# Undergraduate Laboratory Experiences Using Uncertainty Analysis to Validate Engineering Models with Experimental Data*

W. G. STEELE and J. A. SCHNEIDER
*Department of Mechanical Engineering, Bagley College of Engineering, Mississipi State University, Mississipi State, MS 39762–5925 USA. E-mail: steele@me.msstate.edu*

*Traditionally, the goals of engineering laboratory instruction have been to introduce students to various measurement devices along with associated methods of interpreting results in the context of experimental uncertainties. There is usually an emphasis on the demonstration of fundamental engineering principles in applications-oriented projects. Often, theoretical engineering models are used to compare predicted outcomes with the experimental results in order to demonstrate the appropriateness and/or limitations of the theoretical model. When making these comparisons, the uncertainty associated with an experiment's measurements is usually included; however, there is seldom consideration of the uncertainty associated with the theoretical model calculations. Students in the Mechanical Engineering (ME) program at Mississippi State University (MSU) are applying the concept of engineering model validation using uncertainty analysis. In this paper, their experiences are used to illustrate how this approach has been implemented in the undergraduate laboratory classes. The methodology is developed for model validation, and a case study from our senior mechanical engineering laboratory is presented which illustrates how the uncertainty of the model is combined with that from the experimental result to provide a quantification of the model's validity.*

**Keywords:** uncertainty; experimentation; modeling; validation; laboratories

## BACKGROUND

At MSU, this model validation approach was first implemented in the ME undergraduate laboratory program to provide a bridge between the theoretical aspects of the traditional engineering courses and the demonstration of basic principles through experimentation. An appreciation of the errors inherent in experimental results is critical, and uncertainty analysis concepts are integrated into the curriculum in an effort to quantify the validity of the test data. This process provides a logical methodology to interpret test results through the application of uncertainty analysis in the planning, design, construction, debugging, execution, data analysis, and reporting phases of experiments [1]. Accuracy of the experiments is investigated along with the appropriateness of a theory or model and its simplifying assumptions. This concept is an extension of the verification and validation research that is currently being done for CFD and other computational design codes [2, 3]. The approach is communicated at undergraduate level through a three-laboratory sequence consisting of Experimental Orientation (EO), Experimental Techniques I (ET I), and Experimental Techniques II (ET II).

In the undergraduate curriculum, EO gives the students an introduction to the use of instrumentation for basic measurements, to the acquisition and processing of the measurement data, and to the concept of uncertainty associated with the instrumentation selection and the measurement process. The second course in the sequence, ETI, concentrates on identification of the key parameters needed to guide the design of experiments using uncertainty analysis. These concepts merge into ETII, which provides the opportunity for students to combine knowledge gained in both EO and ETI, and to compare theoretical predictions with the measured outcome. The complete description of these courses has been presented elsewhere [4].

For the undergraduate ETII class, students select from a range of mechanical systems. Some of the systems are based on potential classroom demonstration units such as a pump test stand, a tensile test machine, or a heat exchanger apparatus. The mechanical system can also be applied to modeling the physics of games such as predicting variability in the ultimate trajectory of softballs, water rockets, or golf balls. Or the mechanical systems can be chosen to augment ongoing research projects to explore the application of uncertainty analysis in the understanding of experimental results and associated engineering models.

In the next section, some experiences from the ETII laboratory at MSU are summarized to show the application of model validation using experimental data. Following this section, the methodology of model validation is given along with an example.

### EXPERIENCES

One ETII project considered the prediction and measurement of the efficiency of a residential gas furnace. The students developed an energy balance model for the system that included the air temperature difference from the inlet to the exit, the air flow rate, and the natural gas flow rate and heat input. The air temperatures were measured with thermocouples, the air flow was measured with a volumetric flow meter, the gas flow rate was measured with an orifice, and the gas heating value was found from standard tabulated values. The uncertainty of each of these inputs was used with model expressions to find the uncertainty of the model predictions.

The efficiency was measured directly with a flue gas analysis device. Initial comparisons of the measured and predicted efficiencies were not good, but the uncertainty for the model was high. The controlling variable for the model uncertainty was the exit temperature. The students modified the exit temperature measurement from a single thermocouple to a thermocouple grid to determine the average air exit temperature with less uncertainty. This improvement gave good comparisons of the predicted and measured efficiencies considering the uncertainties of both.

