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Learning Modules for Statics*

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Classroom implementation of a new approach to teaching statics is presented here. This new approach, which was explained in a companion paper [1], is based on the need to address the important concepts of statics first in isolation, and the recognition that students do not perceive the forces between rigid, inanimate objects. Hence, statics instruction was revised to focus on individual concepts in the context of situations in which all the forces are readily perceived. In this paper we demonstrate an implementation of this approach that draws upon widely accepted classroom techniques to promote active learning: peer-to-peer collaboration, integration of assessment and feedback into classroom activities, and the use of concrete physical manipulatives. With the development of learning modules, which feature objects to manipulate or examine, PowerPoint presentations and concept questions, the authors have transformed this instructional approach into practical classroom tools. The details of classroom implementation, extensive excerpts from the modules, and an assessment of the efficacy of this approach are presented.

INTRODUCTION

IN A COMPANION paper [1] to this one, we elaborated upon the motivations for our substantial reinventing of statics instruction. In short, we are conscious of the distinct concepts which students need to confront in statics, and we seek to expose students to one new concept at a time. In addition, we recognize that students often have difficulty perceiving the forces between inanimate objects. Hence, we treat each of the core concepts of statics within the context of forces which students can readily perceive: forces exerted by hand and forces that are made evident through the deformation or motion they induce. In this paper, we give extended examples of in-class instructional activities that dovetail with the progression of concepts and contexts described in the companion paper, and which are consistent with many approaches to improving learning that have been demonstrated in other subjects.

GENERAL APPROACHES TO IMPROVING LEARNING

In this section we describe several generally accepted techniques for improving learning outcomes that are relevant to our instructional approach. Students learn in part through a process of constantly comparing their understanding and predictions with observations of the world. Making comparisons with observations is one way of obtaining feedback, which is necessary to refining one's understanding. Increasingly, it is appreciated that assessment, in this broad sense of testing one's knowledge and understanding, must not be relegated to the end of the semester or even to several times during the semester; assessment must be fully integrated into learning [2].

Debating important questions with peers also has benefits. In these circumstances, students have to generate and articulate explanations of their reasons, and they have to argue their point of view; generating explanations can increase learning [3, 4]. Students must also listen to and comprehend the arguments of others. Finally, students have to weigh possibly opposing arguments and ultimately arrive at what they believe is the best answer. Collaboration between students, if harnessed appropriately and focused on salient issues, can be a powerful tool in learning [5].

Finally, for many subjects in the sciences or technologies, physical referents or manipulatives can serve to enhance learning. The use of manipulatives in a lecture environment accommodates students with a greater range of learning styles, as compared with only traditional lectures. As an example relevant to our implementation, students that learned about pulleys on real pulley systems were better able to solve real-world problems compared to students who learned from line diagrams [6].

A classroom that integrates the above three elements of assessment, collaboration and manipulatives is one that actively engages students. Particularly in large classes, students learn more when they are actively engaged in learning [7–9].

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INTEGRATING MANIPULATIVES, ASSESSMENT, AND COLLABORATION INTO THE STATICS CLASSROOM

We have integrated the elements described above to form a set of classroom activities, each designated as a 'learning module'. Each learning module addresses a relatively small number of concepts, from the concept sequence described in the companion paper, through a combination of desktop experiments or demonstrations, PowerPoint presentations and concept questions. The purpose of each learning module is to acquaint students with the concept(s) in the context of a real artifact and to help students make firm connections between the symbolic representations of statics and the physical features they represent.

If students are ultimately to apply statics to objects, then, we believe, statics must be learned, from the very beginning, in the context of real objects. The objects in our classrooms play two roles: they are the bodies whose equilibrium is considered, and often they are the bodies against which we exert forces and which exert forces on us. Objects may be physical artifacts which students can hold, the human body itself, or objects that are more conveniently depicted with, say, digital images. When the learning module involves an object, there is either a single copy of the object for the instructor to demonstrate in front of the class, or more often there are enough objects for every two or three students to share a copy. Many modules take advantage of a particular object that has been specially designed to facilitate learning of a number of concepts (e.g. the couple, static equivalency, 2-D and 3-D equilibrium, pin joints). This object, shown in (Figs 1–2), is referred to as the 'L'.

Students are actively engaged in relating the object to the concepts by contemplating a series of conceptual questions that are interspersed in the PowerPoint presentation. Such questions help students both to focus on the key ideas and to gauge their current level of understanding. Students are invited to vote on each of the questions. This can be done through raising one of a set of colored note cards or through a modern electronic Classroom communication system. The performance on such questions offers the instructor real-time insights into the understanding of students, which can affect further instruction.

While, in some instances, nearly all students will answer correctly, there is far more commonly a distribution of answers. In such cases, the instructor can suggest that students discuss the question with their neighbors. In addition, if appropriate, students are invited to seek to answer the question by manipulating the object they share. Much of the power of the technique proposed here resides in the combination of peer discussion and discovery learning associated with manipulating the object in pursuit of conceptual understanding.

The instructor controls the pace of the learning module through the PowerPoint presentation. Besides concept questions, typical presentations contain slides presenting basic information or theory, answers to questions, and depictions of how the situations contemplated are represented symbolically in statics with free-body diagrams and vectors.

DESCRIPTIONS OF SELECTED LEARNING MODULES WITH EXAMPLES OF POWER POINT PRESENTATIONS

In the following sections, we explain the progression of concepts in several key modules. In each case, selected slides from the PowerPoint Presentations are shown. In the electronic version of this paper when one clicks on the module title the full presentation opens.

Balancing simple objects with fingers and nutdrivers: Equilibrium under actions of concentrated forces and couples

The first five modules presented in this paper focus on balancing objects with fingers or smooth rods (which are modeled as applying normal concentrated forces) and nutdrivers (which are modeled as applying couples). As explained in detail in the companion paper [1], our goal is to

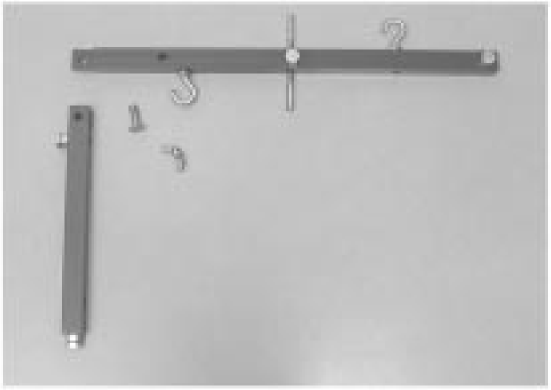


Fig. 1. 'L' disconnected.

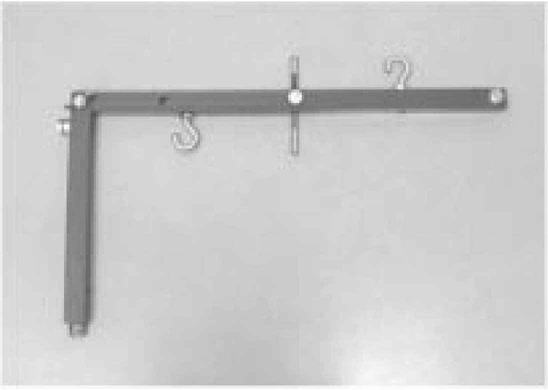
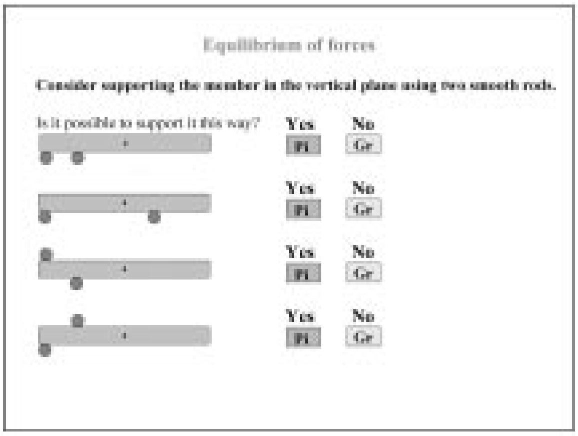


Fig. 2. 'L' connected.



