} \end{cases}$$

(1)

The humidity safety device

To detect an overly high level of relative humidity in the wet air at an early stage, an on/off hygrostat HBC 10 is mounted in the supply
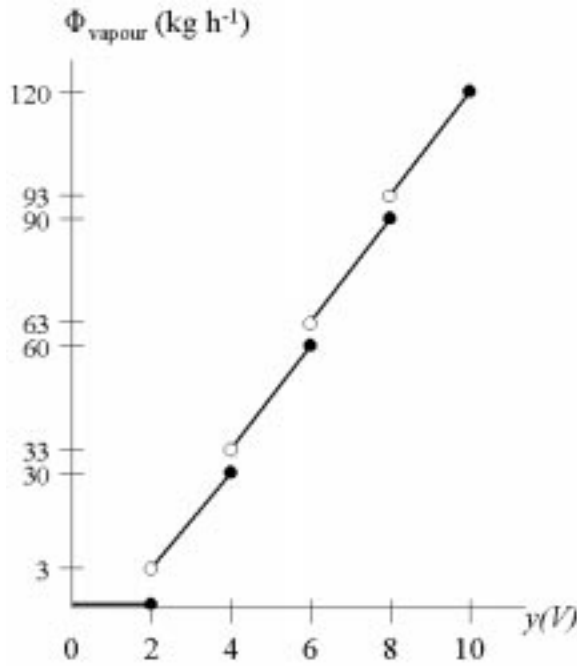
Fig. 2. Graph of the transfer function of the control elements. For clarity the vertical axis is locally compressed.

channel. At a relative humidity of 95% at 20°C this safety device switches off the Condair units. A reset, eventually a hard reset, of the Condair units is then necessary.

To describe the transfer functions of the mixing processes in the supply channel and in the printing works, the following definition of the absolute humidity of wet air is given. The absolute humidity, c, of an amount of wet air, enclosed in a volume V, is defined as

$$c = \left(\frac{\text{mass of the vapour contained in the volume } V}{\text{mass of the dry air contained in the volume } V}\right)$$

(2)

The transfer function of the function block, vapour-dry air mixing in supply channel due to

the Condair units, is

$$c_c = \frac{\Phi_{vapour}}{\Phi_{in,dry}} \qquad (3)$$

Taking into account the absolute humidity, c_b, of the ambient supply air, the wet air in the supply channel has the absolute humidity $c_c + c_b$.

The controlled system

The controlled system, i.e. the air in the printing works, may be considered as a one-compartment system with two air inflows, including the supply air, and one air outflow, the exhaust air.

To derive the transfer function of the controlled system, a more detailed way of illustrating the direction of the signals is useful. Figure 3 shows the block diagram of the air conditioning system.

The function block PRW 'air mixing in the printing works' has two incoming signals: $c_c + c_b$ due to the supply channel SUC, and c_b due to the five floor air conditioning units, the windows and the chinks. The absolute humidity of the mixed air in the printing works is denoted by c. Because the problem, sketched in the introduction, is not caused in any way by the humidity produced by the printing process, the contribution of this vapour to the humidity in the printing works is neglected.

The transfer function of the printing works is the solution of the following differential equation:

$$\frac{\Phi_{in,dry}}{\Phi_{out,dry}} c_c + c_b = \frac{m}{\Phi_{out,dry}} \frac{dc}{dt} + c \qquad (4)$$

In this formula m denotes the mass of the dry air in the printing works. Taking $\rho = 1.293 \, \text{kg m}^{-3}$ as the density of dry air, $m = 29.89 \times 10^3 \, \text{kg}$ is obtained.