Another ETII experiment involved the use of an alternating fatigue device to determine the fatigue life of Aluminum 6061 T-6 for various stress levels. The engineering model used was the standard stress versus number of cycles to failure fatigue chart for this material. The initial comparison of the test and model results for this experiment was poor. The estimated values of stress were based on the displacement of the specimen and on the specimen's dimensions. This approach led to reasonably large uncertainty values for the stress. Also, the number of cycles to failure was determined with a mechanical counter, which had an uncertainty of 5 percent. An improvement to the test was made by gluing a strain gage to the specimen and by using a computer data acquisition system to monitor the strain (stress) and to measure the number of cycles to failure. This improved test procedure yielded results with less uncertainty that agreed well with the published data.

### METHODOLOGY

Some of the previous work related to the application of uncertainty analysis in undergraduate engineering laboratory courses is documented in References 1 and 4–9. These efforts have been directed primarily toward quantifying the uncertainty of the experimental results. The methodology for applying uncertainty analysis to experimental results is summarized below, where the nomenclature has been updated to the latest accepted version.

In nearly all experiments, the measured values of different variables are combined using a data reduction equation (DRE) to achieve some desired result. A general representation of a data reduction equation is:

$$r = r(X_1, X_2, ..., X_J) \qquad (1)$$

where $r$ is the experimental result determined from $J$ measured variables $X_i$. Each of the measured variables contains systematic (fixed) and random (varying) errors. Errors in the measured values then propagate through the DRE, thereby generating systematic and random errors in the experimental result, $r$. Uncertainty analysis is used to estimate the random and systematic standard uncertainties of the result, $s_r$ and $b_r$, respectively, and the corresponding expanded uncertainty of the result, $U_r$.

If it is assumed that the degrees of freedom for the result are large (>10), which is very appropriate for most engineering applications, then the "large sample assumption" [6] applies and the 95 percent confidence expression for $U_r$ is

$$U_r = 2\sqrt{b_r^2 + s_r^2} \qquad (2)$$

The systematic standard uncertainty of the result is defined as

$$b_r^2 = \sum_{i=1}^{J} \theta_i^2 \, b_i^2 + 2 \sum_{i=1}^{J-1} \sum_{k=i+1}^{J} \theta_i \, \theta_k \, b_{ik} \qquad (3)$$

where

$$\theta_i = \frac{\partial r}{\partial X_i} \qquad (4)$$

The systematic standard uncertainty estimate for each $X_i$ variable is the root-sum-square combination of its elemental systematic standard uncertainties

$$b_i = \left[ \sum_{j=1}^{M} b_{i_j}^2 \right]^{1/2} \qquad (5)$$

where $M$ is the number of elemental systematic standard uncertainties for $X_i$ and where each $b_{i_j}$ is the standard deviation level estimate of the systematic uncertainty in variable $X_i$ resulting from error source j. The standard deviation level systematic uncertainty estimate for an error source is usually achieved by making a 95 percent confidence estimate of the limits of the error for that source and dividing that estimate by 2 [6]. The

second term in Equation (3) accounts for systematic errors that have the same source and are correlated. The factor $b_{ik}$ is the covariance term appropriate for the systematic errors that are common between variables $X_i$ and $X_k$ and is determined from [10] as

$$b_{ik} = \sum_{\alpha=1}^{L} b_{i_{\alpha}} b_{k_{\alpha}} \qquad (6)$$

where variables $X_i$ and $X_k$ share $L$ identical systematic error sources. The random standard uncertainty of the result is defined as

$$s_r^2 = \sum_{i=1}^{J} \theta_i^2 s_i^2 \qquad (7)$$

where $s_i$ is the sample standard deviation for variable $X_i$ (sample standard deviation of the mean if $X_i$ is a mean value or sample standard deviation if $X_i$ is a single reading).

This same basic methodology can be applied to the engineering model for estimating the uncertainty associated with the calculated result from the model. The engineering model has input values that have uncertainties. These uncertainties cause an uncertainty in the calculated result. The model may also have an uncertainty based on how well it matches the physics of the experiment. This additional uncertainty cannot be estimated before the validation process and is therefore the primary reason for doing a validation study on the engineering model.

