

# The CD Lens for Optical Engineering

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*Compact disks as a device for the teaching of optics have uses other than traditional diffraction experiments. Like a Fresnel zone plate that can focus light and form images of distant objects, the circular grating-like structure of a compact disk (CD) renders itself a chromatic lens. This paper suggests an interesting experimental activity that can be easily reproduced for early physics classes. The optical converging nature and chromatic characteristics of focal lengths of the CD lens are experimentally presented and theoretically interpreted.*

## INTRODUCTION

The use of interference-based optical devices is becoming a very basic and important part of the optics industry. Solid-state lasers, photonic band-gap devices, chip-scale spectrometers and fiber-optic communications components are just some examples of modern devices that use interference to solve specific technological needs. It is highly desirable that the engineering students can be provided with related information, ideas, and materials for improving and updating their background and their understanding of the fundamental optics, and that the optical engineers, as educators, can be assisted in their teaching and in their effectiveness in developing the talents of students.

Since about a decade after the invention of the first type of compact disks (CDs); i.e., compact disks—digital audio, numerous science instructors have devoted increasing attention to the use of a CD for classroom demonstrations or laboratory activities on diffraction patterns of a laser beam or optical spectroscopy of various light sources [1–14]. Treated as a reflecting diffraction grating of 625 lines/mm, a standard CD that has an angular resolution of 0.16 arc min for the first-order diffraction [7] provides a simple, as well as inexpensive, approach for effective displays of visible emission spectra. Among the above cited works, Lee and co-workers presented an interesting comparison between diffraction patterns of a normally or obliquely incident helium–neon laser beam obtained with a CD–read-only memory (CD–ROM), unused and data-written CD–recordable (CD–R), and a digital versatile disk (DVD–ROM) [3]. Recent articles by Gluck [5] and Knauer [7] have individually explored the transmission mode of CD gratings used in optical demonstra-

tions. The former suggested a distinct use of a transparent CD, enabling one to observe the wavelength-dependent focusing of sunlight [5]. Reporting his rough observation of exploiting the whole disk as a lens, Gluck pointed out that sharper focusing *may* be achieved by masking the disk in order to restrict the range of illuminated radii [5]. Inspired by Gluck, we carried out this project as a way of doing hands-on science during the winter break in the academic year 2002–2003. The activity entails both outdoor quantitative observations and data analyses. We found the experiment evolved to be not only educational but also intriguing because of its use of everyday materials. A brief report on this project in layman's language, focusing on the operational aspect with some preliminary results, has been published in a recent issue of *Physics Education* as a Frontline note [15]. In this relatively more in-depth paper, we provide more detailed technical information and more physical insights on our investigation of the chromatic properties of a transparent CD–R with a given annular opening of a mask. The averaged wavelengths of major colour bands of white light are deduced from the observed data of corresponding focal lengths and compared with the generally accepted spectral ranges of colours.

## THEORETICAL BACKGROUND

A grating, framework or latticework having an even arrangement of long narrow objects with interstices between them, can be used to disperse light by interference between wave trains from the interstices. Let us consider a diffraction grating of transmission type consisting of a large number of identical parallel fine grooves each separated by a constant distance  $d$ . If a light beam is incident to the grating, a multi-order diffraction pattern can

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be seen in the half-space behind the grating plane. Principal maxima of the diffracted beams in the far field originating from a normally incident beam of wavelength  $\lambda$  are found to occur at different angles  $\theta$  in accordance with the grating equation or formula [16]

$$d \sin \theta = n\lambda, \tag{1}$$

where the integer  $n$  specifies the order of diffraction. It is obvious that the angle of diffraction increases with the increase of the wavelength for a given order. Note that the relation between wavelength and angle of diffraction sets a constraint on the highest allowed order to be observed due to the requirement that  $\sin \theta = n\lambda/d \leq 1$ , indicating that the shorter the wavelength, the greater the possibility for higher-order diffraction to occur. If the grating spacing, also known as the grating constant,  $d = 1.6 \mu\text{m} = 1600 \text{ nm}$ , for instance, the diffracted red light up to the second order can take place, while diffraction up to the fourth order of violet light may be visible.