Equilibrium of forces

Consider supporting the member in the vertical plane using two smooth rods.

Is it possible to support it this way?

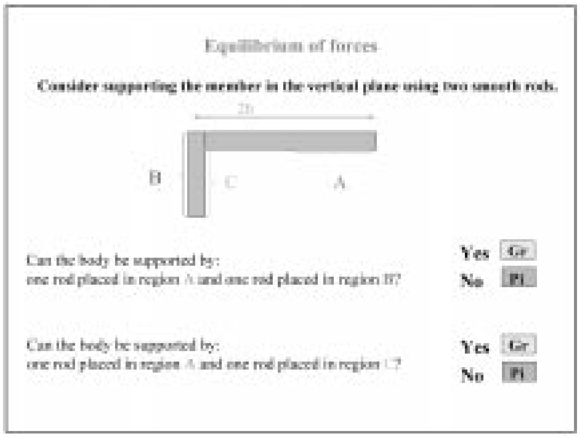
Yes Pi No Gr

Yes Pi No Gr

Yes Pi No Gr

Yes Pi No Gr

Fig. 3. Module I—slide 1/13.



Equilibrium of forces

Consider supporting the member in the vertical plane using two smooth rods.

Can the body be supported by:
one rod placed in region A and one rod placed in region B?

Yes Gr
No Pi

Can the body be supported by:
one rod placed in region A and one rod placed in region C?

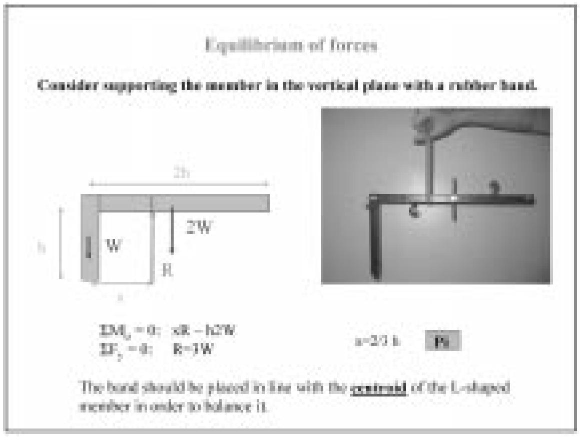
Yes Gr
No Pi

Fig. 5. Module I—slide 8/13.

make forces real by focusing on those forces that can be experienced through the sense of touch. Thus, we separate out and address the basic concepts of statics, including forces, moments, couples, static equivalency, free-body diagrams, and equilibrium in 2-D and 3-D, entirely without the need to invoke difficult-to-perceive forces between contacting inanimate objects. Only after this initial phase of statics are students gradually introduced to contacts between inanimate objects, frictional forces and connections. This gradual transition from manually exerted forces to contacts between inanimate objects prepares students to better understand the loads acting at connections between bodies.

Module I, entitled *Equilibrium of Forces*, addresses the conditions that forces acting on a body must satisfy to maintain the body in equilibrium. In this module, students 'discover' that balancing the downward force of the weight by an upward force is insufficient: one must also prevent the body from rotating. This is linked to the idea of a moment due to a force. Students confront this idea first in the context of a simple rectangular bar (Fig. 3), and then later consider a slightly more complex body, the 'L'. Here is an example in which we take advantage of student intuition as to how to balance an object, and reconcile that intuition with the results of statics. As a by-product, the idea of the centroid arises naturally (Fig. 4). Next, the possibility of using horizontal as well as vertical forces is introduced, along with the idea that there must be balance of forces in both horizontal and vertical directions (Figs 5 and 6). It should be noted that the module has already touched upon two common mistakes that students make: they simply neglect to include the requirement of moment balance and they allow for forces in some direction (say horizontal one) to be left unbalanced.

Module II, entitled *Couples in 2D*, turns to the extremely important idea of the couple. We take the couple to be a collection of forces which produce a net moment, but no net force. This concept is most useful when the forces forming the couple are applied to points in such a way that it is difficult to discern the individual forces; thus only their net effect is of interest. We believe that failure to distinguish the concepts of force, moment due to force about a point, and couple is at the root of many students' errors in statics. The idea of a couple is approached by having students first recognize that there are motions which a single force cannot produce, no matter



Equilibrium of forces

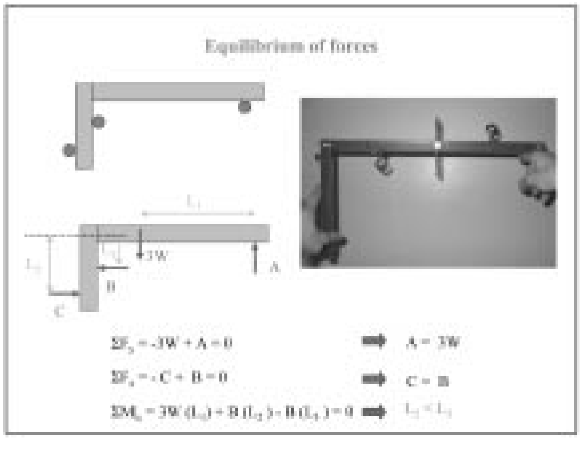
Consider supporting the member in the vertical plane with a rubber band.

$\Sigma M_o = 0$: $xR - h2W$
$\Sigma F_y = 0$: $R=3W$

$x=2/3\ h$ Pi

The band should be placed in line with the centroid of the L-shaped member in order to balance it.

Fig. 4. Module I—slide 7/13.



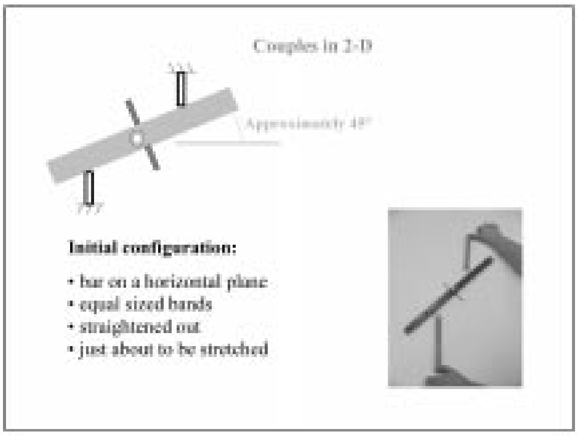
Equilibrium of forces

$\Sigma F_y = -3W + A = 0$ ➡ $A = 3W$

$\Sigma F_x = -C + B = 0$ ➡ $C = B$

$\Sigma M_o = 3W(L_o) + B(L_2) - B(L_3) = 0$ ➡ $L_2 < L_3$

Fig. 6. Module I—slide 12/13.