Considering the model result, m, to be a function of $K$ input values, $Y_i$, as indicated by Equation (8)

$$m = f(Y_1, Y_2, \ldots, Y_K) \qquad (8)$$

the uncertainty of the model result would then be determined from the uncertainty propagation equation [6] as

$$U_m = 2\left[\sum_{k=1}^{K} \theta_k^2 s_k^2 + \sum_{k=1}^{K} \theta_k^2 b_k^2 + 2\sum_{k=1}^{K-1} \sum_{j=k+1}^{K} \theta_k \theta_j b_{kj}\right]^{1/2} \qquad (9)$$

where $\theta_k$ is the derivative of the model with respect to each input quantity $Y_i$ and the $s_k$ and $b_k$ factors are the random and systematic standard uncertainties for the model input variables. Note that property values and empirical coefficients will have uncertainties as well as the input variables.

To determine the validation of the model with respect to the result of the experiment, $r$, a comparison error, $E$, is defined as

$$E = r - m \qquad (10)$$

The uncertainty associated with this comparison error is

$$U_E^2 = U_r^2 + U_m^2 \qquad (11)$$

The basic concept in the validation process is a comparison of $E$ and $U_E$. If $|E|$ is less than $U_E$, then the comparison is within the noise level of the uncertainty, and the level of validation of the model is $U_E$. If $|E|$ is much larger than $U_E$, then there is probably justification for improving either the selection of the governing equations for the model or the initial simplifying assumptions [3]. In this case, the sign $E$ gives some indication of the correction needed to the model.

Another benefit of the validation process is the determination of how each of the sources of uncertainty affects the uncertainty of the comparison error. The uncertainty percentage contribution, UPC, for each error source is determined as

$$\frac{\theta_i^2 b_i^2 \times 100}{(U_E/2)^2} \; ; \; \frac{2\theta_i \theta_k b_{ik} \times 100}{(U_E/2)^2} \; ; \; \frac{\theta_i^2 s_i^2 \times 100}{(U_E/2)^2} \qquad (12)$$

where the sum of all of the UPC's is 100 percent. The factors in Equation (12) come from Equations (3), (7), (9), and (11). These parameters show the percentage contribution that each error source makes on the square of the total uncertainty of the comparison error. A review of the UPCs will identify which uncertainties are controlling the total uncertainty and which are having a negligible effect. This information can be used to identify where improvements need to be made in the uncertainties of the experimental variables or in the model input variables to reduce the magnitude of the uncertainty of the comparison error.

In the next section, the validation process is illustrated with an example experiment from the ME laboratory course at MSU.

*Example*
One assignment for an ETII team was to determine experimentally and theoretically the head loss, h, for a straight section of a plastic pipe. The flow apparatus is shown in Fig. 1. Pressure differentials across both the orifice plate flow meter and the straight pipe section were measured using a manometer and a differential pressure transducer.

For the experiment, the result was the measured head loss in the pipe, $\Delta hp_r$, over a range of flow rates. For the engineering model, the following expression was used to predict the pipe head loss, $\Delta hp_m$,

$$\Delta hp_m = f\frac{8L\left[C(\Delta ho)^{0.5}\right]^2}{g\pi^2 d^5} \qquad (13)$$

where $L$ is the length of the pipe, $C$ is the orifice flow coefficient, $\Delta ho$ is the orifice head loss, $g$ is the acceleration of gravity, $d$ is the pipe diameter, and $f$ is the friction factor. The Haaland relationship [11] was used for the friction factor