A standard CD has 20 625 turns of a single spiral of total length over 5 m, each track spaced by  $1.6 \mu\text{m}$  [17]. The grooves on the CD are, therefore, nearly circular, allowing one to regard the disk as a circular diffraction grating. Now consider an expanded and collimated light beam passing normally through a transparent CD whose surface is masked with a black, matt paper disk from which a concentric annular zone with an adequate width has been cut out to provide an entry for the light beam. The CD will bring the diffracted light of a given wavelength to several foci located at certain distances that are determined by the allowed orders of diffraction. See Fig. 1.

Based upon the grating formula, the first-order diffraction is governed by the equation

$$d \sin \theta = \lambda. \tag{2}$$

Because the penetrating light has a circularly symmetric distribution on the CD, the first-order

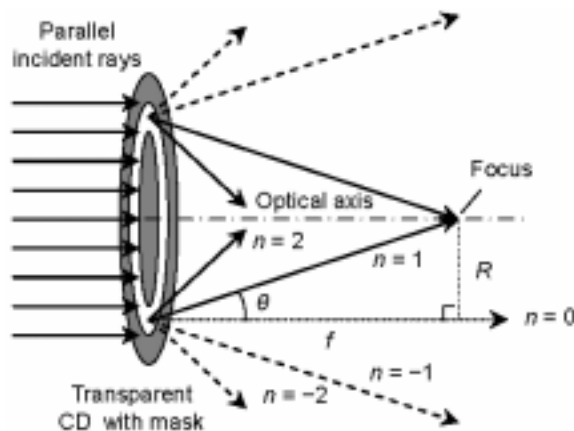


Fig. 1. Geometry for the derivation of the focal-length formula; i.e., Equation (3), used in locating images. This figure illustrates image formation of diffracted rays of a single component of normally incident white light.

diffracted rays of wavelength  $\lambda$  from the impinging light at radius  $R$  form a divergent frustum of conic shell and a right circular cone shell of radius  $R$  whose apex is located at a common point; i.e., the focal point, of superposition of diffracted light on the optical axis. The focal length  $f$  is given by

$$\lambda = dR(f^2 + R^2)^{-1/2} \tag{3}$$

or

$$f = [(d/\lambda)^2 - 1]^{1/2} R. \tag{4}$$

Equation (4) is the focal-length formula for a masked CD acting as a chromatic lens. The focal length varies linearly with the radius of the annular opening on the mask. One can also see that the focal length is dependent on wavelength, suggesting that the CD possesses obviously chromatic aberration. The CD that is acting as a lens actually has the property of forming a real inverted image of a point on the axis, as does a typical converging lens, but by a process of diffraction instead of refraction. It is also due to focusing by wavelength-dependent diffraction that the chromatic aberration of the CD lens is much more severe than a traditional lens made of optical glass. While the focal length of a typical glass lens is slightly shorter for light of a shorter wavelength, the focal length of the CD lens decreases as the wavelength is increased for a fixed radius in use. Because  $\lambda/d < 1$ , it is of interest to examine the binomial expansion of Equation (4). Some simple steps of manipulation lead to

$$f = R(d/\lambda)[1 - (\lambda/d)^2]^{1/2} = [d/\lambda - (\lambda/d)/2 - (\lambda/d)^3/8 - \dots]R, \tag{5}$$

indicating that the CD lens is even more chromatic than a Fresnel zone plate, which can focus light through diffraction and whose focal length is inversely proportional to the wavelength [16].

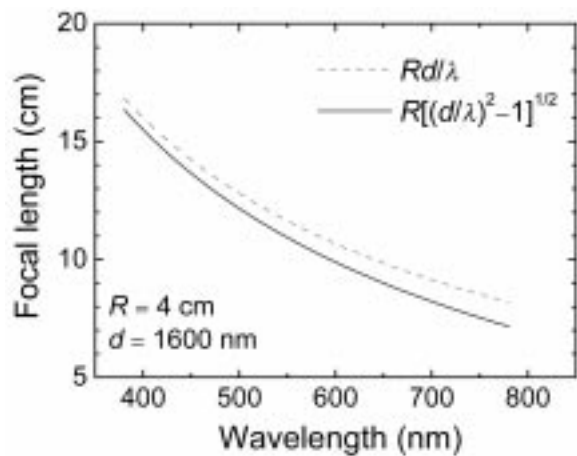


Fig. 2. Chromaticity of a CD lens from 380 to 780 nm. The solid curve shows the wavelength-dependent focal length given by Equation (4) and the dashed curve represents the approximation of order  $-1$  of the binomial expansion of the focal-length formula.

