

Linking Undergraduate Mathematics Education with Engineering*

MARK H. HOLMES

Department of Mathematical Sciences, Rensselaer Polytechnic Institute, Troy, NY, USA

E-mail: holmes@rpi.edu

ROBERT L. SPILKER

Department of Biomedical Engineering, Jonsson Engineering Center 7032, Rensselaer Polytechnic Institute, Troy, NY, USA

To help students grasp the intimate connections that exist between mathematics and its applications in engineering, a library of interactive learning modules was developed. This library covers the mathematical areas normally studied by undergraduate students and is used in engineering courses at all levels. Moreover, the library is designed not just to provide critical connections across disciplines but also to provide longitudinal subject reinforcement as students progress in their studies. In the process of developing the modules, a complete editing and publishing system was constructed that is optimized for automated maintenance and upgradeability of materials. The result is a single integrated production system for web-based educational materials. Included in this is a rigorous assessment program, involving both internal and external evaluations of each module. As will be seen, the formative evaluation obtained during the development of the library resulted in the modules successfully bridging multiple disciplines and breaking down the disciplinary barriers commonly found in their math and engineering courses.

Keywords: engineering mathematics; mathematics library learning modules; disciplinary barriers

INTRODUCTION

THE ABILITY OF undergraduate engineering students to succeed is fundamentally dependent on their understanding of basic mathematical tools and concepts. In conjunction with this there is a need for better integrating the mathematical sciences into engineering and improving instruction in mathematics through incorporation of disciplinary perspectives that arise in engineering. Unfortunately, typical institutional separation of courses often makes it difficult for students to grasp the intimate connections that exist between mathematics and its applications in other disciplines. To help make these connections clearer a library of interactive, web-based learning modules linking important mathematical topics with contemporary applications in various engineering fields has been developed. This library covers the mathematical areas normally studied by undergraduate students, including calculus, linear systems, and probability and statistics. The same modules are also used in many engineering courses at all levels, from the freshman to senior year. Linking subjects in this way has provided critical connections across disciplines and has also provided longitudinal subject reinforcement as students progress in their studies.

The 40+ modules developed to date are part of Project Links, an NSF-supported undertaking based at Rensselaer Polytechnic Institute, with collaboration from the University of Delaware, Hudson Valley Community College, Siena College, and Virginia Polytechnic Institute. To date, approximately 70 faculty have actively participated in the program (a complete listing, ordered by department, can be found on the Links website [1]). Included in this effort has been an innovative and comprehensive evaluation program to assess the viability and effectiveness of the modules. For this we collaborated with the Evaluation Consortium from the University of Albany School of Education.

The paper begins with an overview of the project, including a discussion of its objectives and organization. After this the process used to develop and test the modules will be described. This will include the evolution of the learning outcomes in a module as well as the technical requirements needed to construct an effective technological learning tool. After this, the evaluation outcomes will be presented along with conclusions on what transpired.

OVERVIEW OF PROJECT

Project Links was conceived to tie crucial topics in mathematics with one or more contemporary

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applications in engineering and science. The four main objectives of Project Links are:

1. To stimulate greater cooperation in educational development across traditional disciplinary boundaries.
2. To encourage interactive teaching and learning strategies and to produce instructional materials for use in workshop or studio-type courses.
3. To create a library of interactive learning materials that link topics in mathematics with applications in engineering and science.
4. To continue our efforts in the application of contemporary technology for educational purposes and to encourage the widespread distribution of the results.

The central component of this effort was the production of instructional web-based modules that exploit the Internet and its attendant technologies, including the Java programming language. For example, in regard to item (2), the modules were designed to be used in a studio classroom, with an instructor present, with significant student-to-student interaction, and with many open-ended challenges included (a listing of the modules is given in Fig. 1). At the same time, the modules were not developed to replace textbooks, professors, or entire courses. Rather, they were designed to give the instructor the flexibility to emphasize certain well-contained topics that are a one- to three-day part of a regular course.