THE ORIGINAL SITUATION IN THE PRINTING WORKS

Until this point, only a part of the block diagram corresponding to the P&ID has been considered. In the original situation the relative
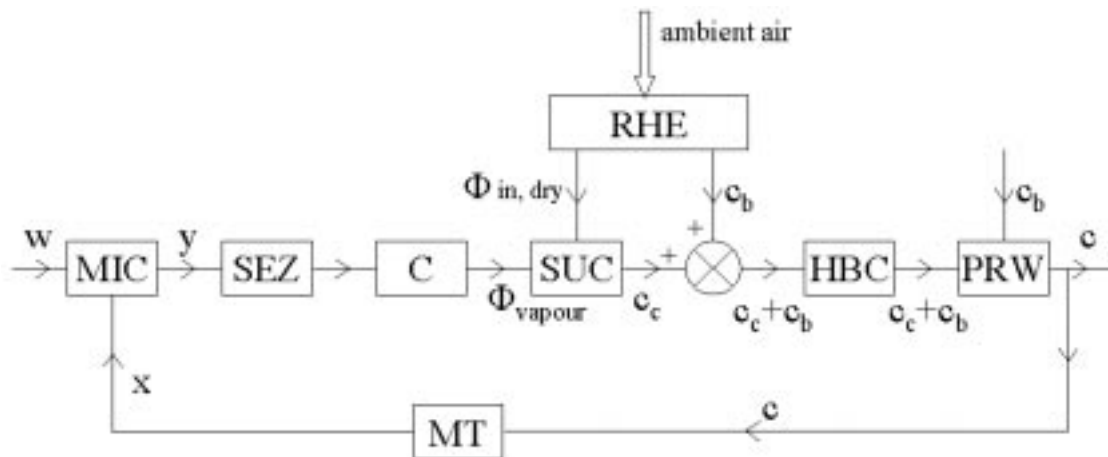

Fig. 3. Block diagram of the air conditioning's control system.

humidity of the air in the printing works was controlled by feedback. A controller (MIC) and a sensor (MT) have to be incorporated to get a closed-loop control system.

The controller

The employed P-controller, equipped with an input range adapter, has an input range from 20% to 95% relative humidity, and a modulating output signal from 0 V to 10 V. The controller was set at a desired relative humidity, w, of 50% with an 8.0% proportional band. In feedback practice the value 50% is fixed in the centre of the domain where the controller is acting in a linear manner. The absolute value of the controller gain, K, can be calculated as $\frac{5}{3} * 10^2$ V, and the bias, B, has the magnitude 5 V. To fix the sign of K the process chain must be examined. The composite transfer function of all the blocks in the process chain, with fixed c_b, is increasing, i.e. the relative humidity, x, increases as the output, y, of the controller increases. For a properly operating closed-loop system, the controller transfer function has to be decreasing, and the controller is set as inverse acting, hence $K > 0$. The above discussion can be summarized in the formula:

$$y = \frac{5}{3} * 10^2 (w - x) + 5.0 \quad \text{with } w = 0.50 \quad (5)$$

The proportional domain of the controller is the interval [0.47, 0.53].

The relative humidity sensor

Next, the transfer function of the relative humidity sensor is described. The relative humidity sensor acts as a transducer, where the input signal is the absolute humidity c and the output signal is the relative humidity x. The transfer function can be calculated by assuming that the wet air to be measured is an ideal gas.

Thus, the transfer function is derived as follows:

$$x(c) = \frac{p_{bar}}{p_{\max}(\vartheta)} \frac{c}{\frac{18}{29} + c} \quad (6)$$

where:

$p_{bar} =$ atmospheric pressure $= 101.325$ kPa;
$p_{\max}(\vartheta) =$ saturation pressure of vapour at temperature θ;
$c =$ absolute humidity of the wet air;
the quotient 18/29 is the molecular mass of water divided by the molecular mass of dry air;
the atmospheric pressure is assumed to be constant and has the value of the standard atmospheric pressure.

The following values have been used: $p_{\max}(15^{\text{o}}\text{C}) = 1.71$ kPa en $p_{\max}(20^{\text{o}}\text{C}) = 2.34$ kPa. Essentially, the calculations can now be started.

Identification of the problem

If the system is under control, i.e. the offset is zero, and the state of the system is stationary,

the external relative humidity c_b can be calculated. Offset zero gives $c = 0.00725$ kg kg^{-1} and $y = 5$ V. With (1), (3) and (4), it follows that $c_b = 0.00631$ kg kg^{-1} and hence the relative humidity of the atmosphere is 59.6% (15$^{\text{o}}$C).

Suddenly, the relative humidity of the atmosphere drops to 41.2% (15$^{\text{o}}$C). The air in the printing works is also influenced by this disturbance. It is easy to show that there will be a problem with the air conditioning. It is assumed that the state of the system settles to a constant value without intervention of the HBC 10.