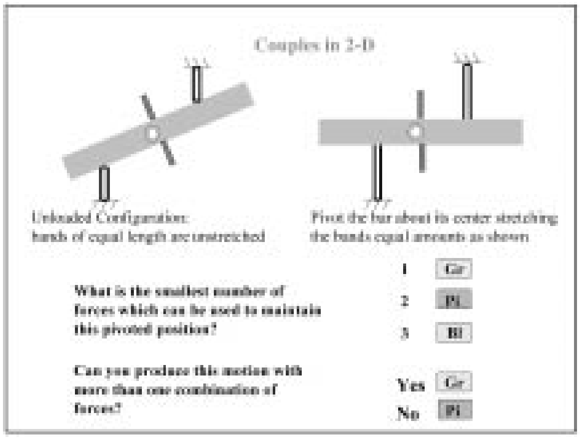


Couples in 2-D

Initial configuration:

- bar on a horizontal plane
- equal sized bands
- straightened out
- just about to be stretched

Fig. 7. Module II—slide 1/12.



Couples in 2-D

Unloaded Configuration:
bands of equal length are unstretched

Pivot the bar about its center stretching
the bands equal amounts as shown

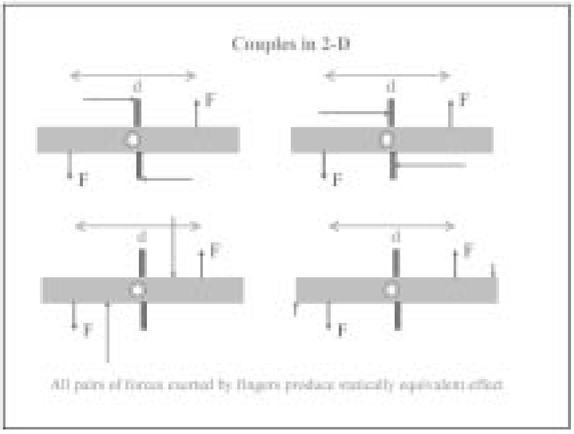
What is the smallest number of forces which can be used to maintain this pivoted position?

1 Gr
2 Pi
3 Bl

Can you produce this motion with more than one combination of forces?

Yes Gr
No Pi

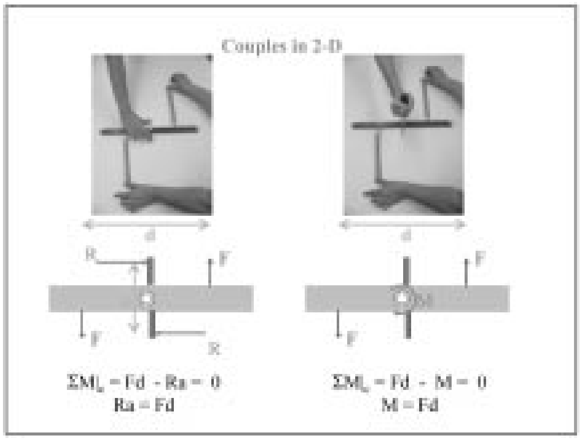
Fig. 8. Module II—slide 4/12.



Couples in 2-D

All pairs of forces exerted by fingers produce statically equivalent effect

Fig. 9. Module II—slide 8/12.

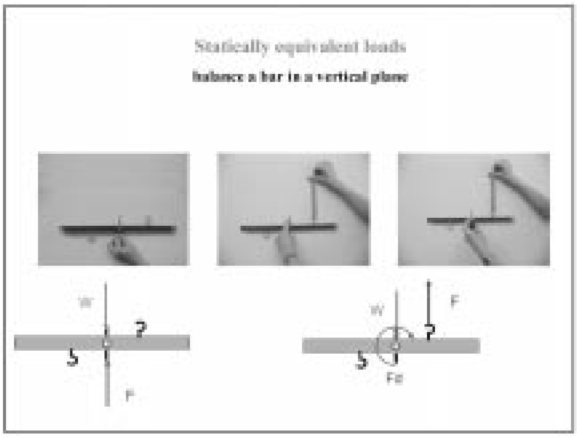


Couples in 2-D

$\Sigma M_c = Fd - Ra = 0$
$Ra = Fd$

$\Sigma M_c = Fd - M = 0$
$M = Fd$

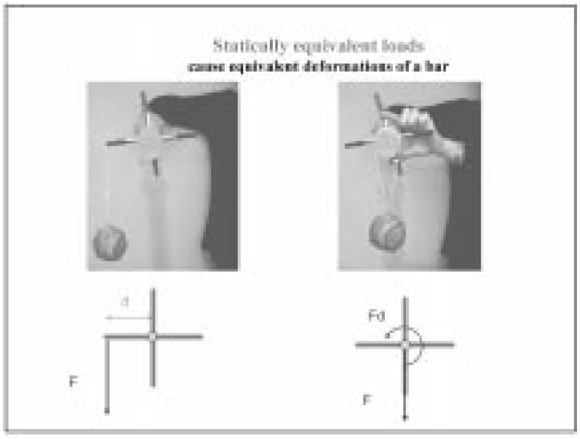
Fig. 10. Module II—slide 10/12.



Statically equivalent loads

balance a bar in a vertical plane

Fig. 11. Module III—slide 5/9.



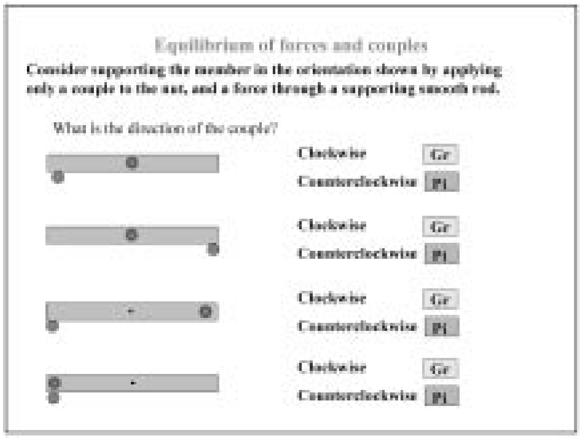
Statically equivalent loads

cause equivalent deformations of a bar

Fig. 12. Module III—slide 9/9.

where it is applied (Figs 7 and 8). Then, once a pair of forces is seen to be adequate, it also is seen that many pairs are equally capable (Fig. 9). This makes the physical meaning of static equivalence concrete. With that preparation, students then find that a nutdriver (or equally a screwdriver or other such implement) is also capable of producing the turning effect (Fig. 10). Students are open to the idea that the nutdriver is applying 'something' which creates a tendency to rotate, but no tendency to translate; namely, a couple. The convenience of this representation is thus more apparent.

Module III, entitled *Static Equivalence*, focuses students on the concept of static equivalence and seeks to highlight the distinctions between forces, moment of forces, and couples. We attempt to do this by taking advantage of various views of static equivalence: different combinations of forces that produce the same motion, different combinations



Equilibrium of forces and couples

Consider supporting the member in the orientation shown by applying only a couple to the nut, and a force through a supporting smooth rod.

What is the direction of the couple?

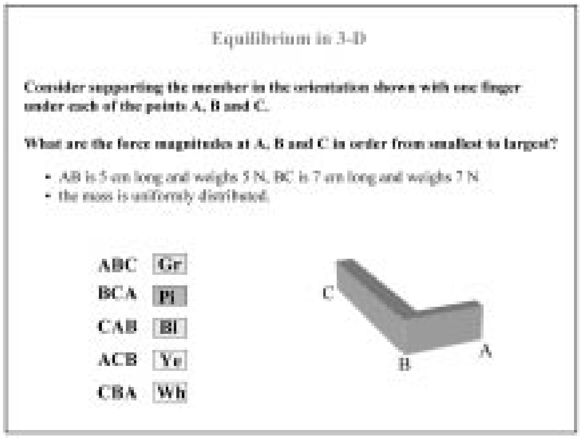
Clockwise Gr
Counterclockwise Pi

Clockwise Gr
Counterclockwise Pi

Clockwise Gr
Counterclockwise Pi

Clockwise Gr
Counterclockwise Pi

Fig. 13. Module IV—slide 2/6.