The library of modules, and how they are used, reinforces many current ideas on how to successfully integrate technology into the learning environment. According to the National Research Council Committee on Developments in the Science of Learning, there are five ways that technology can be used to establish an effective learning environment: by using real-world problems, by providing scaffolding support, by increased feedback, by building communities of learners, and by expanding opportunities for teacher learning [2]. As will be seen later, all of these are central components of the project. Similarly, the project closely reflects the criterion measures identified in the National Science Education Standards [3]. For example, Science Education Program Standard C states: "The science program should be coordinated with the mathematics program to enhance student use and understanding of mathematics in the study of science and to improve student understanding of mathematics." This is one of the core objectives of the project. A detailed comparison of how Project Links aligns with the standards set by this group, the National Science Education Standards for Teaching, the NCTM Content and Process Standards, and the ISTE Standards for Students can be found at the project website [1].

To successfully integrate faculty from multiple disciplines to produce effective technologically enhanced learning tools requires careful organization and planning. Accordingly, the develop-

ment and testing of each module involved the close interaction of three teams: the faculty team (who were responsible for the content), the technical support team (who were responsible for template development and interface design), and the evaluation team (who were responsible for usability tests and external reviews).

One reason for separating the faculty from the technical component of module development is that the creation of educationally oriented content presents new challenges and opportunities. We wanted the faculty, at least in the beginning, not to be restricted by the current state of the technology. The use of interactive learning materials, and the resources available electronically, involves the development and analysis of new learning techniques and strategies. Simply converting textbook material to a web format is insufficient and the role of multimedia objects makes the potential for student interactivity much greater. In a similar vein, we wanted the technical team to push the capabilities of the software and hardware to see what tools and functionality they could develop. As the project matured, these two groups became more integrated and this occurred as the formative evaluation became available and the technical components became standardized.

An issue that arose immediately between the faculty and technical teams was what functionally was necessary and what was actually possible. An early example of this was the expectation, and strong desire, by the faculty to be able to manipulate three-dimensional objects in the Java applets. The fact was, however, that at the time this was not possible in Java, although it was expected it would be in the near future. Accounting for the developing nature of the technology therefore began to be integrated into the design of the modules. Another issue that arose concerned disciplinary differences related to emphasis and presentation of the content. An example here was the desire by the physicists to use scalar notation while the mechanical and electrical engineers wanted vectors. For these reasons, we developed a framework for presenting technical course content that is useful to multiple disciplines. This framework includes learning modules plus an index to shared key concepts. We also built customizable tools that promote a high level of student interactivity, plus authoring tools enabling content creation and maintenance in the rapidly evolving technological environment.

In order to assist in the creation of inherently interactive documents, a complete editing and publishing system was developed by Project Links. This included the design of a consistent interface where the common elements include graphical components, a navigation tool, as well as a general look and feel that remains throughout all the materials. This allowed content developers to focus on the production of materials and how to use them effectively in the classroom. Moreover, our development environment encourages the

