

# Modular Control Laboratory System with Integrated Simulation, Animation, Emulation, and Experimental Components\*

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*A typical sequence for the design of a controller, given the desired objectives, is the following: system modeling, design and mathematical analysis, simulation studies, emulation, and experimental implementation. Most control courses thoroughly cover design and mathematical analysis and utilize a simulation or experimental project at the end of the course. However, animation and emulation are seldom utilized and projects rarely cover the entire controller design sequence. This paper presents a control laboratory system developed at the University of Missouri at Rolla that integrates simulation, animation, emulation, and experimental components. The laboratory system may be applied to a wide variety of controls courses, from undergraduate to graduate. In addition to the simulation and experimental studies, students utilize animation and emulation components. Animation allows the students to visualize, as well as validate, their controllers during the simulation design phase, and emulation allows students to debug their programs on the target processor before experimentally implementing their controllers. Two experiments are presented to demonstrate the modular control laboratory system.*

## NOMENCLATURE

$a_n$  : Normal acceleration ( $m/s^2$ )  
 $a_t$  : Tangential acceleration ( $m/s^2$ )  
 $B_m$  : Motor viscous damping (Nms)  
 $e_{ss}$  : Steady state position error (m)  
 $F$  : Driving force from motor on cart (N)  
 $F_x$  : Reaction force in x-direction between cart and pendulum (N)  
 $F_y$  : Reaction force in y-direction between cart and pendulum (N)  
 $k$  : Spring force constant (N/m)  
 $K_a$  : PWM gain  
 $K_g$  : Motor internal gearbox gain  
 $K_p$  : Proportional controller gain (V/m)  
 $K_t$  : Torque constant (Nm/A)  
 $K_v$  : Voltage constant (V/(rad/s))  
 $I$  : Motor current (A)  
 $J_m$  : Motor inertia ( $kgm^2$ )  
 $l$  : Pendulum length (m)  
 $m$  : Cart mass (kg)  
 $m_p$  : Pendulum mass (kg)  
 $M_1$  : Cart 1 mass (kg)  
 $M_2$  : Cart 2 mass (kg)  
 $R$  : Motor electrical resistance (O)  
 $R_g$  : Motor gear radius (m)  
 $T_f$  : Coulomb friction torque (Nm)  
 $T_m$  : Torque drained from motor (Nm)  
 $T_s$  : Torque applied to shaft (Nm)

$v$  : Cart velocity (m/s)  
 $v_1$  : Cart 1 velocity (m/s)  
 $v_2$  : Cart 2 velocity (m/s)  
 $x$  : Cart position (m)  
 $x_1$  : Cart 1 position (m)  
 $x_2$  : Cart 2 position (m)  
 $x_r$  : Reference cart position (m)  
 $\theta$  : Pendulum angular position (rad)  
 $\omega$  : Pendulum angular velocity (rad/s)  
 $\omega_m$  : Motor angular velocity (rad/s)  
 $\omega_s$  : Shaft angular velocity (rad/s)

## INTRODUCTION

CONTROL THEORY is difficult for most students to understand if the theory is presented only at an abstract level and they are unable to apply it to a real system and visualize the results. Completing the entire controller design cycle and applying the results to a physical system, therefore, helps the students to better understand the theoretical material. A modular control laboratory system developed at the University of Missouri at Rolla that integrates simulation, animation, emulation, and experimental components is introduced in this paper. While control laboratories are typically designed for a specific experiment, the modular laboratory system presented in this paper consists of reconfigurable components, providing a flexible platform capable of many different

\* Accepted 26 July 2005.

experiments. Generally, the students conducting control experiments are required to design the controllers and then simulate their performance before implementing them on the experimental system. A major problem in such a process is that the system performance cannot be visualized during the early portion of the control design phase. Animation transforms static data into dynamic data that can be visualized, providing students with a better understanding of the system performance. Emulation is the step between simulation and experimental implementation where the control program is executed on the target processor; however, the physical system is replaced with a digital simulation. This step allows the students to debug their programs before the experiment.

The laboratories in this paper are conducted on a modular platform. A modular control laboratory consists of mechanical and software components that can be easily reassembled for different experiments, thus, providing a cost-effective system. As an example, Hagan and Latino [1] built a modular laboratory at Oklahoma State University. New components designed by the students can be added to the system, providing flexibility for the control experiments. In control education, more and more modular systems have been utilized [2].

There has been an abundance of work in developing hardware control laboratories. Traditional apparatuses include inverted pendulum, tank system, and ball-and-beam [3]. Malmborg and Eker [4] developed a double tank system, where the objective was to maintain a constant liquid level, and implemented a PID controller, a time-optimal controller, and a logic-based switching strategy. Mori et al. [5] performed one of the first studies that investigated the stabilization of an inverted pendulum. This has become one of the most popular experiments for control laboratories. This experiment was extended by Yamakita et al. [6] and Sasaki et al. [7] who developed systems of double inverted pendulums and applied robust control, and by Meier et al. [8] who studied the stabilization of a triple-inverted pendulum. Whelan and Ringwood [9] implemented a ball-and-beam experiment where vision was used to measure the ball's position and velocity. Sridharan [10] extended this experiment by creating a ball on a beam on a roller. A variety of new devices have also been implemented in control laboratories. For example, Chapuis [11] utilized a model helicopter in the laboratory to analyze flight controller performance, and Zhao et al. [12] designed and built an electric prototype vehicle SMARTREV to serve as a platform for research and education in vehicle control. Horacek [13] conducted a summary on building control laboratories, concentrating on the equipment, scale models, and supporting software environment.

The hardware system described in this paper is based on the classical inverted pendulum setup. With the movement of one cart, Mori et al. [5]

successfully swung up a pendulum with a bang-bang controller and balanced it with a LQR controller. Furuta et al. [14] presented a computational strategy to obtain the time optimal control for this nonlinear system. Astrom et al., [15] used an energy control method to improve system performance. Other methods include Lyapunov optimal feedback control [e.g. 16], sliding mode control [e.g. 17], and fuzzy control [e.g. 18–21]. In this paper, two isolated inverted pendulum systems are combined, which can be reconfigured to provide a wide variety of experiments.

