# Accuracy and Uncertainty: A False Dichotomy in Engineering Education. A Case Study from Civil Engineering\*

## AMIN ELSHORBAGY

Centre for Advanced Numerical Simulation, Department of Civil & Geological Engineering, University of Saskatchewan, Saskatoon, SK, Canada S7N 5A9. Email: amin.elshorbagy@usask.ca

Predictive uncertainty is an important concept that civil engineering students should understand. The students need to realize that uncertainty is inevitable in spite of the efforts made to make models, algorithms, and analysis techniques as accurate as possible. In this paper, the issue of uncertainty is addressed through an illustrative example from the field of surface water quality management. The example demonstrates that simple probabilistic analysis can be effective for both walking the students through the issue of uncertainty and realistically quantifying the uncertainty for real-life civil engineering applications.

Keywords: uncertainty; probabilistic analysis; hydrologic engineering; design accuracy; water quality management

# INTRODUCTION

ACCORDING TO WEBSTER'S DICTIONARY. engineering is 'the application of science and mathematics by which the properties of matter and sources of energy in nature are made useful to people in structures, machines, products, systems, and processes.' To many engineers and engineering students the word 'mathematics' and the traditional perception of engineering imply accuracy and precision. Engineering students continuously deal with and are concerned about systems and their accuracy. These concerns involve measuring instruments, analysis techniques (e.g., optimization algorithms), or simulation and prediction models. Chimeno et al. [1] correctly observed and reported that engineering students never think about the measurement uncertainty and that the first barrier to overcome is to convince the students that the measurement uncertainty is an important factor. When students are asked to use a few measuring tapes to measure the height of one of the students, they will realize that slightly different measurements may be reported by the same student using various measuring tapes and that slightly different measurements might be reported by different students using the same measuring tape.

When adopting simulation models to simulate or predict a specific variable or phenomenon, the predictive uncertainty is inevitable. The model uncertainty may stem from uncertainty about its structure, parameters, or input data [2]. In water resources analysis, most design values are quantiles derived from frequency curves [3] that assume a certain probability distribution. There are inevitable uncertainties about the probability distribution itself as well as the values derived from the distribution. Engineers and engineering students may unjustifiably use such values with a few significant digits to proceed with the design process to produce what they believe to be a 'precise' design. They may ignore the fact that engineering measurements, analysis, and modeling can be accurate but may never be precise and will certainly be overshadowed by uncertainty.

The aim of this paper is to help expose and eliminate, through a case study about surface water quality management, the misconception that accuracy and uncertainty are dichotomous in engineering practice. The simplified case study, which can be presented to civil engineering students, will demonstrate the possibility of being highly uncertain in spite of using well established standard engineering practices.

#### THE FALSE DICHOTOMY

In hydrologic design, it is common to rely on the intensity–duration–frequency (IDF) curves [3] to identify the rainfall design value needed for a hydro-technical structure. The region-specific IDF curves are constructed assuming that rainfall is a pure random variable. Even though a value read off the IDF curves is subject to a significant level of uncertainty, it is common to ignore such uncertainty, especially at the undergraduate level. This is just one example of many possible examples across various fields of civil engineering. Failing to

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explicitly address the issue of uncertainty and discuss it with the students is an unjustified case of presenting the design process as a process that encapsulates what we perfectly know. It is also an indication of the extremely narrow approach to engineering education [4]. Engineering students need to understand that, as future designers, they must present a complete picture of what is perfectly known and what is uncertain because this is important to decision makers.

As M. Wald noted in his editorial, the shift in engineering education from science to vocational education is inevitable [5]. In order to make future decision makers and regulators willing to accept risk and uncertainty-based analysis and designs, and thus introduce uncertainty and frequencybased environmental regulations, engineering educators must ensure the introduction of such concept in the engineering curriculum [6]. In spite of the importance of teaching uncertainty in engineering, there have been a few articles aimed at addressing this issue [1, 7], and they focused only on measurement uncertainty. There is a gap in engineering education literature with respect to teaching uncertainty in engineering analysis and design. This paper is an attempt to help fill this gap by presenting an illustrative example from the field of surface water quality management.