TOPIC	Indicator	MODULE
Mechanical Oscillations	● ○ ○ ○ ○ ○	Vibrating Strings
	● ○ ○ ○ ○ ○	Forced Spring Mass
	● ○ ○ ○ ○ ○	Spring Mass
	○ ○ ● ○ ○ ○	Fourier Series
	○ ○ ● ○ ○ ○	Linear Pendulum
	○ ○ ● ○ ○ ○	Spring Pendulum
	○ ○ ● ○ ○ ○	Nonlinear Pendulum
	○ ○ ○ ○ ○ ○	Multiple Spring Mass System
Probability and Statistics	○ ○ ● ○ ○ ○	Means and Variances
	○ ○ ○ ○ ○ ○	Continuous Random Variables
	○ ○ ○ ○ ○ ○	Inventory Control
	○ ○ ○ ● ○ ○	Conditional Probability
	○ ○ ○ ● ○ ○	Poisson and Exponential Distributions
	○ ○ ○ ○ ○ ●	Random Variable Relations
Electricity and Magnetism	● ○ ○ ○ ○ ○	Electric Potential
	○ ○ ○ ○ ○ ○	Electric Field
	○ ○ ○ ○ ○ ○	Gauss's Law
	○ ○ ○ ● ○ ○	Faraday's Law and Induction
	○ ○ ○ ● ○ ○	Resistance & Capacitance
	○ ○ ○ ● ○ ○	Electromagnetic Oscillations
○ ○ ○ ○ ○ ●	Ampere's Law	
Transport Phenomena	● ○ ○ ○ ○ ○	Drag Forces on Solid Objects
	● ○ ○ ○ ○ ○	Mass Transport
	● ○ ○ ○ ○ ○	The Gradient
	○ ○ ○ ● ○ ○	Lake Pollution
	○ ○ ○ ● ○ ○	Sequential Batch Reactions
	○ ○ ○ ○ ○ ○	Heat Conduction I
	○ ○ ○ ● ○ ○	Heat Conduction II
	○ ○ ○ ○ ○ ●	Chemical Kinetics and Equilibria
	○ ○ ○ ○ ○ ●	Continuously Stirred Tank Reactor
	○ ○ ○ ○ ○ ●	Flux and Surface Integration
Linear Systems	● ○ ○ ○ ○ ○	Bicycle
	○ ○ ● ○ ○ ○	Matrix Kit
	○ ○ ○ ● ○ ○	Crystallography
System Design	● ○ ○ ○ ○ ○	Constrained Optimization
	○ ○ ○ ● ○ ○	Curvature and Curve Design
	○ ○ ○ ○ ○ ●	Boundary Value Problems for ODEs
Graph Theory	○ ○ ● ○ ○ ○	Industrial Drilling
	○ ○ ● ○ ○ ○	Networking
	○ ○ ● ○ ○ ○	Sperner's Lemma
	○ ○ ○ ○ ○ ○	Graph Isomorphism

Fig. 1. Listing of modules, from the Project Links web-page, by general engineering topic. There is also a listing by general math topic on the website.

re-use of pre-built interactive objects, hence minimizing the content developers' need for technical expertise. At the same time, our environment is optimized for automated maintenance and upgradability of materials. The result is a single integrated system. The development of this software environment will be discussed below and a focus on the use of this environment in the classroom will also be presented.

MODULE DESIGN AND OBJECTIVES

The modules are designed for small-group student interactions with an instructor nearby. They are a self-contained conceptual unit, intended for use over one to three days in the normal course of the term. They rely heavily on hypertext construction, animations, and interactive Java applets. However, a complete module also includes an Instructor's Manual, which presents the design intent of the module, recommendations for use in the classroom, and handouts designed to accompany the interactive module. We have found that the handouts, which are mostly worksheets based on the module subject matter, are a critical component of the learning process in the classroom. There are several reasons for this. One is the multi-modality achieved when combining writing with the interactive and collaborative nature of the module [4]. A second reason involves the properties of a collaborating group, which includes the topic discussed, the insights that emerge, and the

collaboratively determined framing of the module's outcomes [5]. Together these are opportunistically emerging processes and the worksheets provide a scaffolding to help the group access and then master the material.

Many questions and examples in each module are purposely left open-ended to encourage communication and self-discovery. They are also designed to encourage students to think creatively in how they approach problem-solving and how the concepts developed are transferable to related situations. This is done, in part, by providing multiple contexts for learning the underlying concepts, and having the examples that are used in the modules based on real-world situations. To achieve this, the developers incorporated actual experimental results, demonstrations, or design problems. The modules use videos, real-time experiments run over the web, animations of experimental results, and data-reduction.

A typical example illustrating the above ideas can be found in the spring mass module, where students study the dynamic behavior of a weight at the end of a spring. This module is used by multiple departments, including math (in sophomore differential equations), physics (in mechanics), and mechanical engineering (in mechatronics). A schematic of the system is shown in Fig. 2 along with the experimental apparatus used to generate data for this problem. The module is capable of acquiring and using real-time data and this is typically done by the students in the engineering courses. In math it is usually the instructor who