Figure 2 displays the theoretical curves of the focal length in accordance with Equation (4) and the first-term (namely, order  $-1$ ) approximation in the expansion based on Equation (5).

## EXPERIMENT

The primary items of which an efficient apparatus can be made for this project include a 25-cm-long hollow cylinder with a radius approximately equal to that of a CD; i.e., 6 cm, a ruler stick with a length of at least 20 cm, a white screen attached to a flattened loop that can slide along the ruler, and some transparent CD-Rs, each covered with a distinct, annularly opened black mask. Figure 3 depicts the complete apparatus we constructed and named CDscope. A black unglassy inner surface of the circular cylinder is preferred. We marked a cross at the optical axis, defined by the axis of the cylinder, on the upright screen that is movable inside the cylinder. Although transparent CDs can be obtained with some extra effort by removing the coatings on the regular CD-Rs, as described in the article by Byrne [2], we elected to use those 'protective' disks without the metalized layer available from bulk packs of CD-Rs [7]. Each masked CD ought to be easily mounted to and removed from the front end of the cylinder. When mounting a CD onto the cylinder, try to make sure that the center of the CD coincides with the optical axis. We simply used a tape to fix a CD temporarily on the cylinder. It is worth mentioning that, to facilitate an observation, a mechanism allowing this device to be aimed at the Sun is of help because the sunlight needs to enter in parallel along the axis of the cylinder in the experiment. We constructed the apparatus with a pair of hinges, allowing us to adjust the angle according to the altitude of the Sun (see Fig. 3). In addition, with the aid of some binder clips, we used a piece of thick black cloth to block the stray light when an experiment was being performed.

We collected raw data of focal lengths on the flat roof of a building on campus on four sunny days in February 2003. Gradually sliding the screen to

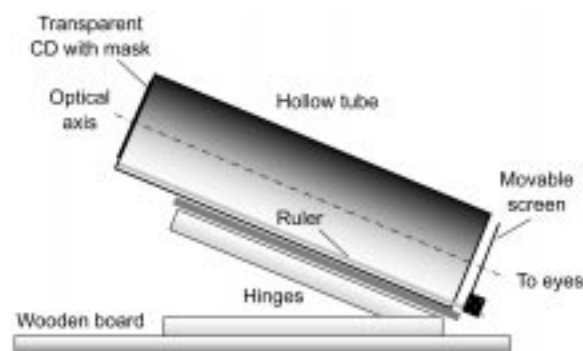


Fig. 3. Schematic of the apparatus, nicknamed CDscope, for observation of wavelength-dependent focusing of sunlight by a CD mounted on the front of the tube.

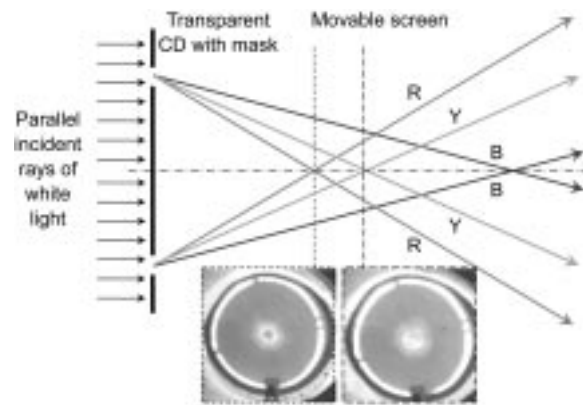


Fig. 4. Formation of foci of colour components. Distinct patterns are revealed at different positions of the movable screen.