$$f = \frac{0.3086}{\left\{\log\left[\left(\frac{6.9}{Re_d}\right) + \left(\frac{\varepsilon}{3.7d}\right)^{1.11}\right]\right\}^2} \qquad (14)$$
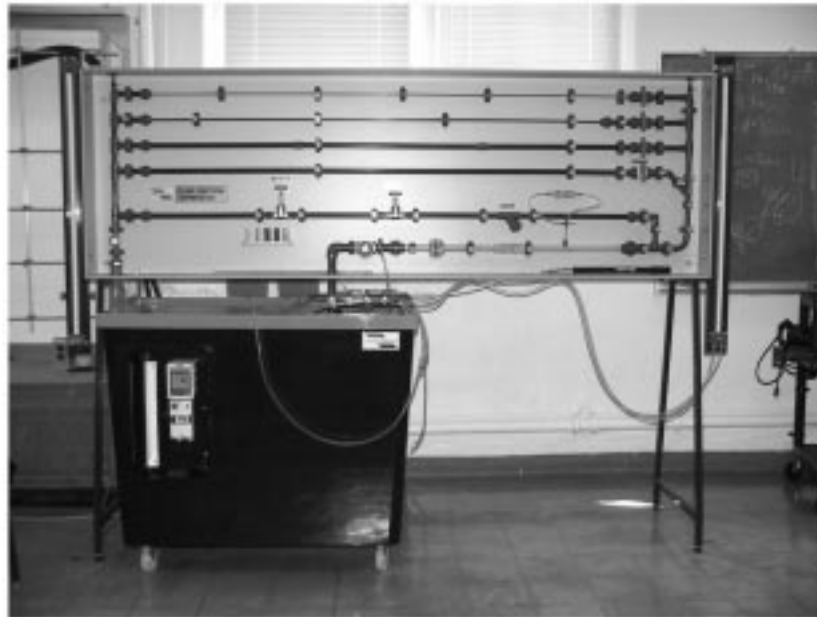
Fig. 1. Fluid friction apparatus

where $\epsilon$ is the roughness of the pipe and $Re_d$ is the Reynolds number based on the pipe diameter

$$Re_d = \frac{4\rho C(\Delta ho)^{0.5}}{\pi d\mu} \qquad (15)$$

with $\rho$ the water density and $\mu$ the water viscosity.

The values of all the experiment's and the engineering model's variables are given in Table 1. Also in the table are the uncertainty estimates for each variable in terms of millimeters of water.

The systematic standard uncertainty for the pipe head loss came from the combination [using Equation (5)] of the elemental systematic standard uncertainties for the pressure transducer of 2.5 mm for zero shift and 2.5 mm for the calibration curve fit. The random standard uncertainty for the pipe head loss was based on the standard deviation of the mean for the 36 readings made with the pressure transducer on each run. This value was nominally constant for each run, so the same value was used for all runs.

The orifice head loss systematic standard uncertainty was based on the accuracy of the manometer used for these measurements. The random standard uncertainty was estimated from the variation

of the water column during readings. The typical variation was about ±4 mm yielding an estimate of the standard deviation of ±2 mm.

The systematic standard uncertainty estimates for the length and diameter were based on the devices used to make the measurements, a scale and a micrometer. The orifice was calibrated before running the experiment using a catch basin of known volume. The systematic standard uncertainty for the flow coefficient is the standard deviation of the mean for the calibration process.

The pipe wall was relatively smooth; however, considering the variation in relative roughness estimates near the smooth wall limit, this value could be uncertain by as much as 50 percent with 95 percent confidence. The water density and viscosity were based on tabulated values at a water temperature of about 291 K, but possible variation in the water temperature of ±2.8 K led to uncertainty estimates of 7 percent for the water viscosity and 0.3 percent for the density, both at 95 percent confidence. The systematic standard uncertainty estimates for these three variables were taken as one-half of the 95 percent confidence estimates.

The results for each run of the experiment and the engineering model are given in Table 2. The

Table 1. Variable values and uncertainty estimates

| Variable | Value | Systematic Standard Uncertainty | Random Standard Uncertainty |
|---|---|---|---|
| pipe head loss ($\Delta hp_r$) | Variable | 3.5 mm | 1.0 mm |
| orifice head loss ($\Delta ho$) | Variable | 1.3 mm | 2.0 mm |
| pipe length ($L$) | 0.99 m | 0.8 mm | — |
| pipe diameter ($d$) | 0.0177 m | 0.006 mm | — |
| flow coefficient ($C$) | $1.18 \times 10^{-3}$ m$^{2.5}$/sec | $9.2 \times 10^{-6}$ m$^{2.5}$/sec | — |
| roughness ($\epsilon$) | $9.1 \times 10^{-8}$ m | 25% | — |
| water density ($\rho$) | 999 kg/m$^3$ | 0.15% | — |
| water viscosity ($\mu$) | $1.056 \times 10^{-3}$ N sec/m$^2$ | 3.5% | — |