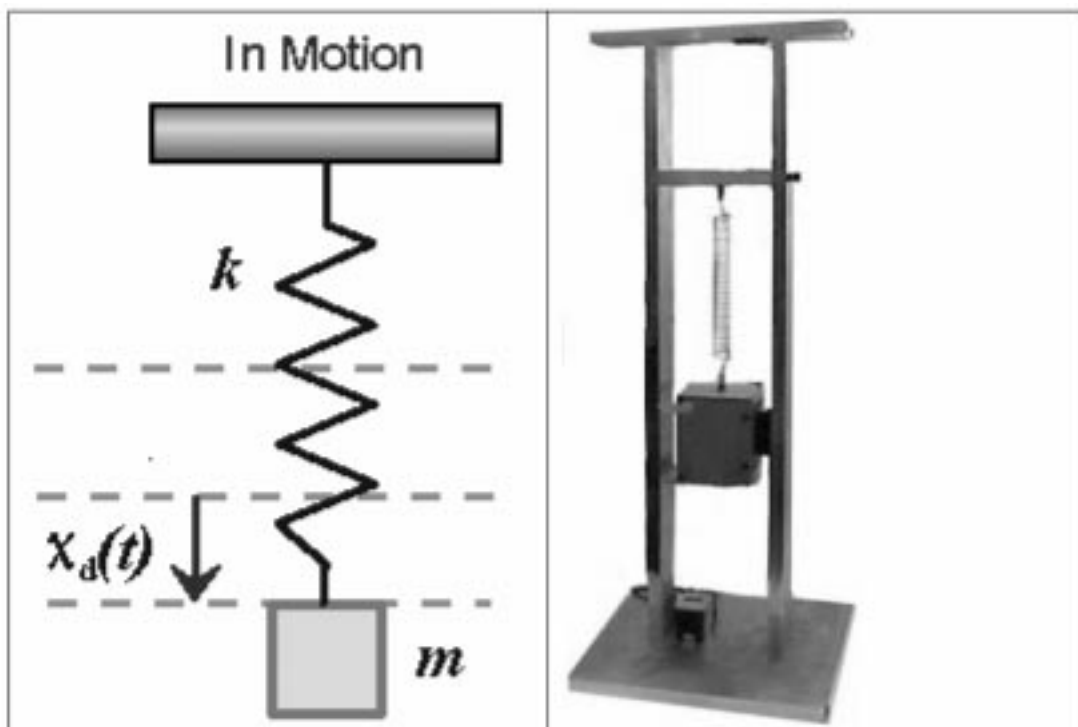


Fig. 2. Problem investigated in the spring mass module. On the left is the ideal lumped parameter model of the spring mass system shown on the right.

carries out the experiment, although we also have pre-recorded videos and data sets built into the module for those who prefer to concentrate on the analytical aspects of the module. After analyzing the mathematical and physical problems for this system, students are then given the images and questions shown in Fig. 3. Each picture is a link to a video showing common objects exhibiting the dynamic behavior of a spring mass system. Based

on what they have learned in the module, they are asked the following:

- Which of these examples are forced spring mass systems?
- Which show significant damping characteristics?
- When modeling these systems, what assumptions must be made to simplify while maintaining essential characteristics in the model?

	<p>A cyclical force is being applied to the fork of the bicycle. Is there a spring, a damper, or both inside?</p>
	<p>All the branches of this tree are moving with their own directions and amplitudes. Can the tree be modeled as a single spring or should it be considered a system?</p>
	<p>This office chair stays upright until you push on it. How much force do you need to apply to lean the chair back?</p>
	<p>A car's suspension uses a spring and a damper together. What kind of damping does the suspension provide: coulomb, viscous, square law, or structural?</p>
	<p>The dial on this spring scale is linear, but is the spring? How accurate is the scale at recording relatively high or low weights?</p>

Fig. 3. Common objects demonstrating spring mass behavior.

These questions are open-ended so they can serve as collaboration and classroom discussion points. More important, however, is that they are an integral component of the learning process as they are reconsidered at multiple places within the module. The reason is that it is essential that the students be able to transfer the fundamental math and physical concepts developed in the module to non-classroom situations and these questions are intended to address this point.

MODULE LAYOUT

A page from the spring mass module is shown in Fig. 4 to illustrate the content and functional navigational schemes. In the upper left-hand corner are the PRIOR/NEXT arrow buttons. This is the path through the module the authors recommend in normal use. The pages used in this path are explained in the materials made available to the instructor. Along the left side of the browser window is the content navigation bar (Objectives, Introduction . . .). This is a clickable list of the main module topics and sub-topics. Each of these may also be reached when using the PRIOR/NEXT arrows, but this list allows the student to jump around, as one would skip through parts of a textbook. A triangular icon appears next to the current topic shown in the frame.