Equilibrium in 3-D

Consider supporting the member in the orientation shown with one finger under each of the points A, B and C.

What are the force magnitudes at A, B and C in order from smallest to largest?

- AB is 5 cm long and weighs 5 N, BC is 7 cm long and weighs 7 N
- the mass is uniformly distributed.

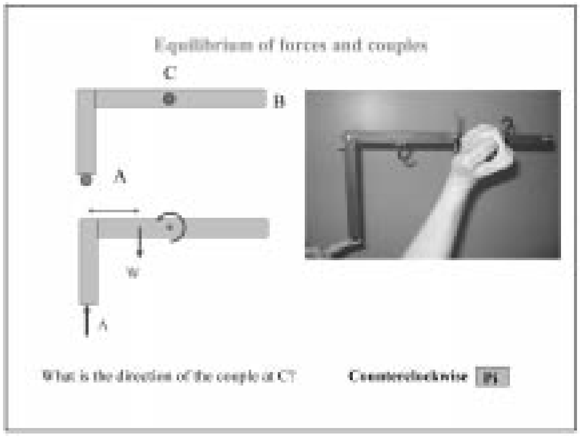
ABC Gr
BCA Pi
CAB Bl
ACB Ye
CBA Wh

Fig. 15. Module V—slide 2/35.

of forces that balance the same applied loading (Fig. 11), and different combinations of forces that produce the same deformation (Fig. 12). Notice that here, as well as in many other modules, we show the physical situation along with the representations in terms of vectors.

In Module IV, entitled *Equilibrium with Forces and Couples in 2D*, students contemplate planar situations where combinations of forces and couples are required for equilibrium. Students develop means of calculating the forces and couples necessary for balancing objects. At the same time, students develop an intuitive, qualitative sense for directions of forces and couples that provide equilibrium, which can also serve as a check on calculations (Figs 13 and 14).

Module V, entitled *Equilibrium with Forces and Couples in 3D*, focuses on the conditions of equilibrium in three dimensions, as seen in the context of the 'L'. The first notion is that balancing an object in such cases requires one to consider rotations (moments) about more than a single axis. Thus, moments due to forces in 3-D should already have been addressed or are addressed in real-time within this module. A second theme in the first part of the module is how many supporting forces are necessary to balance the object. With three forces clearly being sufficient (students have the 'L' to test this out), the relative values of the three forces is addressed (Fig. 15). Students consider both arriving at an answer through a superposition argument (Fig. 16), and systematically exploring the summation of forces and moments. Students are then asked whether a single force is sufficient to support the body (Fig. 17); in answering why it is not, students are led to the notion of center of gravity (Fig. 18). Students next consider bodies on which combinations of forces and couples act. Questions regarding which combinations of supporting forces and couple can produce equilibrium (Figs 19 and 20) led to considerable debate between students; typically the answer is only resolved with a combination of manipulating the 'L' and peer discussion. Subsequent slides reinforce what students have learned earlier in 2-D regarding how a couple contributes to the moment summation, but not to the force summation. There is additional complexity in recognizing that the couple contributes to the moment summation about one axis only.



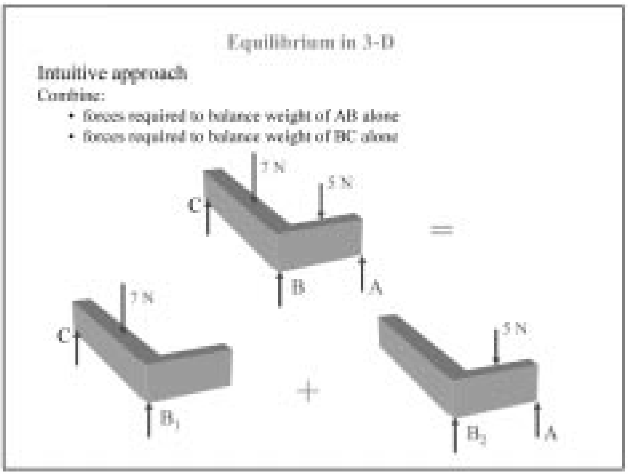
Equilibrium of forces and couples

What is the direction of the couple at C? **Counterclockwise** Pi

Fig. 14. Module IV—slide 5/6.

Equilibrium involving distributed contact forces, with and without friction

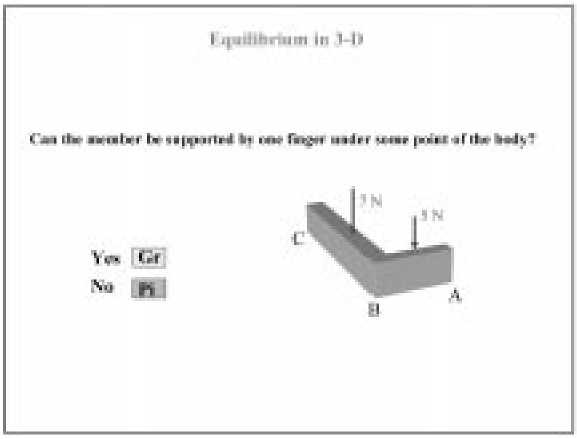
In many practical situations, the interaction between objects occurs not over a small region,



Equilibrium in 3-D

Intuitive approach
Combine:
- forces required to balance weight of AB alone
- forces required to balance weight of BC alone

Fig. 16. Module V—slide 4/35.



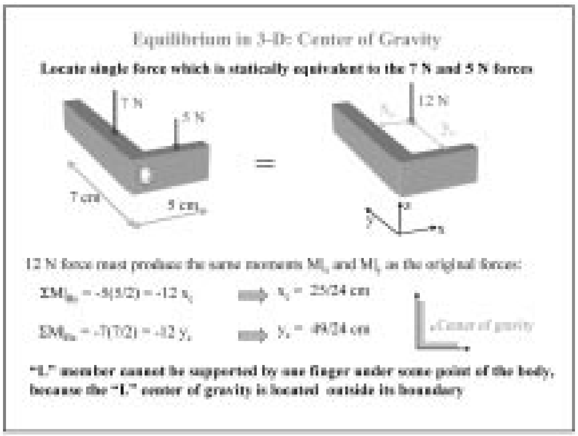
Equilibrium in 3-D

Can the member be supported by one finger under some point of the body?

Yes Gr

No Pi

Fig. 17. Module V—slide 14/35.



Equilibrium in 3-D: Center of Gravity

Locate single force which is statically equivalent to the 7 N and 5 N forces

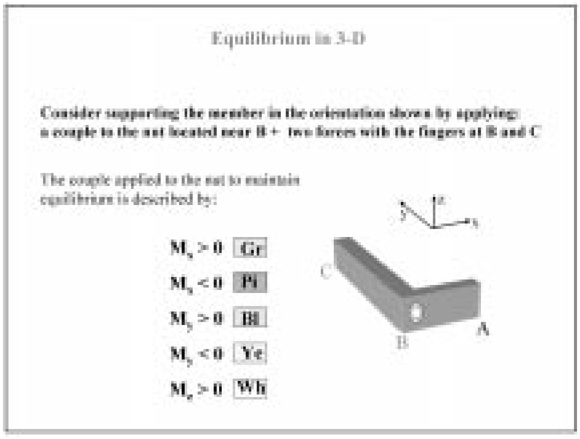
12 N force must produce the same moments M_x and M_y as the original forces:

$\Sigma M_x = -5(5/2) = -12\, x_c \Rightarrow x_c = 25/24$ cm

$\Sigma M_y = -7(7/2) = -12\, y_c \Rightarrow y_c = 49/24$ cm

"L" member cannot be supported by one finger under some point of the body, because the "L" center of gravity is located outside its boundary

Fig. 18. Module V—slide 15/35.