Improvements in computing power have led to marked advancements in virtual laboratories. Clement and Knowles [22] assembled a robotics laboratory station in support of machine vision courses. Overstreet and Tzes [23] provided an Internet-accessible environment for a real-time mechatronic laboratory where the controller was implemented from a remote site. Ko et al. [24] developed a web-based laboratory, using video and audio conferencing, and Swamy et al. [25] presented a solution for the remote control of hardware by using available freeware. Various computer visualization software packages for evaluating the performance of control systems have been developed. Real-Time Simulation and Animation (RTSA) software, introduced by Cheok and Kheir [26], was very effective for presenting concepts of dynamic control systems in instructional and research laboratories. Computer-aided control engineering (CACE) was described by Kheir et al. [27]. Users expressed ideas to the computer by utilizing a graphical user interface and icon manipulation instead of programming in scripted codes. The results were displayed by the computer with color graphics, animation, three-dimensional visualization, etc. Dixon [28] discussed the standardization of computer-aided control system design (CACSD) software tools based on graphical control system simulation software (e.g. Matlab/Simulink). The virtual laboratory components described in this paper are the simulation and animation programs built in Matlab as m-files and the emulation programs constructed in Labview.

The modular control laboratory system developed at the University of Missouri at Rolla is presented in the following sections. Different experiments for undergraduate and graduate control courses are introduced and the utility of the control laboratory system is illustrated via two examples.

## MODULAR CONTROL LABORATORY SYSTEM

The modular control laboratory system presented in this paper is designed such that a variety of experiments, suitable for a wide range of controls courses from introductory undergraduate to advanced graduate, may be easily

constructed. Figure 1 shows the control laboratory system setup. The physical base is a linear track with a length of 1.2 m. One or two carts may be placed on the track. A DC motor (24 V, 1.44 A) and an incremental rotational encoder (4096 counts per revolution with X4 encoding) connected by rotational gears (radii of 0.021 m and 0.0144 m, respectively) are mounted on each cart. The carts may be connected by a spring (999.4 N/m,  $-0.05\sim 0.05$  m) and pendulums (0.073 kg, 0.567 m), that are free to rotate  $360^\circ$ , may be connected to each cart. Encoders (4096 counts per revolution with X4 encoding) are directly attached to each pendulum to measure angular position and a DC motor (24 V, 1.7 A) may be directly attached to each pendulum. Connectors such as screws and couplings are used to attach the components and every laboratory may be easily assembled and disassembled. See Figure 2 for two of the different configurations of the modular control laboratory.

Simulation and animation programs are built in Matlab as m-files. The simulation programs numerically simulate the closed-loop system behavior, including nonlinearities such as saturation.

The animation programs read the cart and pendulum positions, which can be generated from simulation, emulation, or experimental data, automatically set the plot scale, and provide a visualization of the system performance by generating dynamic images of the physical system. The reference and actual values are simultaneously shown to illustrate the controller behavior.

The emulation and experimental implementation programs are developed in the National Instruments Labview programming environment. Labview is a graphical programming environment tailored to measurement and control applications (see Fig. 3). Labview was selected as the programming platform for the control laboratory since it is utilized in several undergraduate courses at the University of Missouri at Rolla. The control programs are executed on a Dell OptiPlex GX400 PC with an Intel Pentium 4 CPU 1.7 GHz processor. Encoder signals are received via a counter-timer board (32 bits, 5 V TTL, 20 MHz) and velocity signals are constructed by processing the encoder signals. Command voltages are sent via an analog output board (12 bits,  $-10$  V to 10 V)

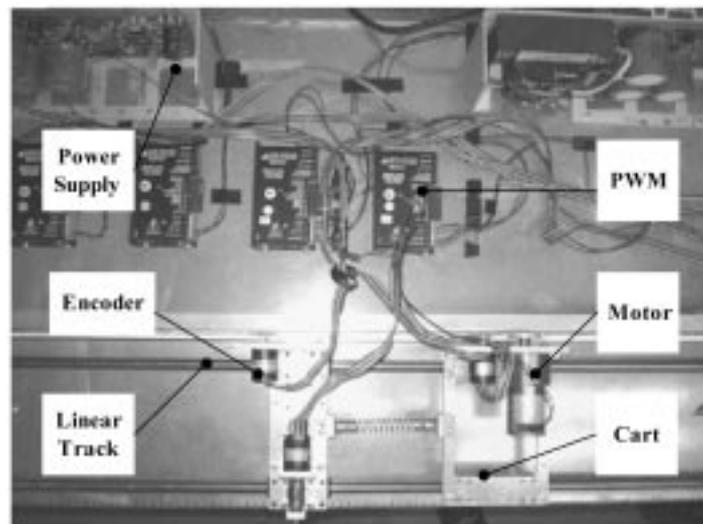


Fig. 1. Modular control laboratory (top view).

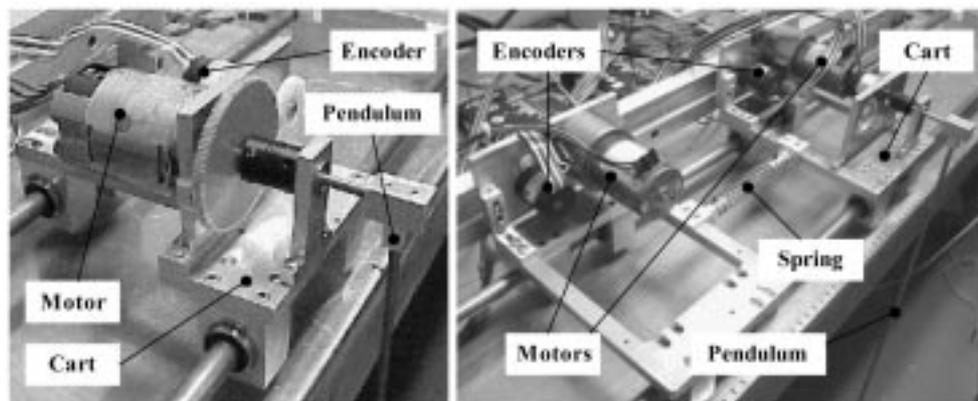


Fig. 2. Two configurations of the modular control laboratory: SISO cart and pendulum (left), and MIMO cart and pendulum (right).