Table 2. Test results

| Run | Orifice Head Loss (m) | $Re_d$ | Pipe Head Loss Measured $\Delta hp_r$ (m) | Data Uncertainty $U_r$ (mm) | Pipe Head Loss from Model, $\Delta hp_m$ (m) | Model Uncertainty $U_m$ (mm) | Comparison Error $E$ (mm) | Comparison Error Uncertainty $U_E$ (mm) |
|---|---|---|---|---|---|---|---|---|
| 1 | 0.08 | 22,623 | 0.14 | 7.4 | 0.13 | 8.1 | 6.1 | 11 |
| 2 | 0.10 | 25,946 | 0.16 | 7.4 | 0.17 | 8.6 | -1.5 | 11 |
| 3 | 0.15 | 31,116 | 0.24 | 7.4 | 0.23 | 10 | 8.1 | 12 |
| 4 | 0.15 | 31,325 | 0.23 | 7.4 | 0.23 | 10 | 1.5 | 12 |
| 5 | 0.19 | 34,730 | 0.29 | 7.4 | 0.28 | 11 | 8.1 | 13 |
| 6 | 0.22 | 37,484 | 0.32 | 7.4 | 0.32 | 12 | 3.3 | 14 |
| 7 | 0.22 | 37,830 | 0.33 | 7.4 | 0.32 | 12 | 7.4 | 14 |
| 8 | 0.25 | 40,373 | 0.37 | 7.4 | 0.36 | 13 | 11 | 15 |
| 9 | 0.26 | 40,974 | 0.37 | 7.4 | 0.37 | 14 | 1.5 | 15 |
| 10 | 0.29 | 43,106 | 0.42 | 7.4 | 0.41 | 15 | 8.4 | 16 |
| 11 | 0.32 | 45,498 | 0.45 | 7.4 | 0.45 | 16 | 6.6 | 17 |
| 12 | 0.33 | 46,032 | 0.47 | 7.4 | 0.46 | 16 | 9.9 | 17 |
| 13 | 0.36 | 48,279 | 0.51 | 7.4 | 0.50 | 18 | 8.9 | 19 |
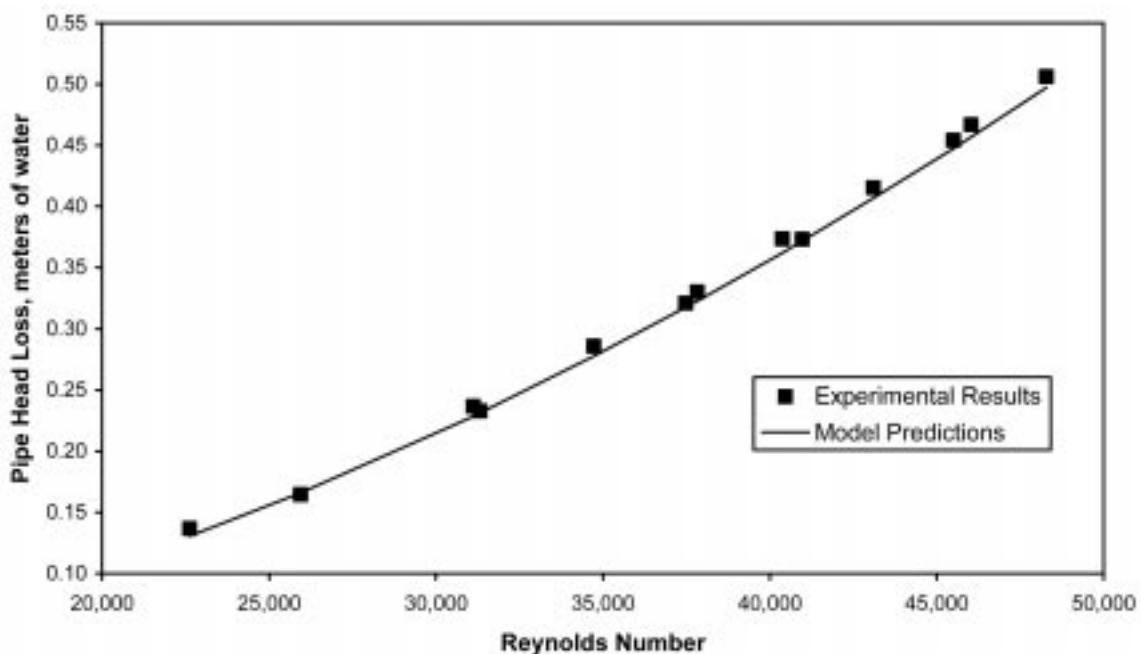