Equilibrium in 3-D

Consider supporting the member in the orientation shown by applying:
a couple to the nut located near B + two forces with the fingers at B and C

The couple applied to the nut to maintain equilibrium is described by:

$M_x > 0$ Gr

$M_x < 0$ Pi

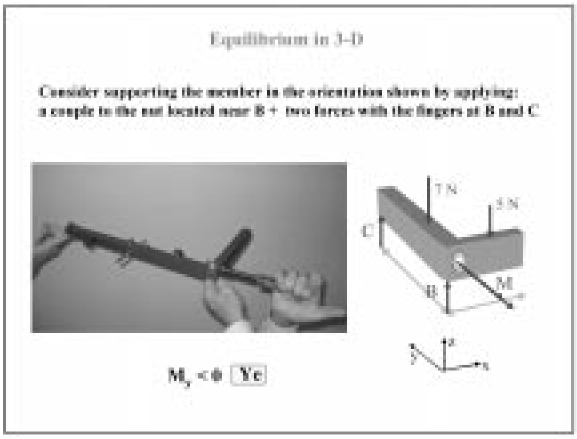
$M_y > 0$ Bl

$M_y < 0$ Ye

$M_z > 0$ Wh

Fig. 19. Module V—slide 24/35.

but over an extended surface. We approach this gradually, first treating situations in which the force is distributed uniformly over the area of contact and is, therefore, statically equivalent to a force acting at the midpoint of contact; later we consider nonuniformly distributed contact forces. In these modules students first focus on contacts between inanimate objects, although we continue to include, as appropriate, situations in which forces are perceived through the sense of touch or are evident by virtue of the deformations they cause.

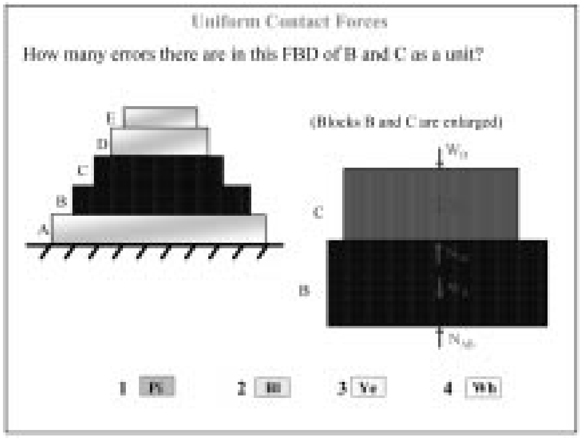


Equilibrium in 3-D

Consider supporting the member in the orientation shown by applying:
a couple to the nut located near B + two forces with the fingers at B and C

$M_y < 0$ Ye

Fig. 20. Module V—slide 25/35.



Uniform Contact Forces

How many errors there are in this FBD of B and C as a unit?

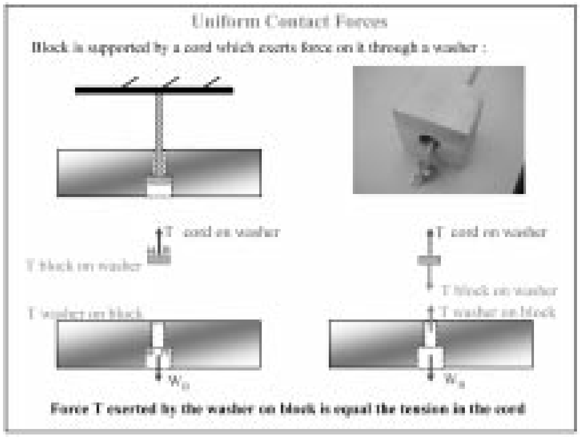
(Blocks B and C are enlarged)

1 Pi 2 Bl 3 Ye 4 Wh

Fig. 21. Module VI—slide 10/22.

Up to this point, contact forces, whether between hand and object or between inanimate objects, have acted exclusively in the normal direction; friction has been neglected. While friction forces are addressed at the end of a typical statics course, it is our contention that connections between bodies (e.g. pin joints), which are based on the neglect of friction, cannot truly be understood without appreciating what friction forces *can* exert. Therefore, in our statics courses we address friction before structures and machines, and we take full advantage of the ability of students to perceive, with the sense of touch, those forces which are exerted by hand.

Module VI, entitled *Equilibrium with Uniform Contact*, first addresses problems with blocks resting on each other; although these involve inanimate contacts, they are sufficiently familiar to students to be accessible. As can be seen in Fig. 21, we can delve into issues that are often subject to confusion. In particular, normal forces are often, without thought, equated to weight, and the force between two contacting bodies is sometimes incorrectly included in the free body diagram in which both bodies are included. With this module, students come to see that a free body

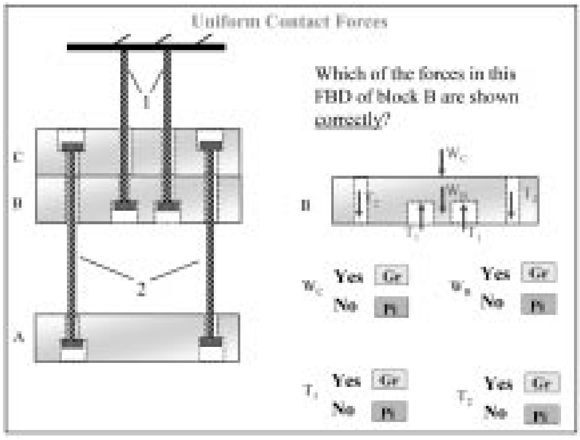


Uniform Contact Forces

Block is supported by a cord which exerts force on it through a washer :

Force T exerted by the washer on block is equal the tension in the cord

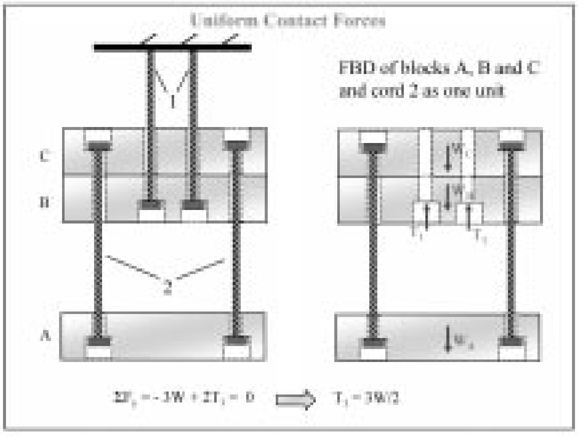
Fig. 22. Module VI—slide 13/22.



Uniform Contact Forces

Which of the forces in this FBD of block B are shown correctly?

Fig. 23. Module VI—slide 17/22.

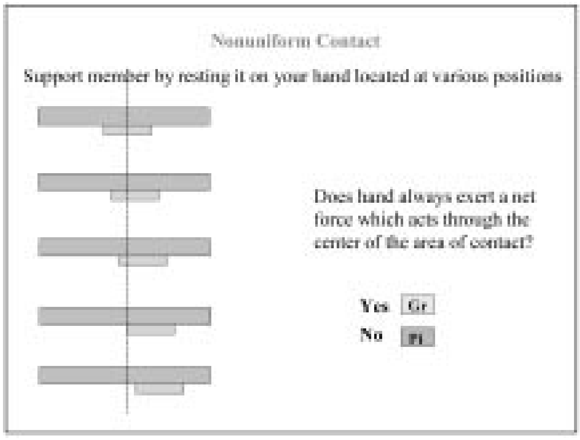


Uniform Contact Forces

FBD of blocks A, B and C and cord 2 as one unit

$\Sigma F_y = -3W + 2T_1 = 0 \Rightarrow T_1 = 3W/2$

Fig. 24. Module VI—slide 22/22.