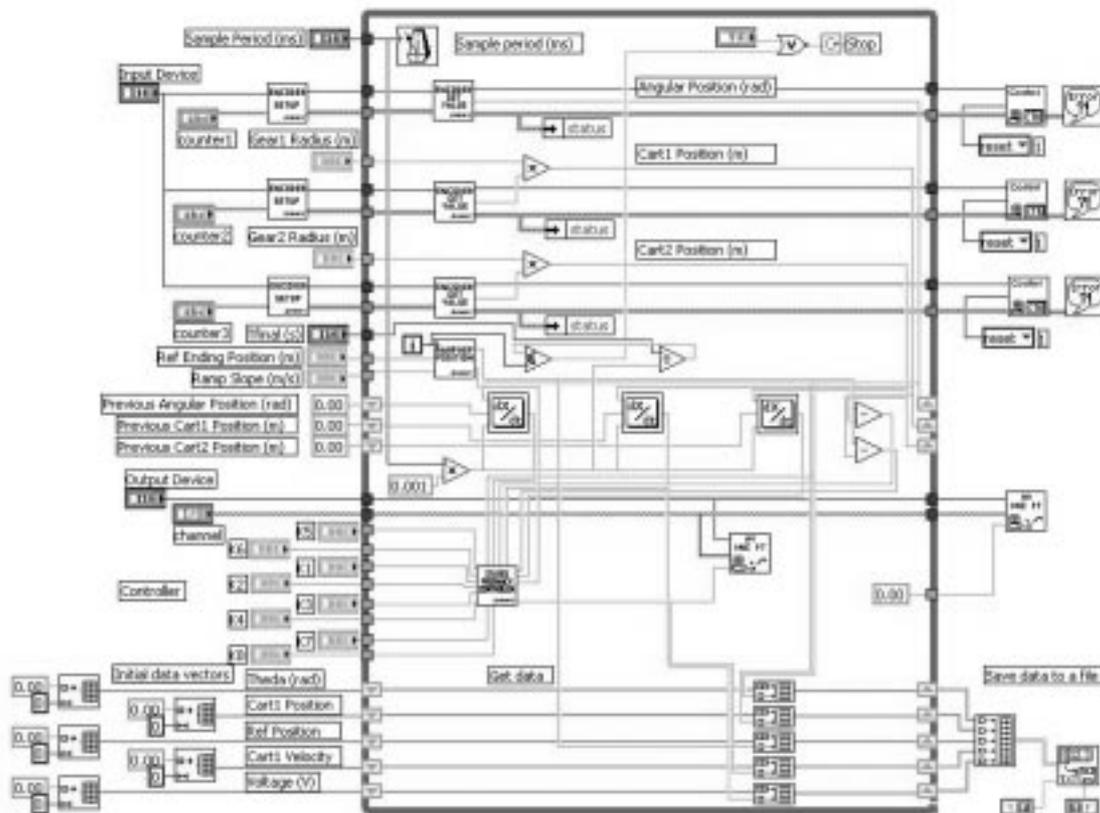
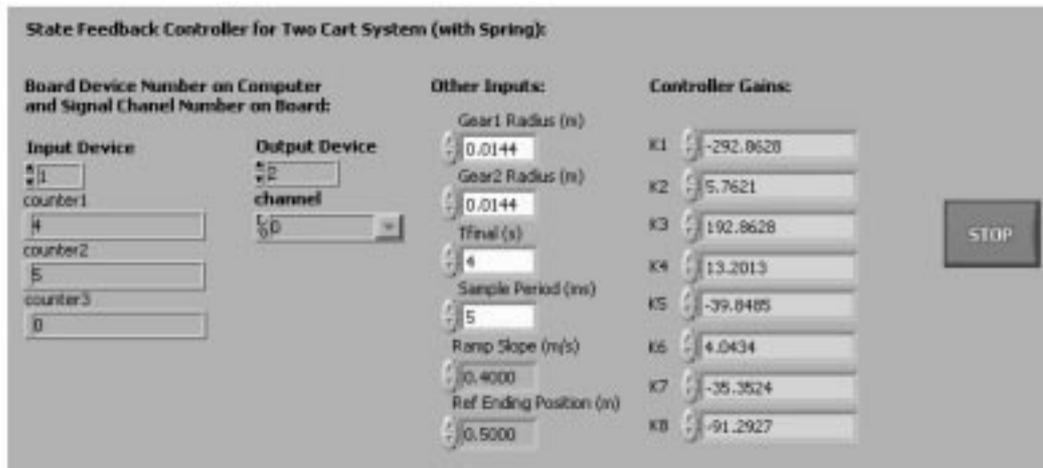


Fig. 3. Two cart-one pendulum laboratory graphical user interface (top) and icon-based sensing and control program (bottom).

to pulse width modulators (PWMs) that amplify the control signals. Two power supplies provide the required power for the four PWMs. Figure 4 provides a schematic of the control laboratory system.

The architectures of the emulation and experimental programs are shown in Fig. 5. The input and output signals are transmitted, via the counter/timer and analog output boards, respectively, between the computer and the physical system. In emulation, however, the controller receives and sends signals to a digital system model programmed in Labview, as well as the counter/timer and analog output boards, respectively. This

model simulates the physical system performance while the physical system is disconnected. Therefore, controller performance is validated on the target processor without the possibility of damaging the physical system.

### DESCRIPTION OF CONTROL COURSES AND EXAMPLE EXPERIMENTS

The modular control laboratory is utilized in several control courses in the Mechanical and Aerospace Engineering and Engineering Mechanics Department at the University of

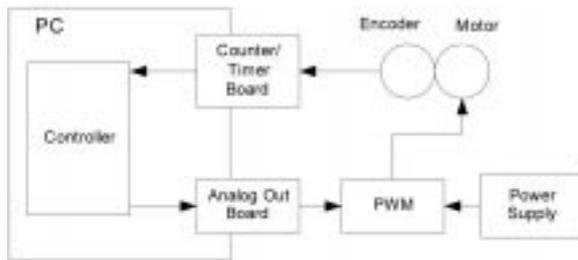


Fig. 4. Control laboratory architecture.