Along the top of the browser window in Fig. 4 is the functional navigation bar, which is used to

branch off of the main topic areas. There are five choices available to the developer during design of the module. They are Concepts, Discover, Applications, Collaboration, and Practice. Any or all may be used from any one page. Again, a small triangular icon appears next to the current branch. Concepts are main topics, usually those that appear on the side navigation bar. Discover pages lead to questions or exercises that allow the student to explore a new area with information acquired from the Concepts pages. Applications are current uses of the topic in real-world situations. Collaboration supplies challenges that must be solved with a partner or by discussion between groups, perhaps with instructor guidance. Practice contains problems that the student must answer to allow the instructor to assess the learning that has taken place. These can have the form of pencil-and-paper worksheet problems, applets, or online submissions.

Towards the bottom left corner of the browser window in Fig. 4 is a listing of reference or help pages. Four pages are available to the student at all times from anywhere in the module:

- Crib-sheet—a summary of important concepts and formulas used in the module
- Library—a list of the multimedia elements and major concept pages included in the module and a link to the site-wide glossary
- Help—help files for students, instructors, installation and technical tips, and known issues and incompatibilities

The screenshot shows a web browser window displaying the 'Spring Mass Module' page. On the left, there is a sidebar with a 'Links' section containing 'Spring Mass' and 'Tracks'. The 'Tracks' section lists various topics: Objectives, Introduction, Real World (Physical System, Actual Dynamic Behavior), Modeling (Physical Model, Math Model), Math Analysis (Analytical Solution, Numerical Solution), Predicted Behavior, Comparisons, and Summary. Below these are buttons for 'CRIB SHEET', 'LIBRARY', 'HELP', 'MAP', 'MODULE HOME', and 'LINKS HOME'. The main content area is titled 'SPRING MASS MODULE' and contains introductory text, a photograph of a bungee jumper, and several paragraphs of text explaining the spring mass system and its application in modeling and real-world examples like bungee jumping.

Fig 4. Typical module page illustrating the layout and navigational systems used.

- Map—a conceptual map of the module material and/or a site map (we employ the Hyperbolic Tree [6], which allows for dynamic links to anywhere in the module)

Furthermore, the front page of any module can always be reached by clicking Module Home at the bottom of the side navigational bar.

As stated earlier, module development is the responsibility of three teams, which work closely together: the content experts, the technical group, and the external evaluators. The content experts are two to four faculty, at least one from math and one from outside math. The technical group, which includes HTML and Java programmers as well as an interface design expert, is responsible for implementing the content and maintaining the website. The evaluators are responsible for carrying out the various external assessment tests that are described later in this paper. There is also a small select committee consisting of members from all three groups that oversees what modules have in common, which includes the interface design, navigation capability and the functionality available in the modules.

CONTENT EXPERTS

Content experts, also known as faculty, provided exceptional ideas for subject matter but required time to learn how to communicate effectively with the other teams. One of the primary communication issues with the technical group centered on the construction and use of storyboards in developing multimedia modules. For those who are unfamiliar with them, storyboards are blueprints to help organize a module in terms of content, navigation and functionality. They usually consist of sketches but also include an outline, flowchart and text describing what should go on each page. We requested that their storyboards consist of two parts: a cohesive view of the module as a whole, and a more specific view of the applets and activities they wanted their students to use. As with any educational endeavor, examples are important and we provide example storyboards on the project website.

When working with the evaluation team, the faculty needed to learn the reasons and benefits of formative assessment, and the value of an observer visiting the class when the module is used. To help with all of this we had several workshops, where the module development process was outlined and where they learned how to develop collaborative teams involving faculty outside their own departments. Interestingly, it was relatively easy for the faculty to find common ground on which they could develop materials for their respective courses. This required an interest in educational innovation, an ability to think a bit differently on what is or is not important in the discipline, and the willingness to discuss this in a group setting.

To supplement this effort we also provided written materials detailing the steps and outcomes for good module development. Nearly all faculty stated at least once that the best way to learn how to develop a multimedia educational module was through an example or model. The items we provided them for this are listed below and all are available on the project website [3]. They were required reading for any and all who participated in the project.