alter its distance from the CD, observers very much enjoyed watching the splendid 'real-time' pattern. A typical view yields multiple rings of continuous spectrum of white light, as shown in Fig. 4. Expectedly from Fig. 4, various patterns with distinct orders of colour rings can be observed at different locations of the movable screen. The effect of the width of an annular mask opening,  $b$ , was examined. We tried masks with  $b = 0.1$ ,  $0.2$ , and  $1.0$  cm and decided to use those with  $b = 0.2$  cm as they gave the best images observed. The contrast of the colourful images depends on the total throughput of the incident light, which, in turn, depends on the total area of the annular window of the mask, while the sharpness of a focused spot depends on the narrowness of the width of the annular zone. Although it is desirable to employ a larger  $b$  in order to exploit its greater light-gathering power, this is, as a matter of fact, at the expense of the sharpness of the images formed by constructive interference of the diffracted rays. All observing runs for  $R = 2.0$ ,  $2.5$ ,  $3.0$ ,  $3.5$ ,  $4.0$ ,  $4.5$ , and  $5.0$  cm were performed between 10:00 a.m. and 2:00 p.m. With  $b$  fixed to be  $0.2$  cm, patterns were clearly seen to turn brighter for openings with greater radii owing to the intensive throughput of total optical power resulting from a mask with a greater opening area of  $2\pi Rb$ . The data used for analysis were acquired on the last day of the experiment to ensure the reliability after the prior three-day exercise.

## RESULTS AND DISCUSSION

The experimental data each taken as an average of four different measurements for a given experimental condition are displayed in the first row for each diameter ring in Table 1. For comparison, listed in the displaced rows are the accepted ranges of focal lengths for various colours of light and values of  $R$  according to Equation (4). This requires the wavelength boundaries of each light colour to be known. These, adapted from reference 18, are presented in the last column in Table 2. One

Table 1. Averaged values each taken from four distinct measurements (first row) in comparison with theoretically predicted ranges of focal lengths for various colour components from normally incident white light (second row). The subscripts R, O, Y, G, B, and V stand for red, orange, yellow, green, blue, and violet, respectively. Numerical data underlined are those beyond the predicted lower bound. All units are in cm

Radius of opening $R$	Focal lengths of different colours												
	$f_R$	$f_O$	$f_Y$	$f_G$	$f_B$	$f_V$	$f_R$	$f_O$	$f_Y$	$f_G$	$f_B$	$f_V$	
2.0	4.4	<u>4.5</u>	<u>4.8</u>	5.5	<u>6.0</u>	<u>6.5</u>	3.7	4.7	5.0	5.6	6.2	7.2	8.2
2.5	5.6	6.0	6.6	7.5	8.5	<u>8.9</u>	4.6	5.8	6.2	6.7	7.8	9.0	10.2
3.0	6.5	7.0	7.5	8.0	<u>9.0</u>	<u>10.1</u>	5.6	7.0	7.5	8.0	9.3	10.8	12.3
3.5	7.0	<u>7.7</u>	<u>8.5</u>	<u>8.7</u>	<u>9.6</u>	<u>11.2</u>	6.5	8.2	8.7	9.4	10.9	12.5	14.3
4.0	7.9	<u>8.9</u>	10.0	10.7	<u>11.5</u>	<u>14.0</u>	7.4	9.3	10.0	10.7	12.4	14.3	16.4
4.5	9.6	<u>10.2</u>	<u>10.5</u>	<u>11.9</u>	<u>13.0</u>	17.0	8.4	10.5	11.2	12.0	14.0	16.1	18.4
5.0	11.0	<u>11.5</u>	13.0	14.0	16.0	18.0	9.3	11.7	12.5	13.4	15.5	17.9	20.4

Table 2. Observed wavelengths  $\lambda_{obs}$  deduced from the  $f$ - $R$  plot