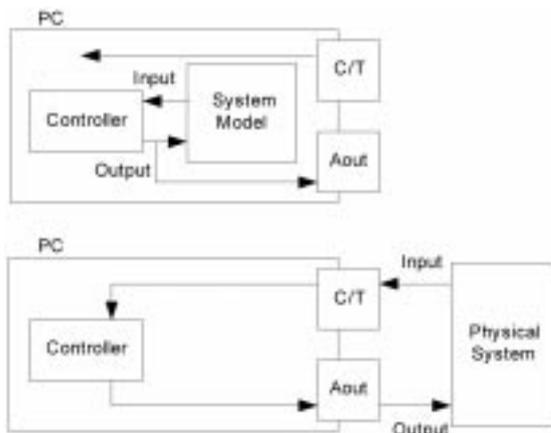


Fig. 5. Emulation structure (top) and experimental structure (bottom).

Missouri at Rolla. The courses follow a similar laboratory process. First, the students characterize the physical system dynamics with a modeling exercise. Differential equations are generated by applying mechanical and electrical first principles. The theoretical principles presented in the course are utilized to design controllers. All differential equations (i.e. physical and control) are transformed into difference equations that are numerically simulated and the results are animated. Nonlinear effects such as quantization and saturation are included. Both simulation and animation help the students analyze the controller performance during the design stage, and mistakes may be detected and corrected. Controllers that are validated via simulation are then implemented in emulation where the controller program is executed on the target processor; however, the physical system is replaced with a digital simulation. After this step, the controllers are implemented on the physical system. A wide variety of experiments may be designed for many different control courses, from undergraduate to graduate.

In the undergraduate control course, concepts from classical control such as Routh arrays, steady-state error, Root Locus Diagrams, proportional (P), integral (I), derivative (D), lead, and lag control, Bode diagrams, and Nyquist diagrams are introduced. Linear systems and linearized systems are considered. Several experiments are possible with the modular control laboratory system. In a cart position tracking laboratory, one motor drives one cart and the students design a controller to

regulate the cart position for ramp inputs. This experiment allows the students to analyze steady-state error, root locus diagrams, and P controllers, and reinforces the concept of system type. A pendulum position-tracking laboratory utilizes one motor that directly drives one pendulum. In this laboratory, the students design a PI controller to regulate the pendulum position at different set points. This nonlinear system reinforces the concept of linearization. In a third laboratory, two carts, where only the first cart has a motor, are connected by a spring and the students design a controller to regulate the position of the second cart. Frequency response, in addition to the concepts listed above, is reinforced. Many other laboratories are possible with the modular control laboratory system.

The introductory graduate control course at UMR concentrates on modern control methods: state space formulation, controllability, observability, pole placement controller design, observer design, linear quadratic regulator (LQR) controller design, and error state-space method. The systems are more complex and multiple-input, multiple-output (MIMO) systems are introduced. In a pendulum-balancing laboratory, a pendulum, which is free to rotate  $360^\circ$ , is mounted on a cart. The objective is to move the cart to maintain the pendulum in the upward position. This laboratory reinforces the concepts of stability, controllability, and observability, and different control algorithms, such as pole placement and LQR control, and linear observers are utilized. Moreover, the students are also required to swing up the pendulum from the downward position to the upward position and then balance the pendulum in the upward position by moving only the cart. In another laboratory, two carts, each of which has a motor, are connected via a spring and the carts move along a prescribed path. This laboratory requires the use of MIMO control techniques. The modular control laboratory can easily be reconfigured for many other graduate level experiments. In all graduate laboratories, the students are required to estimate and reject friction and design observers to estimate velocities.

Undergraduate and a graduate control laboratory experiments are now presented to illustrate the utility of the modular control laboratory system developed at the University of Missouri at Rolla.

#### *Cart position tracking laboratory*

The cart position tracking laboratory was designed for the undergraduate introductory controls course. The objective of this laboratory is for the students to model, simulate, and control the position of a cart that moves on a linear track. The reference is a ramp input where the cart moves a distance of 90 mm at a rate of 30 mm/s and then moves a distance of 90 mm at a rate of  $-30$  mm/s. The motor data file is provided to the students so they can determine the motor parameters (e.g. mechanical inertia, electrical resistance, stall torque).

Ignoring the electrical dynamics of the electrical portion of the DC motor, the model of the cart system is:

$$I = \frac{K_a}{R} V_c - \frac{K_v}{R} \omega_m \quad (1)$$

$$\dot{x} = v \quad (2)$$

$$[J_m K_g^2 + m R_g^2] \dot{v} = -B_m K_g^2 v - K_g R_g T_f + K_g R_g K_t I \quad (3)$$

where

$$K_g \equiv \frac{T_s}{T_m} = \frac{\omega_m}{\omega_s} = \frac{\dot{\omega}_m}{\dot{\omega}_s} = 19.7,$$

$$R_g \equiv \frac{T_s}{F} = \frac{v}{\omega_s} = \frac{\dot{v}}{\dot{\omega}_s} = 0.00794m,$$

and

$$T_f = 0.0035 \text{sgn}(\omega_m).$$

Using a proportional controller, the control law is:

$$V_c = K_p e = K_p [x_r - x] \quad (4)$$

Ignoring Coulomb friction, the open-loop transfer function is:

$$\frac{x(s)}{V_c(s)} = \frac{K_a K_g R_g K_t}{\{R[J_m K_g^2 + m R_g^2]\} s^2 + \{R B_m K_g^2 + K_t K_v K_g^2\} s} \quad (5)$$

Combining Equations (4) and (5), the closed-loop transfer function is:

$$\frac{x(s)}{x_r(s)} = \frac{K_p K_a K_g R_g K_t}{R(J_m K_g^2 + m R_g^2) s^2 + (R B_m K_g^2 + K_t K_v K_g^2) s + K_p K_a K_g R_g K_t} \quad (6)$$

Using the final value theorem, the steady-state error is:

$$e_{ss} = \frac{R B_m K_g^2 + K_t K_v K_g^2}{K_p K_g R_g K_t K_a} \quad (7)$$

For a steady-state error of 0.5 mm, the controller gain is  $K_p = 962.2 \text{ V/m}$ .

The closed-loop system was simulated using equations (1)–(4). Equations (2) and (3) were solved using the Euler integration method. Note that the current was saturated at  $\pm 1.44 \text{ A}$ , and the command voltage was saturated at  $\pm 10 \text{ V}$ . An animation program was provided to the students. After running the simulation, the reference and actual positions are input to the animation program so the students can visualize the cart performance. The simulation results and a screen shot of the animation are shown in Figs 6 and 7, respectively.