Fig. 2. Results for pipe head loss experiment and model

Reynolds number range for this test was 22,000 to 48,000, and the pipe head loss varied from 0.14 to 0.51 meters of water over this range. The comparison of the measured and predicted pipe head loss values is given in Fig. 2. The comparison looks very good, but uncertainties should be taken into account in order to validate the model. This validation comparison is shown in Fig. 3, where the comparison error, *E*, from Equation (10) is plotted along with the uncertainty in the error, $U_E$, from Equation (11). For this test, |*E*| was less than $U_E$ for all runs; therefore, the model is validated at the level of $U_E$ for this pipe diameter and smoothness over this range of Reynolds numbers. The validation uncertainty varies from 11 mm of water at the low Reynolds number end to 19 mm of water at the upper end.

This level of validation is based on the uncertainties of the experiment result and the model

input variables. An investigation of the UPC's for each of the error sources will show which uncertainties are dominating the determination of the error uncertainty. Table 3 gives UPCs for the two limits of the test's Reynolds number range. At the lower Reynolds number, the systematic standard uncertainty of the pipe head loss measurement dominates $U_E$. To reduce this uncertainty, a better transducer with less zero shift and a more linear calibration curve would be needed. The random standard uncertainty of the orifice head loss measurement also is significant at the low Reynolds number limit. At the higher Reynolds number, the orifice flow coefficient uncertainty dominates $U_E$. To reduce this value, the orifice would need to be better calibrated. If these three uncertainties were reduced, then there would be less uncertainty in both the experiment and the model, and the uncertainty in the error would
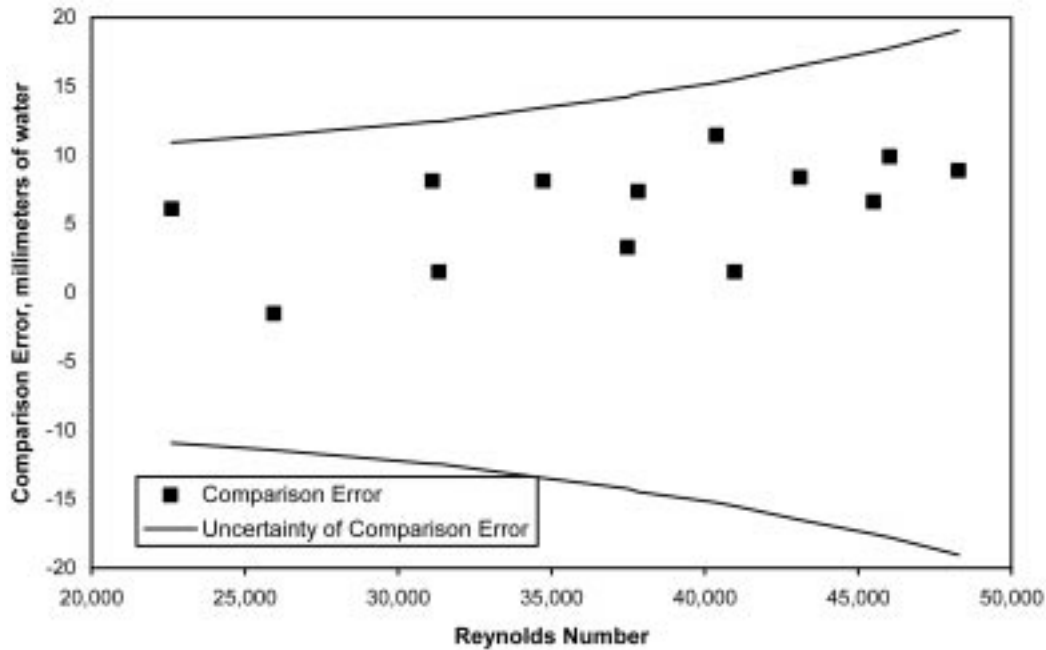
Fig. 3. Validation of pipe head loss model