Color	Slope	Standard deviation	$\lambda_{obs}$ (nm)	Accepted range of wavelength [18]
Red	$2.120 \pm 0.038$	0.368	$683 \pm 10$	630–760
Orange	$2.274 \pm 0.021$	0.207	$644 \pm 5$	595–630
Yellow	$2.486 \pm 0.043$	0.412	$597 \pm 9$	560–595
Green	$2.701 \pm 0.051$	0.495	$555 \pm 9$	490–560
Blue	$3.001 \pm 0.077$	0.743	$506 \pm 12$	430–490
Violet	$3.528 \pm 0.077$	0.739	$436 \pm 9$	380–430

can see from Table 1 that the experimental data are generally consistent with the theoretical results. Among the 42 data, there are 20 falling behind the predictions and none over the predicted ranges. With the distributions of raw data of measurements taken into account, we attribute this result to the systematic error caused by either the device itself or the common operational process. During the experiment, the observers found that they seemed to have trouble discerning orange light from red and that blue and violet were not easily distinguishable as the rings shrank to little spots. These problems appear consequently in the table where the experimental data associated with the orange, blue, and violet light are not very ideal.

Figure 5 illustrates the relation between the observed focal length and the radius of the annular opening according to the experimental data given in Table 1. Each straight line in the figure, expected from Equation (4) for a constant wavelength, represents a least-squares fit to the experimental data. Note that Equation (4) implies the line to pass the origin. The relevant results of the fits, including the slopes, errors, correlation coefficients, and standard deviations, are presented in Table 2. By use of Equation (3), two of the authors (PCL and CKT) calculated the observed wavelength  $\lambda_{obs}$  for each light colour via averaging seven wavelength values derived from the focal lengths listed for each column in Table 1. Instead

of finding experimentally the average of wavelength of each observed colour by substitution into Equation (3) of the experimental data, the other author (WL) performed an independent approach toward the wavelengths. He simply took full advantage of Fig. 5, which yields the

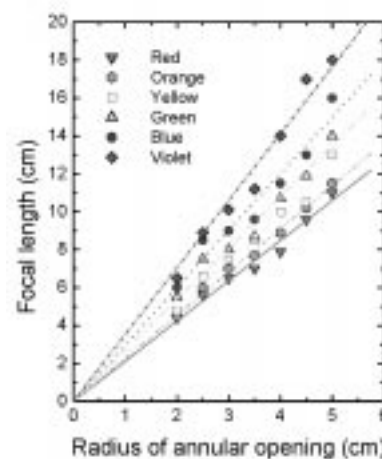


Fig. 5. Focal length,  $f$ , as a function of the radius  $R$  of the annular opening on a mask for various colour components of white light. Each line associated with a set of seven data points represents the results of a linear least-squares analysis with a coefficient of determination  $r^2 > 0.95$ . Observed wavelengths can be computed from the slopes in the diagram via Equation (4).

numerical values of slopes with correlation coefficients  $r > 0.974$ , as shown in the second column of Table 2. The results for  $\lambda_{\text{obs}}$  agreed to within their mutual errors. From the slope of each line,  $c$ , the weighted average of wavelength can be deduced using  $\lambda = d(c^2 + 1)^{-1/2} = 1600(c^2 + 1)^{-1/2}$  nm, given by Equation (4). The results are listed in the  $\lambda_{\text{obs}}$  column in Table 2. The obtained wavelength associated with orange yields a precision of 0.8% and any of the others a precision of  $\sim 2\%$ . For comparison, listed also are the accepted ranges of each major colour in the visible spectrum. They are found in the last column. It is obvious from Table 2 that the experimental values, within their uncertainties, are in agreement with the accepted results in general. The observed wavelengths, however, all seem to suffer an appreciably systematic red shift. Adopting the midpoint of each wavelength range as the accepted value, one can see that the estimated wavelengths are 3–10% higher than the accepted values except for the red one, which is 2% below. The calculated wavelengths for the orange and blue are apparently outside the accepted ranges and the deduced value for the orange differs by at least  $\sim 10$  nm in comparison with the upper limit of the wavelengths of orange light. It is worth mentioning again that all of the observers, when carrying out the experiment, encountered a common difficulty of picking the orange focus out accurately and that they all suffered from a difficulty in discerning blue and violet foci. Their similar procedures; i.e., moving the screen that was, at the beginning, just behind the CD further away to locate an image, may have added to the contribution of the apparent tendency of the shift.