Before the controller was experimentally implemented, emulation was conducted to avoid

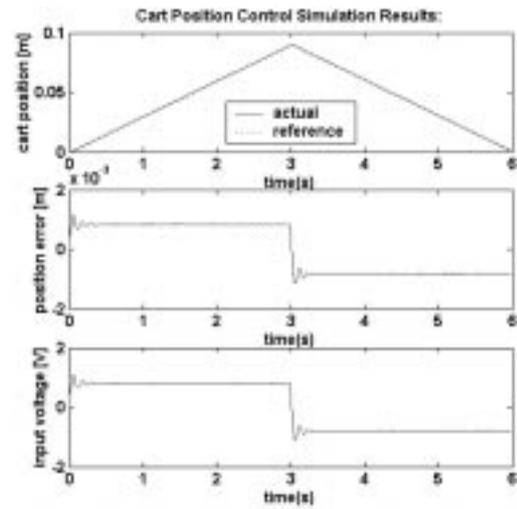


Fig. 6. Cart position tracking laboratory simulation results.

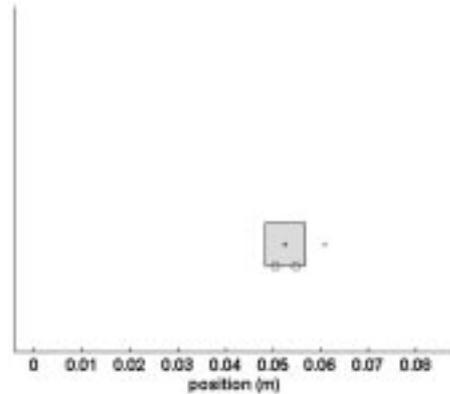


Fig. 7. Cart position tracking laboratory animation screenshot with actual (dot in cart center) and reference (dot to the right of cart) positions.

program conversion mistakes. The system model is the same for both the emulation and simulation programs, therefore, the results are the same (see Fig. 8). The experimental results are shown in Figure 9. The desired steady-state error was not achieved due to the fact that Coulomb friction was ignored. As a comparison, a controller with a gain of  $4811 \text{ V/m}$ , that produces a theoretical steady-state error of  $0.1 \text{ mm}$ , was also implemented (see Figure 10). The high gain controller causes the command voltage to constantly saturate and the system reaches an unwanted limit cycle. The students were able to see the results and the data was emailed to them. The students graphed the data and could also run the data through the animation file.

In this laboratory, the students utilized mathematical tools (e.g., modeling via first principles, transfer functions, final value theorem) they learned in their coursework to model the physical system and design the controllers. The actual physical system was simulated, animated, and emulated, and then the controller was implemented experimentally. In this way, the students were

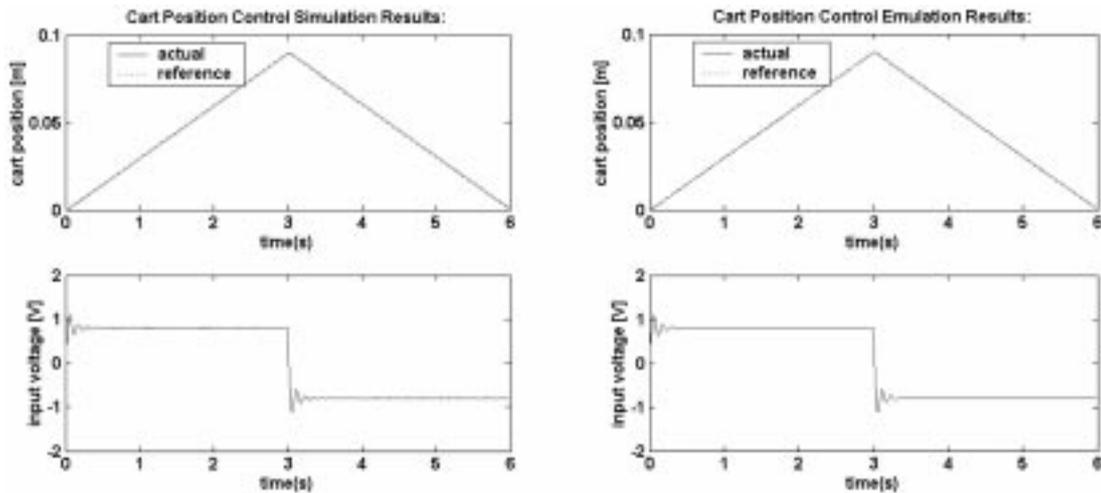


Fig. 8. Comparison between simulation results (left) and emulation results (right) for cart position tracking laboratory.

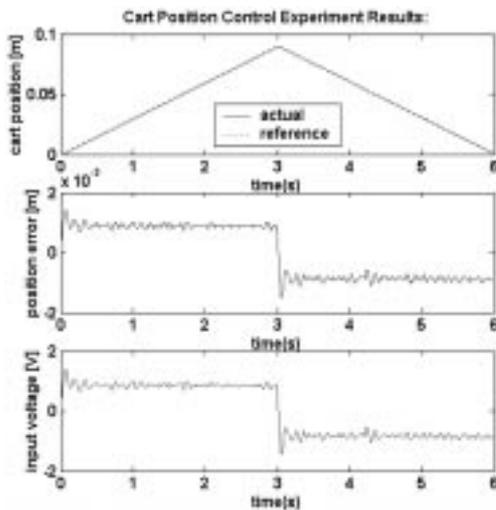


Fig. 9. Cart position tracking laboratory experimental results ( $K_p = 962.2$  V/m).