Table 3. Uncertainty Percentage Contributions

| Variable | Reynolds Number | | | |
|---|---|---|---|---|
| | 22,623 | | 48,279 | |
| | Systematic Standard Uncertainty | Random Standard Uncertainty | Systematic Standard Uncertainty | Random Standard Uncertainty |
| pipe head loss ($\Delta hp_r$) | 42.6 | 3.4 | 15.1 | 1.2 |
| orifice head loss ($\Delta ho$) | 11.0 | 28.1 | 2.8 | 7.1 |
| pipe length ($L$) | 0.0 | - | 0.2 | — |
| pipe diameter ($d$) | 0.2 | - | 0.8 | — |
| flow coefficient ($C$) | 10.5 | - | 54.9 | — |
| roughness ($\epsilon$) | 0.0 | - | 0.0 | — |
| water density ($\rho$) | 0.0 | - | 0.0 | — |
| water viscosity ($\mu$) | 4.2 | - | 17.9 | — |
| | Sum = 100.0 | | Sum = 100.0 | |

be less. Then the comparison range for $E$ would be smaller, but it is very likely that improved pressure transducers and better orifice calibration would reduce the comparison error.

## CONCLUSION

Understanding the limitations of physical models is key to the successful practice of engineering. In the ETII laboratory in ME at MSU, the students are given an opportunity to investigate the validity of models by comparing the predictions with experimental results. The uncertainties in both the model and experimental results are used (1) to assess the model validity, (2) to identify ranges where different or improved models are needed, or (3) to show that improved variable uncertainties are needed to reduce the validation uncertainty.

## REFERENCES

1. W. G. Steele, and H. W. Coleman, Integrating Uncertainty Analysis Concepts into Undergraduate Laboratory Courses, *International Journal of Engineering Education,* **8**, 1992, pp. 147–153.
2. H. W. Coleman, and F. Stern , Uncertainties in CFD Code Validation, *ASME Journal of Fluids Engineering,* **119**, 1997, pp. 795–803.
3. H. W. Coleman, Some Observations on Uncertainties and the Verification and Validation of Simulations, *ASME Journal of Fluids Engineering*, **125**, 2003, pp. 733–735.

4. S. T. Hudson, J. A. Schneider, and W. G. Steele, Uncertainty Analysis for Undergraduate and Graduate Mississippi State University Mechanical Engineering Students, AIAA Paper 2003–0797, 41st AIAA Aerospace Sciences Meeting and Exhibit, Reno, NV (2003).
5. M. Fernandez Chimeno, M. A. Garcia Gonzalez, and J. Ramos Castro, Teaching Measurement Uncertainty to Undergraduate Electronic Instrumentation Students, *International Journal of Engineering Education*, **21**, 2005, pp. 525–533.
6. H. W. Coleman, and W. G. Steele, *Experimentation and Uncertainty Analysis for Engineers,* Second Edition, New York, NY, John Wiley & Sons, Inc., (1999).
7. H. W. Coleman, and W. G. Steele, Engineering Application of Experimental Uncertainty Analysis, *AIAA Journal*, **33**, 1995, pp. 1888–1896.
8. W. G. Steele, R. A. Ferguson, R. P. Taylor, and H. W. Coleman, Computer-Assisted Uncertainty Analysis, *Computer Applications in Engineering Education*, **5,** 1997, pp. 169–179.
9. M. H. Hosni, H. W.Coleman, and W. G. Steele, Application of MathCAD Software in Performing Uncertainty Analysis Calculations to Facilitate Laboratory Instruction, *Computers in Education Journal*, **7,** 1997 pp. 1–9.
10. K. K. Brown, H. W. Coleman, W. G. Steele, and R. P.Taylor, Evaluation of Correlated Bias Approximations in Experimental Uncertainty Analysis, *AIAA Journal*, **34**, 1996, pp. 1013–1018.
11. B. K. Hodge and R. P. Taylor, *Analysis and Design of Energy Systems*, Third Edition, Prentice Hall, Upper Saddle River, NJ (1999).

**W. G. Steele** is William L. Giles Distinguished Professor and Head of Mechanical Engineering at Mississippi State University. His principal area of research is the use of uncertainty analysis in experimentation and design, and he has authored a textbook on the subject (Reference 6). Dr Steele has served on national and international committees concerned with uncertainty analysis standards.

**J. A. Schneider** is an Associate Professor in the Mechanical Engineering Department at Mississippi State University. Her primary area of research is the engineering of the microstructure by control of the processing parameters to obtain the desired mechanical performance of structural materials. Much of her research centers on characterization of the microstructual evolution during either the processing or service life of the material.