Moreover, the subsequent analysis can be simplified by a linear approximation of the transfer function (6) in the point $c_w = c(w) = 0.00725(083)$ kg kg^{-1} because the transfer function (6) is only slightly concave in a neighbourhood of c_w. This can be written as follows:

$$x(c) \approx x(c_w) + \frac{p_{bar}}{p_{\max}} \frac{18}{29} \frac{1}{(18/29 + c_w)^2} (c - c_w)$$
$$(7)$$

The approximated composite transfer function of the sensor (7) and the P-controller (5) is:

$$y = 11\,360(c_w - c) + 5.0 = 87.369 - 11\,360c \quad (8)$$

Equation (4) can now be rewritten as:

$$0.4063 * 10^{-2}y + 0.053117 = 13.9c \quad (9)$$

Solving c and y from the equations (8) and (9) results in $c = 0.006795$ kg kg^{-1} and $y = 10.18$ V. This value of y exceeds the output range of the controller. y will have a saturated output value 10.0 V. With (1) and (3) one calculates $c_c = 0.01089$ kg kg^{-1}. The HBC 10 will stop the Condair units as $c_c + c_b$ tends to the value 0.0152 because this value exceeds both the safety limit 0.0139 kg kg^{-1} of the HBC 10 and even $c_{max} = 0.0147$ kg kg^{-1}.

The result may seem to be exaggerated but the objective was to let students to produce a non-functioning air conditioning calculation independent of any interim rounding errors.

Next, the question is tackled of how long it takes until the HBC 10 stops the air conditioning. In this case it is quite instructive to use the complicated transfer function (6) and not the linear approximations (7) or (8). Now all the equations (1), (3), (4), (5) and (6) are combined. This gives the following result:

$$\frac{c + 18/29}{c^2 + 2.7005c - 0.01840} \frac{dc}{dt} = -\frac{\Phi_{out,dry}}{m} \quad (10)$$

Integrating this differential equation has as a result:

$$(c + 2.7073)^{0.7688}(c - 0.006795)^{0.2312}$$
$$= 0.3639 * \exp(-0.4650 * 10^{-3}t) \quad (11)$$

Finally, the safety function of the HBC 10 is taken into account, calculating y from $c_c + c_b = 0.0139\,\mathrm{kg\,kg^{-1}}$. $y = 8.987\,\mathrm{V}$ and with (5) and (6) one concludes that $c = 0.006900\,\mathrm{kg\,kg^{-1}}$. The solution (11) shows that after 730 s the HBC 10 will cut the operation of the Condair units.

If the sensor transfer function (6) is linearized, the calculated operating time of the Condair units would be $7.5 \times 10^2\,\mathrm{s}$.

THE PRESENT SITUATION AT THE PRINTING WORKS

As indicated in the introduction, the printing works suffered a considerable loss of production because of the occasional breakdown of the air conditioning. The control technician who maintained the installation, created a cheap and effective solution and executed the redesign of the air conditioning.

Redesign of the control system
To eliminate the problem outlined in the previous section, the control technician first removed the relative humidity sensor (measuring x at point A in the P&ID) and fitted an absolute humidity sensor in the supply air channel (measuring c_b at point B in the P&ID). Secondly, he lowered the maximum capacity of each Condair by 20%. Moreover, he replaced the input range adapter of the P-controller by an adapter for absolute humidity measurement with an input range from $0\,\mathrm{g\,kg^{-1}}$ to $20\,\mathrm{g\,kg^{-1}}$. Furthermore, in the controller settings, he put the desired absolute humidity, w, in the printing works at $7.2\,\mathrm{g\,kg^{-1}}$ and the proportional band at 20%. The controller has now become a feedforward controller.

Validation of the redesigned control system
As previously, the calculations are started by supposing that the system is under control (i.e. $c = w$), and that the state of the system is stationary. Further, the Condair units are not functioning (i.e. $\Phi_{vapour} = 0$ and $c_c = 0$) and the output signal of the controller is 2.0 V. Again, the relative humidity of the ambient atmosphere is calculated. The transfer function (4) simplifies to $c_b = c$. Combining these results, it can be deduced that relative humidity of the ambient atmosphere is 68% (15°C).