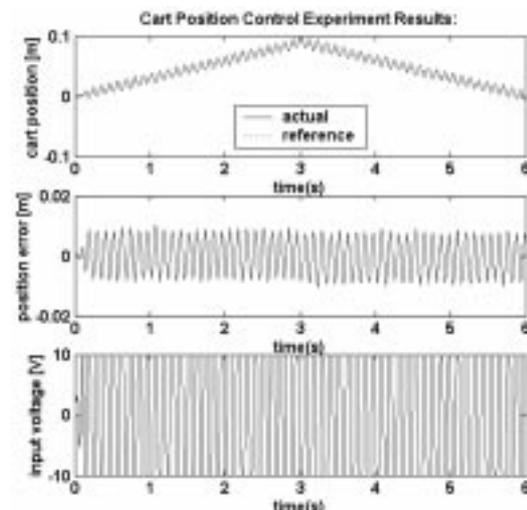


Fig. 10. Cart position tracking laboratory experimental results ( $K_p = 4811$  V/m).

able to go through the entire controller design cycle and understand the physical significance of the mathematics they learned in their coursework. The integration of the simulation, animation and emulation components with the experimental portion of the laboratory provided a positive experience for the students. The match between the simulation, emulation, and experimental results allowed the students to gain a physical insight into the system dynamic equations. Also, the animation was very useful in allowing students to understand the physical system and comprehend the meaning of a ramp input. This laboratory also taught the students about real-world effects that must be taken into account (i.e. Coulomb friction and control signal saturation). During their coursework, students are presented with idealized linear systems. In this laboratory, the Coulomb frictional effects, present in both the simulation and in the experiments, prevented the students from reaching the desired steady-state error. For the large gain controller, the effects of

saturation became apparent in both the simulation and experiments.

#### Two cart/one pendulum laboratory

A laboratory utilizing two carts and one pendulum was designed for the introductory graduate control course. Two carts are connected with a spring and a pendulum, which is free to rotate  $360^\circ$ , is mounted on one cart (cart 2). A motor and gear are assembled on the other cart (cart 1). The objective of this laboratory is for the students to move the carts a fixed distance and then bring the system quickly to rest. The reference position is a ramp with a slope of  $0.4$  m/s and an end position of  $0.5$  m. Two separate coordinate systems are fixed to each cart, respectively. The reference positions given for cart 1 and cart 2 in their own coordinate systems are the same. In addition, the pendulum is required to remain down and should come to rest quickly (Fig. 11). The students also need to implement a reduced-order observer to estimate the immeasurable states (i.e. cart and pendulum

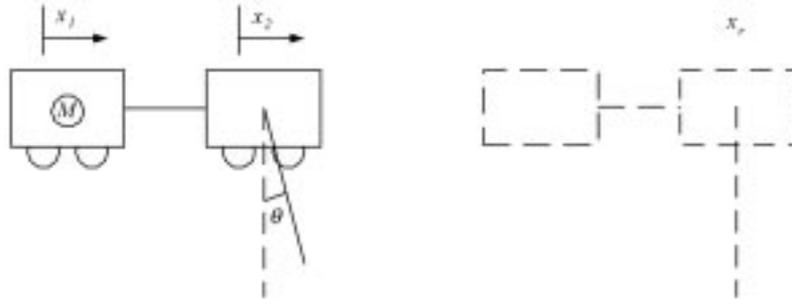


Fig. 11. Two cart-one pendulum laboratory schematic.

velocities and Coulomb friction). Like the cart laboratory, the students determine the motor parameters with a data file provided to them.

The dynamic model of the two cart one pendulum system is:

$$\begin{aligned} & \left[ m_p + M_2 - \frac{3}{4}m_p \cos^2(\theta) \right] \\ & \dot{v}_2 = -kx_2 + kx_1 + \frac{3}{4}m_p g \sin(\theta) \cos(\theta) \\ & + \frac{m_p l}{2} \omega^2 \sin(\theta) \end{aligned} \quad (8)$$

$$\begin{aligned} & \left[ \frac{m_p l}{2} \cos(\theta) - \frac{2(m_p + M_2)l}{3 \cos(\theta)} \right] \\ & \dot{\omega} = -kx_2 + kx_1 + \frac{m_p l}{2} \omega^2 \sin(\theta) \\ & + \frac{(m_p + M_2)}{\cos(\theta)} g \sin(\theta) \end{aligned} \quad (9)$$

$$\begin{aligned} (J_m K_g^2 + M_1 R_g^2) \dot{v}_1 = & -B_m K_g^2 v_1 - k R_g^2 x_1 \\ & + k R_g^2 x_2 + K_t R_g K_g I - K_g R_g T_f \end{aligned} \quad (10)$$

$$\dot{x}_1 = v_1 \quad (11)$$

$$\dot{x}_2 = v_2 \quad (12)$$

$$\dot{\theta} = \omega \quad (13)$$

where the current is given in Equation (1). Substituting for the current, equation (10) becomes:

$$\begin{aligned} (J_m K_g^2 + M_1 R_g^2) R \dot{v}_1 = & -(B_m K_g^2 R + K_g^2 K_t K_v) v_1 \\ & - k R_g^2 R x_1 + k R_g^2 R x_2 + K_t K_a R_g K_g V_c \\ & - K_g R_g R T_f \end{aligned} \quad (14)$$

The system is linearized about  $\theta = 0$  and  $\omega = 0$ ; therefore,  $\sin(\theta) \approx \theta$ ,  $\cos(\theta) \approx 1$ , and  $\omega^2 \approx 0$ . Equations (8) and (9), respectively, become:

$$\left( \frac{m_p}{4} + M_2 \right) \dot{v}_2 = -kx_2 + kx_1 + \frac{3}{4}mg \theta \quad (15)$$

$$-\left( \frac{1}{6}m_p l + \frac{2}{3}M_2 l \right) \dot{\omega} = -kx_2 + kx_1 + (m_p + M_2)g\theta \quad (16)$$

In order to use state feedback control to drive the errors to zero, the system states are redefined as  $e_1$ ,  $v_1$ ,  $e_2$ ,  $v_2$ ,  $\theta$ , and  $\omega$ , where  $e_i = x_r - x_i$  ( $i = 1, 2$ ), and  $x_r$  is the same for both carts in their respective coordinate systems. Rearranging,  $x_i = x_r - e_i$  ( $i = 1, 2$ ) and  $\dot{x}_i = \dot{x}_r - \dot{e}_i$  ( $i = 1, 2$ ). Note that  $x_r$  is a ramp input, thus,  $\dot{x} \neq 0$ . Substituting  $x_i = x_r - e_i$  ( $i = 1, 2$ ) into equations (14)–(16):

$$\begin{aligned} (J_m K_g^2 + M_1 R_g^2) R \dot{v}_1 = & -(B_m R + K_t K_v) K_g^2 v_1 \\ & + k R R_g^2 e_1 - k R R_g^2 e_2 + K_t K_a R_g K_g V_c \\ & - K_g R_g R T_f \end{aligned} \quad (17)$$

$$\left( \frac{m_p}{4} + M_2 \right) \dot{v}_2 = ke_2 - ke_1 + \frac{3}{4}m_p g \theta \quad (18)$$

$$-\left( \frac{1}{6}m_p l + \frac{2}{3}M_2 l \right) \dot{\omega} = ke_2 - ke_1 + (m_p + M_2)g\theta \quad (19)$$