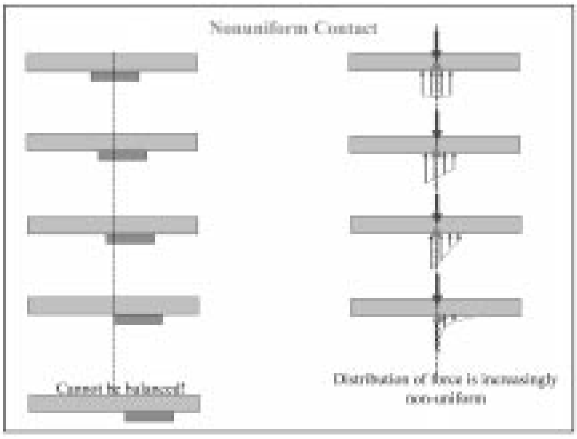


Nonuniform Contact

Support member by resting it on your hand located at various positions

Does hand always exert a net force which acts through the center of the area of contact?

Fig. 25. Module VII—slide 1/20.

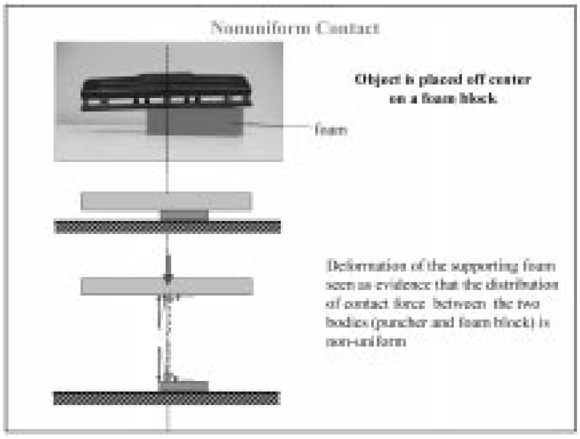


Nonuniform Contact

Cannot be balanced!

Distribution of force is increasingly non-uniform

Fig. 26. Module VII—slide 6/20.



Nonuniform Contact

Object is placed off center on a foam block

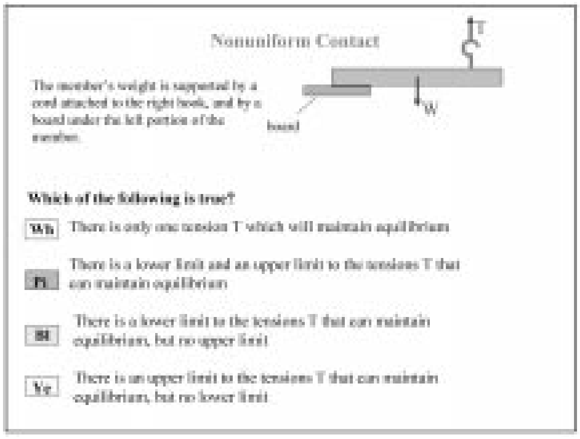
Deformation of the supporting foam seen as evidence that the distribution of contact force between the two bodies (puncher and foam block) is non-uniform

Fig. 27. Module VII—slide 5/20.

diagram can include a single part or several distinct parts.

An even richer set of problems can be considered by also including cords, another situation of inanimate objects exerting forces with which students are at least superficially familiar. Students are shown a realistic embodiment of the connection between cords and blocks, and acquainted with the detailed way in which the individual parts exert forces on one another (Fig. 22). Then, students are confronted with a variety of problems involving sets of parts, gaining experience with the ideas that the normal force between bodies is an unknown to be determined and that one can collect bodies in different ways to form free body diagrams (Figs 23 and 24).

Module VII, entitled *Equilibrium with Nonuniform Contact*, addresses situations in which bodies exert on one another force distributions which are not uniform. Thus, the presumption in the


Nonuniform Contact

The member's weight is supported by a cord attached to the right hook, and by a board under the left portion of the member.

Which of the following is true?

- **Wh** There is only one tension T which will maintain equilibrium
- **Pi** There is a lower limit and an upper limit to the tensions T that can maintain equilibrium
- **Bl** There is a lower limit to the tensions T that can maintain equilibrium, but no upper limit
- **Ye** There is an upper limit to the tensions T that can maintain equilibrium, but no lower limit

Fig. 28. Module VII—slide 8/20.

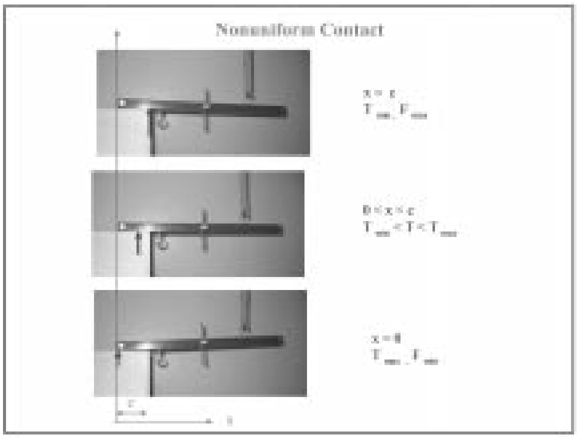
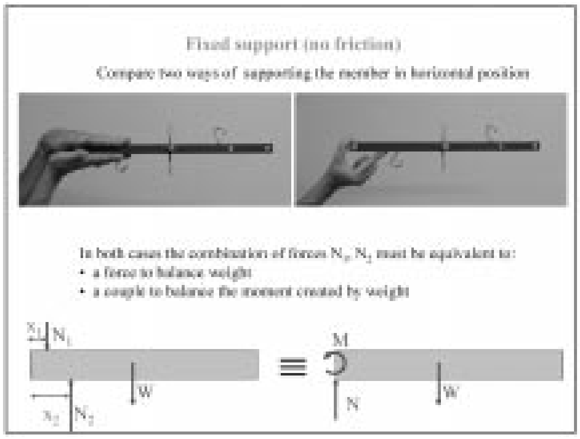

Nonuniform Contact

Fig. 29. Module VII—slide 9/20.

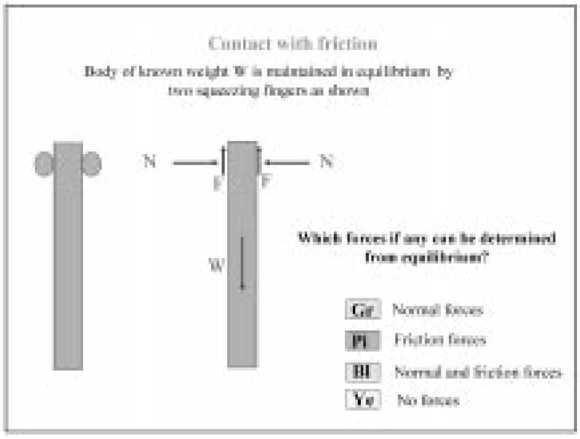

Fixed support (no friction)

Compare two ways of supporting the member in horizontal position

In both cases the combination of forces N_1, N_2 must be equivalent to:

- a force to balance weight
- a couple to balance the moment created by weight

Fig. 30. Module VII—slide 16/20.