We qualitatively observed second-order focused images at points closer to the disk. A primary obstacle, in addition to the images being fainter, existed that hindered us from taking the measurements effectively. This was due to the third-order images of shorter wavelengths merging with the

second-order images of longer wavelengths, as predicted theoretically by the grating formula.

The primary effort in the demonstration is to determine the wavelength dependence of the focal length. To emphasize the value of everyday-life materials in the activity, we elected to rely upon subjective judgment of the wavelength based on the perception of colour. Improvements can be made without changing any existing work or results. For example, one can make the use of several narrow-band optical filters that allow a small number of wavelengths to pass: use 'red,' 'green,' or 'blue' filters in front of the CD or screen to measure the location of the focal length.

## CONCLUSIONS

We have described an educational demonstration using readily available and inexpensive materials to illustrate how interference may be used to create a lens. We have also demonstrated that a CD acts as a chromatic lens whose focal length is very sensitive to the wavelength. In particular, assuming that  $\lambda_{\text{obs}} = \text{constant}$  for all the eyes judging the major colours, we detail a simplified analysis which produces an estimated value for each representative wavelength. There exhibits a fairly good agreement between the experimental results and the accepted values, although the experimental results seem to suffer a systematic error, giving rise to a bathochromic shift. Two sets of rings of continuous spectra of white light were qualitatively observed.

This project is certainly in coherence with the missions of hands-on science. We emphasize that the use of everyday materials and outdoor observation, such as those used in the present experiment, has the positive effect of capturing engineering students' interest. This demonstration could be used in its existing form or be easily modified for the classroom.

## REFERENCES

1. H. Brouwer, Line spectra using a CD disc, *Journal of Chemical Education*, **69** (1992), p. 829.
2. A. Byrne, Compact disk spectrometers revisited! *The Physics Teacher*, **41** (2003), pp. 144–145.
3. H.-Y. Chen, K.-X. Yang and W. Lee, Diffraction experiments with compact disks, *Chinese Physics Education* (Taipei), **5** (2001), pp. 46–51.
4. M. G. Cornwell, CD means colourful diffraction, *Physics Education*, **28** (1993), pp. 12–14.
5. P. Gluck, Compact disk optics, *The Physics Teacher*, **40** (2002), pp. 468–469.
6. J. E. Kettler, The compact disk as a diffraction grating, *American Journal of Physics*, **59** (1991), pp. 367–368.
7. T. Knauer, A compact disk transmission spectroscope, *The Physics Teacher*, **40** (2002), pp. 466–467.
8. H. Kruglak, The compact disc as a diffraction grating, *Physics Education*, **26** (1991), pp. 255–256.
9. H. Kruglak, Diffraction demonstration with a compact disc, *The Physics Teacher*, **31** (1993), p. 104.
10. R. C. Mebane and T. R. Rybolt, Atomic spectroscopy with a compact disc, *Journal of Chemical Education*, **69** (1992), p. 401.
11. C. Nöldeke, Compact disc diffraction, *The Physics Teacher*, **28** (1990), pp. 484–485.
12. J. Tellinghuisen, Exploring the diffraction grating using a He–Ne laser and a CD-ROM, *Journal of Chemical Education*, **79** (2002), pp. 703–704.
13. F. Wakabayashi, K. Hamada and K. Sone, CD-ROM spectroscope: A simple and inexpensive tool for classroom demonstrations on chemical spectroscopy, *Journal of Chemical Education*, **75** (1998), pp. 1569–1570.

14. V. Zanetti and J. Harris, Spectra of three light sources with a CD, *The Physics Teacher*, **31** (1993), pp. 82–83.
15. W. Lee, P.-C. Lin and C.-K. Tseng, Compact disc doubles as a chromatic lens, *Physics Education*, **39** (2004), pp. 384–386.
16. E. Hecht, *Optics*, 4th edn, Addison Wesley, New York (2002), pp. 466 and 488.
17. J. A. Cope, The physics of the compact disc, *Physics Education*, **28** (1993), pp. 15–21.
18. G. J. Shugar and J. T. Ballinger, *Chemical Technicians' Ready Reference Handbook*, 3rd edn, McGraw-Hill, New York (1990), p. 715.

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