Note the terms with  $x_r$  are canceled. A control algorithm for exogenous signals (i.e., references and disturbances) is applied [29]. The state space description is  $\dot{z} = Az + BV_c$  and  $y = Cz$ , where  $z$ ,  $A$ ,  $B$ , and  $C$ , respectively, are:

$$z = \begin{bmatrix} e_1 \\ e_2 \\ \theta \\ \dot{x}_r \\ v_1 \\ v_2 \\ \omega \\ T_f \end{bmatrix} A = \begin{bmatrix} 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 253 & -253 & 0 & -43.4 & 0 & 0 & 0 & -242 \\ -773 & 772 & 0.42 & 0 & 0 & 0 & 0 & 0 \\ 2040 & -2040 & -27.1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} B = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 2.65 \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad (20)$$

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \end{bmatrix} \quad (21)$$

An LQR controller is designed. The control law is given by:

$$V_c = -Gz \quad (22)$$

The weighting matrices, selected via trial and error, are:

$$Q = \text{diag}[5000 \ 1 \ 5000 \ 1 \ 2000 \ 1 \ 1 \ 1] \quad R = 1 \quad (23)$$

The Matlab function **lqr** is used to calculate the gain matrix:

$$G = [-292.86 \ 5.76 \ 192.86 \ 13.20 \ -39.85 \ 4.04 \ -35.35 \ -91.29] \quad (24)$$

The closed-loop pole locations are located at 0, 0,  $-7.6$ ,  $-38.9$ ,  $-1.8 \pm 4i$ , and  $-4.3 \pm 30i$ .

The pendulum and cart positions are measured via three separate encoders and the reference position is predefined; thus,  $e_1$ ,  $e_2$ ,  $\theta$ , and  $\dot{x}_r$  are measurable. The other states, namely,  $v_1$ ,  $v_2$ ,  $\omega$ , and  $T_f$ , must be estimated. As a result, a reduced order observer is designed. The unmeasurable states are estimated by  $\hat{z}_2 = Ly + p$ , where  $p$  is described by:

$$\dot{p} = F\hat{z}_2 + (A_{21} - LC_1A_{11})C_1^{-1}y + HV_c \quad (25)$$

where  $\hat{z} = [\hat{v}_1 \ \hat{v}_2 \ \hat{\omega} \ \hat{T}_f]^T$ ,  $F = A_{22} - LC_1A_{12}$ ,  $H = B_2 - LC_1B_1$ , and the matrices  $A_{11}$ ,  $A_{12}$ ,  $A_{21}$ ,  $A_{22}$ ,  $B_1$ ,  $B_2$ , and  $C_1$  (i.e., the corresponding submatrices of  $A$ ,  $B$ , and  $C$ ) are:

$$\begin{aligned} A_{11} &= \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} & A_{12} &= \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -242 \end{bmatrix} \\ A_{21} &= \begin{bmatrix} 253 & -253 & 0 & 0 \\ -773 & 773 & 0.42 & 0 \\ 2040 & -2040 & -27.1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \\ A_{22} &= \begin{bmatrix} -43.4 & 0 & 0 & -242 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} & B_1 &= \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \\ B_2 &= \begin{bmatrix} 2.65 \\ 0 \\ 0 \\ 0 \end{bmatrix} & C_1 &= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \end{aligned} \quad (26)$$

The desired observer closed-loop pole locations, selected by trial and error, are  $-6$ ,  $-6.5$ ,  $-7$ , and  $-7.5$ . The Matlab function 'place' is used to calculate the observer gain matrix:

$$L = \begin{bmatrix} 28.9 & 0 & 0 & 0 \\ 0 & -6 & 0 & 0 \\ 0 & 0 & 6.5 & 0 \\ 0.22 & 0 & 0 & 0 \end{bmatrix} \quad (27)$$

When applying the observer to the system,  $v_1$ ,  $v_2$ ,  $\omega$ , and  $T_f$  are estimated and the control law becomes:

$$V_c = -G\hat{z} = -G[e_1 \ e_2 \ \theta \ \dot{x}_r \ \hat{v}_1 \ \hat{v}_2 \ \hat{\omega} \ \hat{T}_f]^T \quad (28)$$

Equation (1), where  $\omega_m$  has been replaced by  $K_g^*v/R_g$ , and Equations (8)–(13) are used to simulate the closed-loop nonlinear system. The differential equations were solved using a fourth-order Runge-Kutta integration routine. Again, the command voltage was saturated between  $\pm 10$  V and the current is saturated between  $\pm 1.44$  A. An animation program was developed to provide the students a means to visualize the system performance given simulation, emulation, or experimental data. The simulation results are shown in Fig. 12, while Fig. 13 is a screen shot of the animation.

Similar to the cart position project, the controller is implemented in emulation before it is implemented experimentally. The result is shown in Fig. 14. Because both simulation and emulation use the

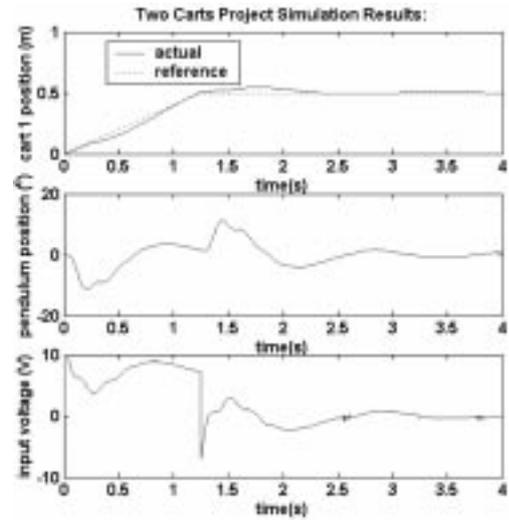


Fig. 12. Two cart-one pendulum laboratory simulation results.