- Design Questionnaire: This consists of a sequence of questions concerning the scope of the content of the proposed module. They were asked to describe the multidisciplinary nature of their proposed module, their constructive use of interactivity and multimedia, and examples of real-world applications. They were also required to provide a draft storyboard for the module. The latter was used to help the content developers work with the technical team.
- Module and Template Overview: This provides an outline of the conceptual units that make up a module, the standard template for the various web-pages within a module, and the functional navigational paths through a module.
- Development Process: This includes explanations and resources to help develop a Links module, including the milestones expected during the development process. For example, it explains when and where assessment is used, how the technical teams will contribute to the effort, and the role of the storyboard in the development process.
- Developer Resources: Information about the resources available to developers, including programmers, media requirements, Java applets, and video tools and requirements.
- Copyright Policy: Steps to take in protecting our material and that of others.
- Navigation Scheme: This provides schematics of the basic functional and content navigation schemes and explains the differences between them.
- Storyboard Examples: A workshop was held so the faculty could learn how to create, and then use, storyboards. These were then used as the examples that we made available to faculty who were new to the project.
- Applet Storyboard Example: We provided examples of storyboards for the various interactive applets that would be used in the module.

ASSESSMENT

Each module passed through a rigorous assessment program, involving both internal and external evaluations. To identify the progress of a module through this process, a version number is assigned to each module and these are shown in Fig. 5, and the various scales are described in Table 1. Alpha and beta tests are conducted by

Table 1. Module version numbers and their meanings for the indicator in Fig. 5

Version	Description
0.0	Module is currently just a concept and is not publicly accessible.
0.2	Module is in prototype format and is publicly accessible.
0.4	Module is partially developed and not yet evaluated.
0.6	Module is completed in the Project Links standard format and is ready for internal alpha and beta testing.
0.8	Module has passed the internal alpha and beta testing and is ready to begin external evaluation for content, usability, and the appropriate use of educational technology.
1.0+	Module is released for public use. It is in the Project Links standard format, and has been evaluated for content, usability, educational technology. It has been revised to reflect changes recommended via the evaluation process.

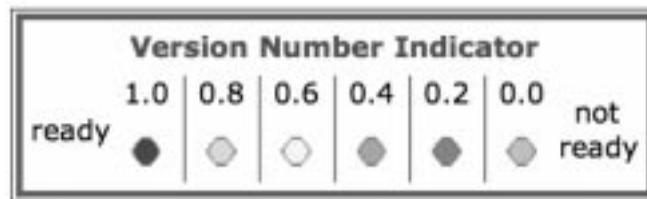


Fig 5. Version numbers, and indicator system, used to identify the progress of modules through the development process. This is on the same web-page as the listing in Fig. 1 and is linked to the information in Table 1.

the Interface Designer, who is a member of the technical group. The test descriptions are as follows.

Alpha testing: The interface designer checks modules on all platforms and browsers for broad editing, and aesthetic and usability issues. Review includes checking for:

- Broad Issues—consistency with the Links format, logical use of the content and functionality navigation, general usability, bugs/technical problems with the animations, videos, applets, etc.
- Editing Issues—titles accurate, links broken, pages missing or incorrectly named, clarity of the text, clarity of graphics/charts, etc.
- Organizational Issues—how is the material presented? Does the module make sense? Can there be more interactivity? Is the medium used effectively?
- Aesthetic Factors—does the module look good? Can it look better?

Beta testing: The interface designer provides a fine-toothed copyedit for grammar, punctuation, misspellings, broken links, broken applets, etc. Beta testing also ensures that the design and usability look good and work smoothly.

The Evaluation Consortium conducted the formal external assessment. This group standardized the process and implemented the evaluation plan. Four types of testing were designed and/or conducted, and these are the Content Review, Usability Testing, Educational Technology Review, and Pilot Testing/Classroom Observation.

These are explained below and some of the findings are presented afterwards.