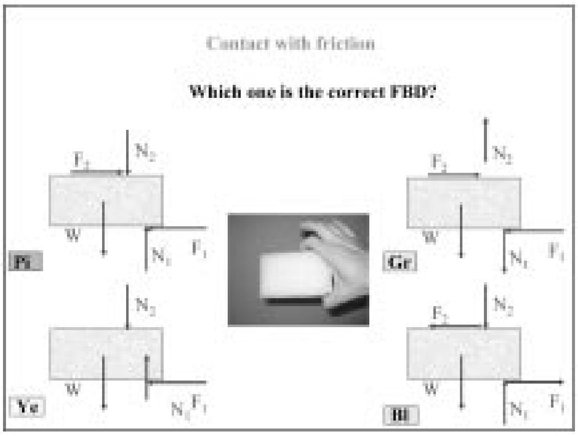

Contact with friction

Body of known weight W is maintained in equilibrium by two squeezing fingers as shown

Which forces if any can be determined from equilibrium?

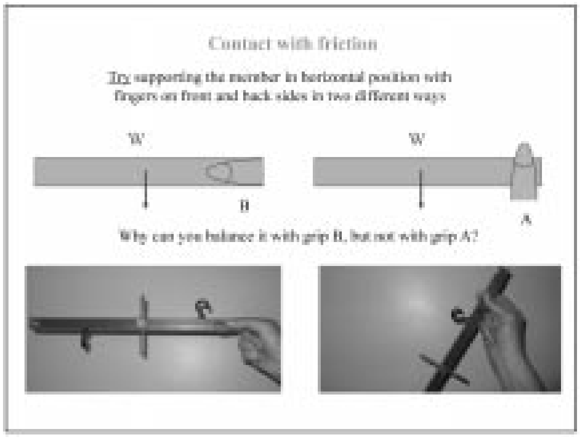
- **Gr** Normal forces
- **Pi** Friction forces
- **Bl** Normal and friction forces
- **Ye** No forces

Fig. 31. Module VIII—slide 2/34.


Contact with friction

Which one is the correct FBD?

Fig. 32. Module VIII—slide 5/34.


Contact with friction

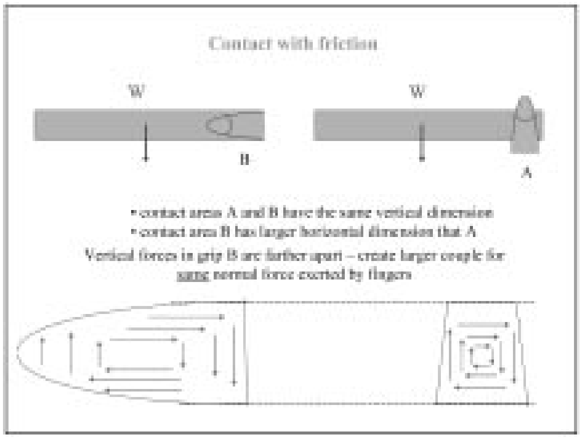
Try supporting the member in horizontal position with fingers on front and back sides in two different ways

Why can you balance it with grip B, but not with grip A?

Fig. 33. Module VIII—slide 13/34.

previous module that the net force acts through the center of the contact between bodies is revisited, first in the context of forces exerted by hand (Figs 25 and 26). After illustrating this non-uniformity with both equilibrium arguments and evidence of deformation (Fig. 27), students can contemplate situations in which the changing location of the net force is critical to the solution and can be determined from equilibrium (Figs 28 and 29). Now, students can also appreciate the basis for the net force and couple that are present in fixed or cantilevered support (Fig. 30).

In module VIII, entitled *Contact with Friction*, we introduce students to the effects of friction forces, first in the context of forces they can exert by hand. One important idea to convey (Fig. 31) is that the friction force between two bodies seeks to maintain equilibrium; the friction force is related to the normal force *only* if slip is impending or occurring (this is an idea that cannot be repeated



Contact with friction

- contact areas A and B have the same vertical dimension
- contact area B has larger horizontal dimension that A

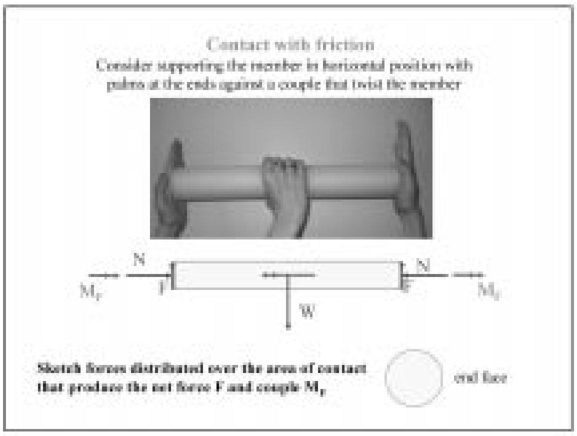
Vertical forces in grip B are farther apart – create larger couple for same normal force exerted by fingers

Fig. 34. Module VIII—slide 20/34.

Contact with friction

Ballerina wears point shoes while executing a pirouette
(to decrease frictional moment by making the area of contact smaller)

Fig. 35. Module VIII—slide 21/34.



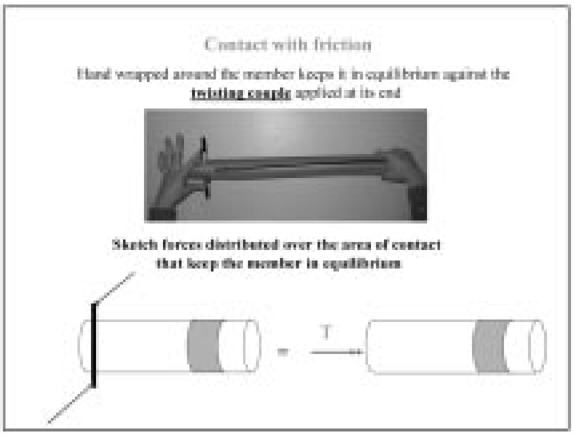
Contact with friction

Consider supporting the member in horizontal position with palms at the ends against a couple that twist the member

Sketch forces distributed over the area of contact that produce the net force F and couple M_y

Fig. 36. Module VIII—slide 9/34.

too often in statics). Next, we consider how friction forces applied at distant points can together create a moment that provides equilibrium (Fig. 32). When bodies contact one another over some area, then the collection of friction forces between them can create couples; the couple created depends on both the forces and on their distribution (Figs 33–37). Finally, such arguments are applied to the loads that a gripping hand can apply to a cylindrical object. By contrasting the

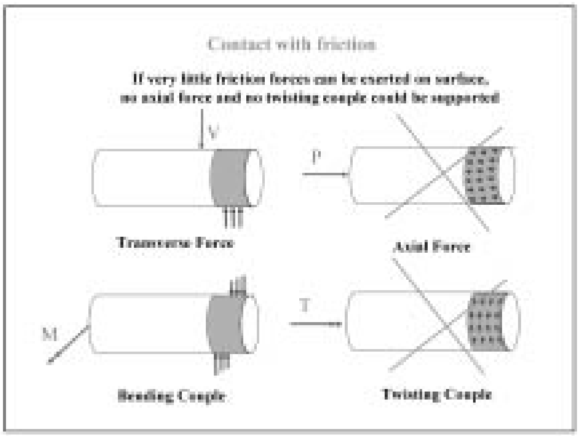


Contact with friction

Hand wrapped around the member keeps it in equilibrium against the **twisting couple** applied at its end

Sketch forces distributed over the area of contact that keep the member in equilibrium

Fig. 37. Module VIII—slide 26/34.



Contact with friction

If very little friction forces can be exerted on surface, no axial force and no twisting couple could be supported

Fig. 38. Module VIII—slide 34/34.

net forces and couples which require friction and those that are the results of only normal forces, the ground is prepared for the standard approximations of a pin joint (Fig. 38).