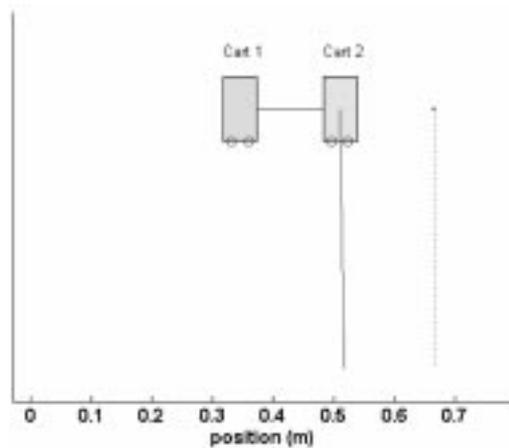


Fig. 13. Two cart-one pendulum laboratory animation screenshot.

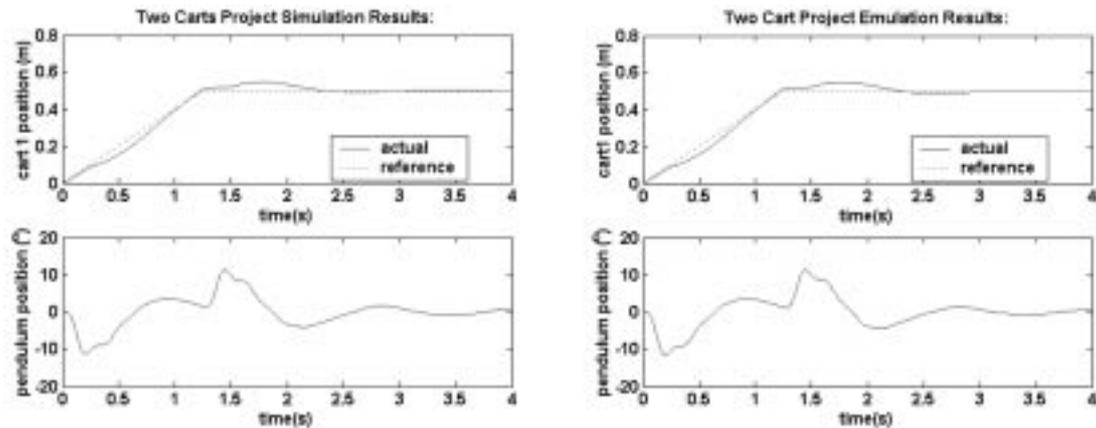


Fig. 14. Comparison between simulation results (left) and emulation results (right) for two cart-one pendulum laboratory.

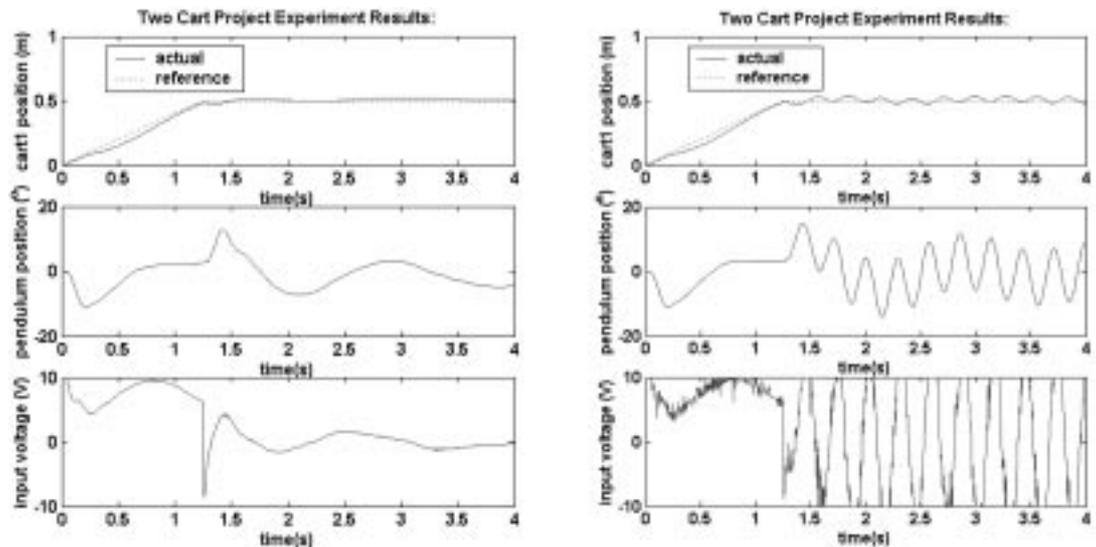


Fig. 15. Experimental results for two cart-one pendulum laboratory (left: with observer, right: without observer).

same system model and the same sample period, the results are identical and, thus, the controller is verified on the target processor. Next, both the controller and the reduced-order observer are experimentally implemented on the physical system. The results are shown in Fig. 15 (left) for the experiment with a reduced-order observer. Figure 15 (right) presents the experimental results when using the same controller where the velocities are estimated by first order backward difference equations. When the observer was not utilized, the encoder quantization drove the closed-loop system unstable.

In this laboratory, the students utilized mathematical tools and techniques (e.g. state-space formulation, LQR control, reduced order observer design) they learned in their coursework to model the physical system and design the controller and observer. This laboratory, as compared to the cart position tracking laboratory, provides the students with an opportunity to investigate a more complex physical system using more sophisticated control techniques. The students were required to linearize the system and they utilized exogenous control

techniques, LQR control algorithms, and observers. Again, the students go through the entire controller design cycle and can understand the physical significance of the mathematics they learned in their coursework. The students also learned how encoder quantization can adversely affect velocity estimation and, hence, closed-loop system performance.

## SUMMARY AND CONCLUSIONS

In this paper, simulation, animation, emulation, and experimental components were integrated to create a modular control laboratory system. The physical components were designed such that a wide variety of laboratory setups may be easily constructed that are suitable for control courses, from undergraduate to graduate. Two laboratories were presented. The animation component augmented the simulations to provide an increased understanding of the course material. The animation was particularly useful in the early control design stage as it allowed for visual analysis. The

emulation component eliminated mistakes encountered when uploading control programs to the target processor. These laboratories also intro-

duced the students to real-world effects that, if not taken into account, can significantly degrade controller performance.

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