Checking that the controller acts indirectly, is also easy. The controller transfer function is:

$$y = 2.5 * 10^3(w - c_b) + 2.0$$

$$\text{with } w = 0.0072\,\mathrm{kg\,kg^{-1}}$$

The proportional domain of the controller is the interval $[4.0\,\mathrm{g\,kg^{-1}}, 8.0\,\mathrm{g\,kg^{-1}}]$. The transfer function (1) of the transducer and the actuator is

replaced by:

$$\Phi_{vapour}(y) = \begin{cases} 0\,\mathrm{kg\,s^{-1}} \\ \quad \text{for } 0\,\mathrm{V} \leq y \leq 2.0\,\mathrm{V} \\ 0.3229 * 10^{-2}y - 0.5625 * 10^{-2} \\ \quad \text{for } 2.0\,\mathrm{V} < y \leq 10.0\,V \end{cases}$$

$$(1')$$

The relative humidity of the ambient atmosphere of 68% (15°C) may be present for $t < 0\,\mathrm{s}$. At time $t = 0\,\mathrm{s}$ this relative humidity suddenly drops to 41.2% (15 °C). The point of interest here deals with the behaviour of the relative and the absolute humidity as a function of time in the printing works. Combining the equations $(1')$, (3), (4) and (12), results in the following differential equation, valid for $t \geq 0\,\mathrm{s}$ and $4.0\,\mathrm{g\,kg^{-1}} \leq c_b < 7.2\,\mathrm{g\,kg^{-1}}$,

$$\frac{m}{\Phi_{out,dry}}\frac{dc}{dt} + c = 0.4192446c_b + 0.0042414$$

$$\text{with } c(0) = 0.0072\,\mathrm{kg\,kg^{-1}}$$

For $c_b = 0.00435\,\mathrm{kg\,kg^{-1}}$ the absolute humidity c evolves for $t \geq 0\,\mathrm{s}$ as:

$$c(t) = 0.0072 + (0.006065 - 0.0072)(1 - \exp(-t/\tau))$$

$$\text{with } \tau = m/\Phi_{out,dry} = 35.8\,\mathrm{min}$$

The relative humidity in the printing works tends towards 42%. Certainly, this is not the result aimed at; rather support of the local floor air conditioning units is needed to reach the eventual 50% (20°C). For all $t \geq 0\,\mathrm{s}$, $c_c(t) + c_b(t) = 0.0121\,\mathrm{kg\,kg^{-1}}$, less than the critical value $0.0139\,\mathrm{kg\,kg^{-1}}$ of the HBC 10.

SOME SPIN-OFF PROBLEMS

Earlier it was stated that this case study allowed the possibility for each team member to undertake a specific task. The way the foregoing discussion has been presented may have hidden this possibility. The following additional problem posed to all teams of students may support this remark explicitly.

Analysis of the control elements
To drive the four Condair units in cascaded operation with a single output signal y from the controller as shown in Fig. 2, three SEZ splitters are needed. Figure 4 zooms in on the function block SEZ and on the function block C of the block diagram 3.

How the two output signals, y_1 and y_2, of an SEZ in Fig. 4 are realised stems from the SEZ product information sheet depicted in Fig. 5. Note that the signal y_1 increases linearly to reach the constant value 10.0 V at $y = $ STB (Fig. 5, left) and that the signal y_2 stays at 0 V as far as 10.0 V STB and then increases linearly over the interval [10.0 V
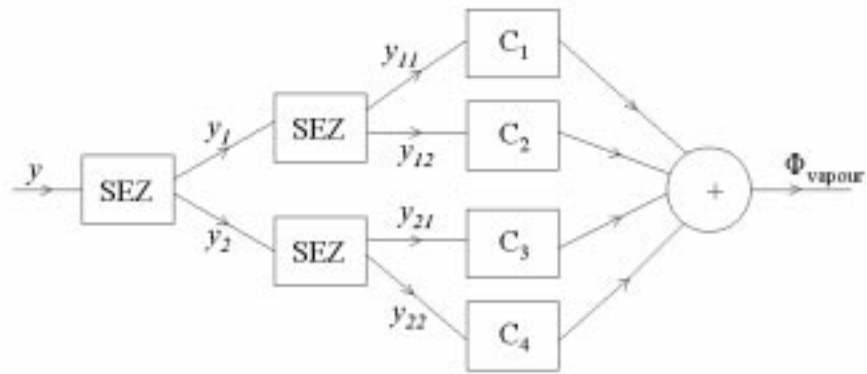
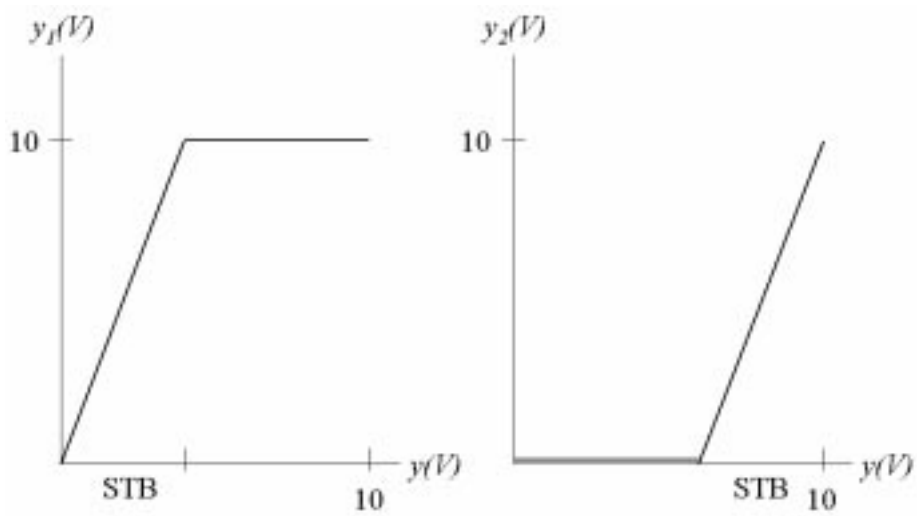Fig. 4. Exploded view of the function blocks SEZ and C.