1. *Usability Testing:* A small number of students are asked to use the module and provide information pertaining to usability from a student’s point of view. An observational checklist and interview protocol is used which includes videotaping students while using the module.
2. *Educational Technology and User Interface Review:* This is a standard review that looks at the module from a technological and instructional point of view and provides validation of the module’s appropriate use of current learning theory. A written review is provided based on a checklist of instruction design concepts.
3. *Pilot Testing and Classroom Observation:* This involves classroom observations and semi-structured interviews with students. The objective is to determine how well the module accomplishes its intended tasks. To date over 1000 students have been observed and interviewed. Data has also been collected from multiple institutions, as well as multiple math and non-math courses (approximately 50).
4. *Content Review:* Once the module has been developed to the satisfaction of the authors and the technical manager, a qualified expert in the subject matter is found outside of the developing institution, and provided with a checklist. In general, content reviewers are asked to work through the module, validating the module content and the accuracy of the materials presented. They are asked to complete a short review of the module delineating content viability.

EVALUATION FINDINGS

Findings from the evaluation program obtained from student perceptions of module usage are presented below. For comparison, the results for Year 1 and Year 2 are given. This is done to indicate the effects of improvements made in the modules based on the first-year assessment. The data reported here are from all modules used during the indicated years and include both math and non-math courses.

Relevance of module

One issue addressed was the perceived appropriateness and relevance of the module content. This information was gathered using in-class observations, paper-pencil surveys and semi-structured interviews participating in classroom activities. The results are given in Table 2. From the data it is seen that there is a marked increase in the second year in the percentage of students who perceived the modules as relevant to the coursework and relevant to the academic area. What is significant is that this is true, whether or not it was a math course. There were also marked increases in all other surveyed responses in this category. This is strong evidence that the students consider the modules to successfully bridge multiple disciplines and break down the disciplinary barriers commonly found in such courses.

Perceived cognitive outcomes

A second assessment issue was the students' perceived cognitive outcomes from using the

modules. This information was gathered using paper-pencil surveys and semi-structured interviews with students. The results are given in Table 3. From their responses it is clear the students have a strong perception that the modules facilitated learning of the content in a variety of ways and there was also a marked increase in all categories in the second year. Two items of particular interest are that 92% agree that using the modules helps them to apply the course content to new problems and 85% agreed that they help to transfer knowledge to problems outside the course. The ability to transfer concepts and methods to new situations is critical in a student's educational development and it is significant that they consider the modules successful in helping them with this.

When the students were queried as to the benefits of using the modules, they reported a variety of direct cognitive benefits, including enhanced learning and problem-solving skills, as a result of module use. More specifically, students reported that the modules provided practice with the use of collaborative skills, hands-on/real world applications, and different types of problem-solving methods. Faculty reported multiple positive cognitive outcomes resulting from module use, including greater understanding of course concepts, making connections between mathematics and engineering concepts, and developing problem-solving skills.

Perceived effective outcomes

A third assessment issue was the perceived effectiveness of outcomes of the modules by the students. This information was gathered using

Table 2. Student perceptions of module relevance (values are a percentage of those who agree with the given statement)

Module Content	Year 2 (n=436)	Year 1 (n=580)
Information presented in the module is relevant to course content	95%	86%
Information presented in the module useful	93%	67%
Information presented in the module is relevant to academic area	91%	68%
Information presented in the module is easy to understand	91%	68%
Information presented in the module is well organized	91%	66%
Content of the module is of interest to students	88%	64%

Table 3. Student perceptions of cognitive outcomes (values are a percentage of those who agree that the modules assisted them in the tasks listed above)

Module Use	Year 2 (n=427)	Year 1 (n=574)
To think about problems in graphical/pictorial ways	95%	77%
To recall course content	92%	64%
To apply course content to new problems	92%	62%
To improve grades	89%	40%
To develop skills in problem-solving in the content area	86%	58%
To develop different ways of solving problems	86%	58%
To work collaboratively with fellow students	85%	68%
To transfer knowledge to problems outside the course	85%	48%
To become motivated to learn course content	83%	40%
To develop an attitude of self-direction and self-responsibility	81%	45%

Table 4. Student perceptions of affective outcomes (values are a percentage of those who agree with the given statement)

Effectiveness Outcomes	Year 2 (n=436)	Year 1 (n=580)
My knowledge has increased as a result of the modules	99%	58%
My confidence in the course area has increased because of the modules	94%	47%
The module content is motivating	94%	42%

paper-pencil surveys and semi-structured interviews with students. The results are given in Table 4. As with the other two categories, the responses in Year 2 are quite positive and, in this case, are approximately twice those that were observed the year before.