EXPERIENCE IN THE CLASSROOM AND IMPACT ON LEARNING

The materials described here have been implemented in our classrooms for the past two years. One module per week is used in class, on average, with modules taking from ten minutes up to, in rarer cases, nearly an entire lecture period. In the case of Steif's class, homework assignments have also been revamped to reflect the initial focus on situations in which forces are readily perceived by students. Thus, there are a number of problems involving balancing objects similar to those considered in class. Only after week 6 of the class (in a 14-week semester) do students start solving 'traditional' statics problems that might involve mechanical hardware and structures, with supports and connections.

It has been our sense that these materials have markedly changed our classes for the better. We have also sought to develop ways of quantifying the impact of our new approach. This is not a trivial task. This has been a complete revamping of the topic sequence, as well as the activities in class. The significant improvement we perceive is not one that could be attributed to a single learning episode; rather, it is due to the cumulative effects of confronting concepts one at a time, in the context of objects to which those concepts are relevant, and with students given constant opportunities to test their understanding and refine that understanding through interaction with peers and the instructor. Indeed, one could argue that the approach must produce improved learning outcomes, since it utilizes several techniques that are known to be effective in their own right.

Perhaps the only fair means of quantitative assessment might be to have one of the authors teach two sections of statics simultaneously, one with the new approach and one with the traditional approach. While one of the authors had an opportunity to do this, after having experienced the markedly heightened level of student engagement that author could not in good conscience teach half of the students in a manner that intuition strongly suggested was inferior.

One means of quantifying the improvement in conceptual understanding which the new approach produces is based on a Statics Concept Inventory, recently developed by one of the authors [10]. This is a multiple-choice test with questions testing key concepts in statics, including the forces in free body diagrams, static equivalence, forces at connections and at frictional contacts, and conditions of equilibrium. In fact, one motivation for developing this test was to have a tool for quantifying the effectiveness of various instructional interventions. Recent psychometric analysis of the current version of this test has shown it to be reliable and internally valid. This test was administered to students in the Department of Mechanical Engineering at CMU, both at the beginning and just after completion of the statics course. The mean and standard deviation for the pre-test was 10.6 and 4.1, and the mean and standard deviation for the post-test was 20.3 and 3.5; the maximum score is 27. Based on the 105 students who took both the pre- and post-tests, the probability that the difference between the pre- and post-tests is due to random variation is $p < 0.001$.

For a sense of the significance of the increase in conceptual understanding as indicated by scores on the inventory, we consider work done on the widely recognized Force Concept Inventory (FCI), which tests conceptual understanding in freshman level Newtonian mechanics. Hake [11] suggested using ‘normalized gain’ to gauge the improvement on the Force Concept Inventory. ‘Normalized gain’ is defined as the actual improvement in the class average scores from pre- to post-test normalized by the maximum possible improvement.

$$\mathit{Normalized\ Gain} = [(\mathit{Post\text{-}test}) - (\mathit{Pre\text{-}Test})] \div [(\mathit{Max\ Possible}) - (\mathit{Pre\text{-}test})]$$

Hake analyzed the results from over 6,000 students taking the FCI. He found that introductory physics courses taught with a ‘traditional’ teaching style had normalized gains of 0.23 ± 0.04, in spite of the wide range of student initial abilities. By contrast, the normalized gains associated with courses Hake designates as ‘interactive engagement’ courses are in the range 0.48 ± 0.14, a statistically significant difference. After examining and discarding a number of possible explanations for these results, Hake concluded that the teaching methods employed in the courses accounted for the difference.

For the students in the CMU class of Fall 2003, who participated in the instructional sequence and class activities described in this paper and its companion, the normalized gain on the Statics Concept Inventory was 0.59, solidly in the range which Hake designates as ‘interactive engagement’. Of course, to solidify this argument it would be helpful to have a study which indicates that this level of normalized gain on the Statics Concept Inventory is indeed a sign of considerable conceptual advancement.

To gauge the reactions of students to our new approach, we surveyed students, sometimes regarding the overall approach, and sometimes regarding individual modules. Results of surveys that were conducted in Fall 2003 are given in

Table 1. Results of student survey at Miami University. Responses 0 to 4 correspond to strongly disagree (0) and strongly agree (4)

	4	3	2	1	0	Average
The focus on forces and couples that you can exert (e.g. manipulating objects with fingers and nut drivers; maintaining your own equilibrium) help your understanding of the concepts of statics.	31	14	1	0	0	3.65
PowerPoint presentations help your understanding of the concepts of statics.	28	17	1	0	0	3.59
Asking of concept questions (questions you voted on with colored cards) in lecture helped your understanding.	27	16	3	0	0	3.52
Demonstrations (e.g. friction; moment of inertia) have been helpful.	32	12	2	0	0	3.65

Table 2. Results of student survey at Carnegie Mellon University. Students were asked how much each of the classroom elements contributed to their learning in statics. Responses correspond to: nothing (0); very little (1); modest amount (2); quite a bit (3); a lot (4)

	4	3	2	1	0	Average
Voting on questions with colored cards	18	28	33	15	4	2.42
PowerPoint presentations (aside from voting)	13	29	37	16	3	2.34
Manipulating objects in class	26	26	35	9	2	2.67

Tables 1 and 2. As can be seen, student reaction to this instructional approach has been very positive.

SUMMARY AND CONCLUSIONS

In this paper we have shown how a new instructional approach to statics is implemented. This approach, which was described at greater length in a companion paper [1], combines two core ideas. First, that there are distinct concepts which students need to learn in statics, and that students should confront each of these concepts initially in isolation. Second, as appreciated from the literature on physics education, students often have difficulty perceiving the forces between inanimate objects. Hence, we seek to isolate each of the core concepts of statics, and to address them within the context of forces which students can readily perceive: forces exerted by hand and forces that are made evident through the deformation or motion they induce. The implementation of these core ideas takes advantage of several widely accepted classroom techniques that promote active learning: peer-to-peer collaboration, integration of assessment and feedback into classroom activities, and the use of concrete physical manipulatives. Implementation is carried out through 'learning modules', which include objects to manipulate or examine, PowerPoint presentations and concept questions. The notion of learning modules is applicable to a wide range of engineering and scientific subjects.

Excerpts from the learning modules corresponding to a significant portion of our sequence of topics in statics are presented in this paper, along with explanations of the ideas developed in each portion of the modules. Since the instructional approach presented here is focused on improving students' overall understanding of concepts in statics, we have sought to judge the effectiveness of the approach by measuring student gains on a recently developed Statics Concept Inventory [10]. We have found that the normalized gain—the actual improvement relative to maximum possible improvement—was quite high, in the range that Hake [11], who studied results of the Force Concept Inventory, associated with active engagement courses. Between that quantification and the very positive response of students in surveys, we have concluded that our approach has merit and that its further development and assessment are worthwhile.

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Paul S. Steif received undergraduate and graduate degrees from Brown University and Harvard University in engineering mechanics. He is currently Professor of Mechanical Engineering at Carnegie Mellon University. He has been active as a teacher and researcher in the field of engineering mechanics, for which he has received a number of awards. Steif is currently involved in research to study student learning in basic engineering subjects, to measure student conceptual progress, and to construct educational materials that facilitate learning. Many of these developments have reached an international audience, including educational software which is published with widely selling textbooks.

Anna Dollár received her Ph.D. in applied mechanics from Krakow University of Technology in Krakow, Poland. From 1995 to 2000 she was an assistant professor at the Illinois Institute of Technology in Chicago. In 2000 she joined Miami University in Ohio as an associate professor. At IIT she received departmental and university Excellence in Teaching Awards. At Miami she received the School of Engineering and Applied Science Outstanding Teacher Award. Her research focuses on mechanics of solids and engineering education.