Before introducing the illustrative example, it might be useful to define uncertainty analysis as the technique used to estimate the interval about a result within which the true value is thought to lie with a certain degree of confidence [7].

### **ILLUSTRATIVE CASE STUDY**

The Total Maximum Daily Load (TMDL) program recently emerged as the fundamental approach to meet water quality standards in water bodies. The TMDL process usually refers to the plan to develop and implement the TMDL of a quantifiable pollutant to achieve compliance with a surface water quality standard [8]. Development of TMDLs for different pollutants at the watershed scale enables managers to enforce constraints on the allowable level of pollutant input. If the level of pollutant input or a water quality parameter in a water body violates the recommended value from the TMDL study, a pollutant load reduction in the watershed could be proposed [9].

Elshorbagy et al. [9] and Ormsbee et al. [10] have proposed a methodology for pH TMDL development. Their idea is to convert the pH standard unit into a quantifiable hydrogen ion load, and therefore recommend load reduction to ensure the pH level in the stream does not fall below 6.0 in acidity prone watersheds. The protocol relies on a regression relationship between streamflow (m<sup>3</sup>/s) and hydrogen ion concentration (g/m<sup>3</sup>) based on measured pH. The recommended load reduction is based on meeting the standards at a chosen single value of flow: the critical flow  $(Q_c)$ . The relationship between hydrogen ion activity and pH can be expressed as follows:

$$\{H^+\} = 10^{-pH} \tag{1}$$

where pH is the negative log of the H+ ion activity in mol/l. The H+ TMDL that results in at least a pH level of 6.0 is determined based on the following equation [10]:

$$TMDL = 2.45 \times Q. \tag{2}$$

where *TMDL* is in g/day and Q is the streamflow in  $(ft^3/s)$ .

#### Deterministic load reduction requirement

The methodology proposed by Ormsbee et al. [10] for determining the required load reduction relies mainly on the assumptions that (i) the H+ concentration can be linked to the streamflow using a simple regression relationship without error between streamflow and H+ concentration, (ii) a designated critical flow (Qc) can be set a priori and used to estimate the corresponding H+ concentration using the regression equation, and (iii) a load reduction can be recommended based on the difference between the estimated TMDL (Equation (2)) and predicted load. Clearly, the deterministic methodology ignores possible prediction errors that are represented by the model residuals and parameters, and both the estimated TMDL and the predicted load are evaluated at a single value of streamflow (e.g.,  $Q_c$ ), ignoring the effect of the natural variability of streamflows.

#### Probabilistic load reduction requirement

Any TMDL program has to be designed in the face of several types of uncertainty [11]. The difficulties of water quality modeling and analysis are aggravated by uncertainties inherent in many steps throughout the modeling exercise. The following uncertainties should be brought to the attention of the students. First, the water quality measurements are usually insufficient for reliable calibration and validation of models. The regression models used for the case study under consideration are no exception. Second, the impairment, evaluated based on concentrations that exceed a certain threshold, is dependent on flow. Flow is a random variable, and those days when sampling occurred may not represent the hydrologic conditions over a long period of time. Finally, violations and compliance evaluated by a model are subject to uncertainties due to the parameters and/or structure of the model.

The U.S. EPA guidelines for state water quality assessments can be considered percentile-based standards. They recommend listing a water body as impaired if more than 10% of the samples from that water body violate the water quality standards [12]. An effective way of developing a percentilebased TMDL has been proposed by Borsuk et al.



Fig. 1. The probability distribution of the regression residuals represents the frequency of violation.

[13] and adopted in this study. The residuals of the regression models are fitted by a Normal distribution, and then a longer set of residuals (e.g., 1000 values) is generated using Monte Carlo simulation. A predicted concentration value, identified using the regression model at a certain flow value (e.g.,  $Q_c$ ), can be replaced by a corresponding set of 1000 instances according to Equation (3).

$$C_i = \bar{C} \pm R_i \tag{3}$$

where  $C_i$  is one of the possible concentration values,  $\overline{C}$  is the mean concentration assessed using the regression model, and  $R_i$  is one of the residual values. Based on the generated set of concentrations,  $C_i$ , forming a distribution  $C_c$ (Fig. 1), the percentage of values violating the standards can be calculated (e.g., 20% of the values are higher than the permissible concentration).