CONCLUSION

The assessment results from Year 2 provide strong evidence that, from the students' point of view the modules are accomplishing what they were designed to do. In particular, the modules successfully bridge multiple disciplines and break down the disciplinary barriers commonly found in their math and non-math courses. Moreover, the modules help apply the course content to new problems and they help transfer knowledge to problems outside the course.

It is also evident that significant improvements were made between the first and second year. This observation generates the question of what exactly was done to achieve this improvement. A partial list of the changes made in the program is given below.

- Instructor Workshop—we held a workshop for the instructors at the end of Year 1 to discuss the

evaluation results. This began with a round-table like discussion of what did and did not work, and what changes were needed in each module so they were effective learning tools.

- Content Improvement—all faculty received assessment reports of the modules and they then spent the summer between Year 1 and Year 2 making improvements based on this information and what was discussed during the Instructor Workshop.
- Technology Upgrade—the modules push the technological envelope and the laptops used in the second year were significantly better than those used in the first year.
- Worksheets—even with interactive computer materials involving collaborative projects, the addition of pencil-and-paper worksheets that the students completed as they worked through the module appear to have helped them to more actively engage with the subject matter.

All four of the above listed changes were potentially important factors for achieving the improved student responses in the second year.

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REFERENCES

1. Project Links, *Mathematics and its Applications in Engineering and Science: Building the Links* (2005). (Available at <http://links.math.rpi.edu/>)
2. J. D. Bransford, A. L. Brown and Rodney R. Cocking, *How People Learn: Brain, Mind, Experience, and School*, National Academy Press, Washington, DC (1999). (Available at <http://www.nap.edu/books/0309070368/html/>)
3. National Committee on Science Education Standards and Assessment, *National Science Education Standards*, National Academy Press, Washington, DC (1996). (Available at <http://www.nap.edu/readingroom/books/nses/html/>)
4. S. L. Oviatt, *Multimodal Interfaces: The Human-Computer Interaction Handbook: Fundamentals, Evolving Technologies and Emerging Applications*, ed. by J. Jacko and A. Sears, Lawrence Erlbaum, NJ, pp. 286–304 (2003).
5. R. K. Sawyer, Improvised lessons: Collaborative discussion in the constructivist classroom, *Teaching Education*, **15**(2), pp. 189–201 (2004).
6. Hyperbolic Tree Java Library, SourceForge Open Source Website, 2001 (<http://sourceforge.net/projects/hypertree/>).

Mark H. Holmes is a Professor in the Department of Mathematical Sciences at Rensselaer. He received a B. Sc. at Colorado State University and a Ph. D. in applied mathematics from UCLA. As co-director of Project Links he received the 2001 ASME Curriculum Innovation Award, and the 2000 Premier Award for Excellence in Engineering Education Courseware. He is also the recipient of the 2002 Award for Innovative Excellence in Teaching, Learning

and Technology, and the 2002 Best Paper Award, 13th International Conference on College Teaching and Learning. He has also been a Guggenheim Fellow and won the Y. C. Fung Young Investigator Award (ASME).

Robert L. Spilker is a Professor in the Department of Biomedical Engineering at Rensselaer. He received a B. Sc. at the University of Illinois and a Sc. D. from the Massachusetts Institute of Technology. As co-director of Project Links he received the 2001 ASME Curriculum Innovation Award, and the 2000 Premier Award for Excellence in Engineering Education Courseware. He is a member of the American Society of Mechanical Engineering, where he has served as Chair of the Executive Committee of the Bioengineering Division, the US Association of Computational Mechanics, the Orthopedic Research Society, and the American Society of Engineering Education. He serves as Past Chair of the Executive Committee of the US National Committee on Biomechanics, and on the Editorial Advisory Board of several journals in the areas of bioengineering and computational mechanics. Dr. Spilker is a Fellow of the American Society of Mechanical Engineers and the American Institute of Medical and Biological Engineering.