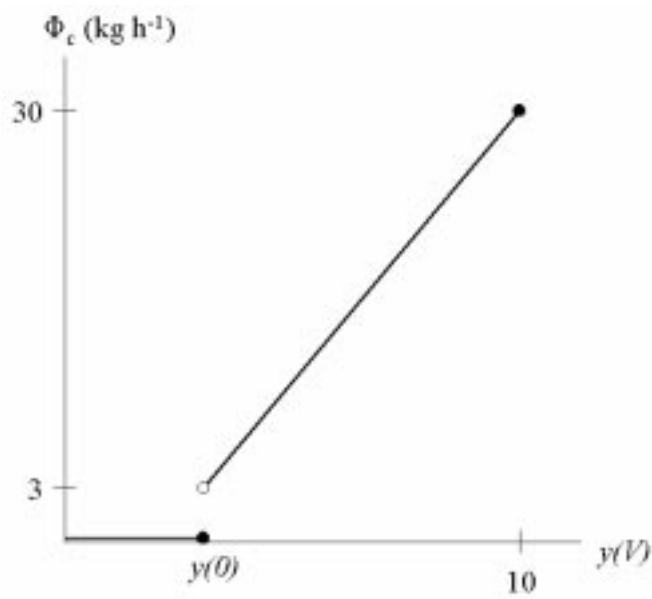Fig. 5. Graph of the transfer functions of an SEZ.

Fig. 6. Graph of the transfer function of a Condair unit.

STB, 10.0 V] to the maximum value 10.0 V (Fig. 5, right).

As for the Condair units, the graph of the transfer function of each Condair is shown in Fig. 6. A Condair unit produces vapour at a flow rate, Φ_c, as soon as y exceeds the ignition value $y(0)$. For $y = 10.0$ V a Condair produces the maximum flow rate of 30 kg h^{-1} of vapour.

The control technician wants a convenient installation of the SEZs and the Condair units by choosing one value for STB of the first SEZ, one value for STB for the two parallel SEZs and one ignition value for all the Condair units. The problem here is to determine these three values necessary to drive the four outputs of the Condair units.

It can be shown that these values are uniquely determined: the first SEZ has STB = 6.0 V, the two parallel SEZs have STB = 20/3 V and the ignition value for each Condair unit must be set at 5.0 V.

Further problems

So far a pocket calculator was sufficient to solve the case study. With students experienced in simulation software teachers can go down a number of interesting bypaths:

- First, investigate the behaviour of the wet air in the printing works if the non-simplified transfer function of the transducer and the actuator is used.
- Second, examine the behaviour of the wet air if the atmospheric pressure gradually changes.
- Third, investigate the phenomenon of condensation of the wet air if the maximum capacity of each Condair is gradually lowered.
- Fourth, explore the sensitivity of the air conditioning's control system to variations of the external parameters $\Phi_{in,dry}$ and $\Phi_{out,dry}$.

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