The percentage estimated based on the aboveoutlined methodology is a single prediction of the frequency of standard violations  $(F_c)$  at a specified flow. Information about the uncertainty in the prediction of  $F_{\rm c}$  is highly useful because it provides a realistic expectation of the chances of compliance with the percentile-based standards [13]. This uncertainty is a quantitative index that represents the probability distribution of the  $F_{\rm c}$  values. Such a distribution can be obtained by perturbing the values of the regression model parameters (slope m and intercept b). A set of m and b can be generated based on the mean value and the standard error of the regression coefficients, maintaining the correlation between them [14]. The generated distribution of  $F_{\rm c}$  values allows for computing the 90% confidence interval (CI), and the confidence of compliance (CC). The CC is the probability that the violation (i.e., the  $F_c$ ) does not exceed a pre-specified percentile, such as the 10% indicated by the U.S. EPA guidelines. More generally, the overall violation and compliance across all flows can be estimated using historical flow values or generated flow by a second set of Monte Carlo

Table 1. Flow and pH monitoring results in the Beech Creek Watershed

	Site P1		Site P2	
Date	Flow in cfs (m <sup>3</sup> /s)	pН	Flow in cfs (m <sup>3</sup> /s)	pН
10/24/2000 11/7/2000 11/9/2000 3/27/2000 4/20/2001 8/13/2001 8/22/2001	0.01 (0.0003) 0.06 (0.0017) 0.44 (0.013) 0.46 (0.013) 0.02 (0.0006) 0.01 (0.0003) 0.00	2.74 3.12 3.49 3.15 3.30 2.94	0.03 (0.0009) 0.18 (0.005) 1.90 (0.054) 0.94 (0.027) 0.53 (0.015) 0.13 (0.0037) 0.10 (0.003)	2.60 2.89 3.06 3.31 3.19 2.85 2.93
1/9/2002	0.15 (0.004)	3.58	0.51 (0.015)	4.50

simulations. The above-mentioned methodology is repeated using a set of 3650 values of flow (equivalent to 10 years of daily values) instead of a single-valued flow ( $Q_c$ ). Values are averaged over all flows.

#### Application to the Beech Creek watershed

The Beech Creek watershed in Western Kentucky is used in this paper. The 1998 303(d) list of waters for Kentucky [15] indicates that 3.4 miles of Beech Creek, from the headwaters to the confluence with Pond Creek in Muhlenberg County, does not meet its designated uses for both contact recreation (swimming) and aquatic life. The Beech Creek watershed is entirely contained within Muhlenberg County, in south-western Kentucky. The Beech Creek watershed contains three main land uses: resource extraction (mining and disturbed land area), forest, and agriculture. Several non-point loading sources were identified in the Beech Creek watershed. In order to provide a more recent characterization of the pH levels in the watershed, the data shown in Table 1 were collected at the sites indicated in Fig. 2.

#### Results and analysis of the deterministic approach

The use of the deterministic approach to TMDL development, as briefly explained above and



Fig. 2. Beech Creek watershed sampling sites.

Table 2. Lowest ten-year mean annual flows and corresponding TMDLs (after [10])

Sub-basin	Area in mi <sup>2</sup> (km <sup>2</sup> )	Q in cfs (m <sup>3</sup> /s)	TMDL in lbs/ day (g/day)
Total	4.12 (10.55)	2.56 (0.073)	0.014 (6.36)
1	1.25 (3.2)	0.78 (0.022)	0.004 (1.82)
2	2.87 (7.35)	1.78 (0.051)	0.010 (4.54)

detailed by Elshorbagy et al. [9], results in a TMDL of 0.014 lbs/day (6.36 g/day) of hydrogen ions at critical flow at the mouth of the watershed (Table 2). The critical flow is the lowest ten-year annual discharge as proposed by the Kentucky Division of Water (KYDOW). The TMDLs for each individual sub-basin were obtained using a simple mass balance technique. For a mass balance to be obtained, the load at the watershed outlet must equal the summation of the incremental load from each sub-basin (Fig. 2). Therefore, the outlet load is distributed throughout the watershed based

on sub-basin area. This process gives the larger sub-basins a larger incremental load; likewise, it gives the smaller sub-basins a smaller incremental load. The load allocations for each sub-basin are simply equivalent to the associated incremental TMDLs shown in Table 2.

Based on a physical inspection of the watershed, it is hypothesized that the decrease in pH in the stream is directly related to the oxidation of pyrite that occurs as runoff flows over the spoil areas associated with previous mining activities in the basin. Using the most recent monitoring data, inductive (regression) models were developed for each monitoring site. For brevity, only the model developed for sub-basins 2 (site P2) is shown in this paper (Fig. 3). A natural log transformation was applied to both flow and concentration values to obtain a linear relationship. The developed relationship may be used to predict ion concentrations in the stream on the basis of streamflow. As can be seen from Fig. 3, there is an inverse relationship between flow and hydrogen ion concentration, indicating a dilution effect at higher flows. It can be reasonably concluded that non-point sources are important because the dilution at higher flows is not as significant as it would be if a constant source was the only source of acidity, in which case the regression model would have a slope of -1.0.

It can be seen from Fig. 3 that the lower pH limit of 6.0 (corresponding to an ion concentration of  $0.001 \text{ g/m}^3$  or -6.9 on the log-scale) is violated at all reasonable flows, including the critical flow. Corresponding predicted hydrogen ion loads could be calculated by multiplying flows and concentrations. Application of this approach yields the predicted loads at critical flow for each site, as shown in Table 3. Note that for an independent tributary the incremental load is equal to the cumulative load for that tributary. In contrast, a sub-basin that has flows entering from adjacent or upstream sub-basins requires a mass balance application to find the incremental load. For example, the incremental load for sub-basin 2 is determined by subtracting the load for sub-basin 1 from the cumulative load for sub-basin 2.



Fig. 3. Flow vs H+ concentrations at site P2.

# Beech Creek Watershed

Table 3. Predicted H<sup>+</sup> loads (after [10])

Sub-basin	Cumulative $Q$ in cfs (m <sup>3</sup> /s)	Cumulative load in g/day	Incremental load in g/day	Required load reduction in g/day
1	0.78 (0.022)	1107	1107	1105
2	2.56 (0.073)	5372	4265	4261

The required load reduction for a watershed is the amount by which the actual in-stream load must be reduced in order to meet the TMDL. This is calculated by subtracting the incremental TMDLs (Table 2) from the incremental predicted loads for each sub-basin (Table 3). This approach allocates the total load reduction for Beech Creek (site P2) between each of the contributing sites in the watershed, so that the entire watershed is rehabilitated and the pH is improved throughout the stream network. Application of this approach yields the values of required load reductions in Table 3.

#### Results and analysis of the probabilistic approach

The probabilistic analysis outlined earlier is first performed to estimate the frequency of standard violations ( $F_c$ ) at the critical flow, and the uncertainty in this frequency. Then a target set of flow values is used to estimate the overall frequency of violations and the associated uncertainty. A set of 3650 values of historical flow was used in this analysis to represent a possible range of flows. The results indicate that there will be 100% violation of standards (i.e., pH < 6.0) within the entire range of ten-year flows because concentrations are far exceeding the permissible level (Fig. 3).

Load (non-point source mass input) reduction scenarios can be perceived as different slopes in the regression-based model, keeping the intercept (i.e., the concentration at flow where  $\ln Q \approx \text{zero}$ ) constant. It should be noted that the concentration of a point source pollution in a stream decreases linearly with increasing flow; i.e. doubling the flow causes the concentration due to the same load to decrease to half (i.e., slope of -1 of the regression line). Different levels of point source pollution are expected to generate similar regression lines (slope of -1) with different values of regression intercept. Based on the same logic, varying the levels of nonpoint source pollution means varying the slope of the regression line while keeping the intercept constant. Knowing that the H+ value of -6.9 secures compliance with the standard, load reduction scenario 1 based on the deterministic approach at  $Q_c$  can be represented as a new line passing through pH = 6.0 at  $Q_c$  (Fig. 4).

The probabilistic analysis based on distribution of residuals is performed with the new line representing the load reduction scenario 1. It is found that, at  $Q_c$  there are 50 % chances of violating the standards (i.e., pH < 6.0) due to the remaining variability. Apparently, this happens when a positive residual (i.e., residuals falling on the upper side of the regression line) causes the concentration to be higher than the deterministic value. The uncertainty about this estimate can be assessed by performing Monte Carlo simulation on the parameter uncertainty. A set of 1000 values of model parameters are generated using Normal distribution for the slope (m). The value of m is used as the mean value while the standard error (0.09) of the slope of the original regression equation (Fig. 3) is used as the standard deviation. Further, the overall exceedance frequency can be estimated using the 3650 range of flows. The expected exceedance ( $F_{\rm c}$ values averaged over the 3650 flow values) is found to be 68%, and the confidence of compliance (percent of times when  $F_c$  is less than 10%) is around 27% (Table 4). This is a nontrivial outcome of the probabilistic analysis that suggests that enforcing the single-valued TMDL based on the deterministic analysis [10] means that the pH level in the stream could be violated 68% of the time. This is not surprising since the critical flow is chosen to be the mean annual flow. Only 34% of the daily flows in the last ten years exceed the critical flow  $Q_{\rm c}$ , thus creating critical conditions (violation of standards) 66% of the time. Apparently, the confidence that pH could meet the standards (CC) is low (27%). The important point in this discussion is that a margin of safety is needed to reduce the risk of violation in the water body.

Table 4.

Load reduction scenario	pH at $Q_c$	Expected exceedances (%)	90% confidence interval	Confidence of compliance
Base case	3.3	100		0.0
Scenario 1	6.0	68	67–69	27
Scenario 2	6.5	54	53-55	42
Scenario 3	7.0	45	43-46	53
Scenario 4	7.5	38	37–40	59



Fig. 4. Four load reduction scenarios at sub-basin 2.

#### DISCUSSION

There is no doubt that engineering, at its core, is decision making [16]. Making decisions under uncertainty is inevitable for engineers during planning, analysis or simulation, design, implementation or construction, and management. Engineering students need to learn that, while applying the most accurate available methods for analysis and design, accuracy and uncertainty are not dichotomous. Walking the students through an example such as the one presented in this paper can help make this concept clear.

The 4261 g/day value of load reduction (Table 3) for sub-basin 2, recommended based on the deterministic approach, has been tested within the context of the probabilistic approach. The expected frequencies of violation (expected exceedances) discussed earlier, and presented as scenario 1 in Table 4, point out the need for considering other load reduction scenarios. Three more scenarios of additional load reduction are considered by changing the slope of the flow-concentration relationships (Fig. 4). The scenarios are designed so that the pH level is increased to 6.5, 7.0, and 7.5 at the critical flows for scenario 2, 3, and 4, respectively. These values correspond to ln ion concentrations of -7.88, -8.97, and -10.13 on the vertical scale of Fig. 4.

The Monte Carlo simulation performed with regard to scenario 1 was repeated with the other three scenarios for site P2. The results of the analysis are summarized in Table 4. For example, the expected exceedances at site P2 can be reduced from 68% (scenario 1) to 38% (scenario 4) by raising the pH level at the critical flow from 6.0 to 7.5. At this point the confidence of compliance increases from 27% to 59%. U.S. EPA guidelines allow up to 10% violation; therefore 10% can be interpreted as the recommended value of expected exceedance. Once the confidence of compliance or the expected exceedance is set in advance, the load reduction requirement can be quantified. The

probabilistic analysis summarized in Table 4 provides a deeper insight and more comprehensive perspective than that offered by the deterministic approach for the pH TMDL development.

Through this real example of surface water quality management, it can be made evident to senior engineering students that the deterministic approach is insufficient because it presents to decision makers and managers only what we know about a situation. The probabilistic approach, through the expected exceedance and the confidence of compliance indices, also presents what we are uncertain about. In engineering vocabulary, this can be translated into a quantifiable margin of safety to increase the recommended load reduction, and to increase the awareness about the possibility of violation even after management measures are taken.

The utility and the impact of the approach presented in this paper for teaching uncertainty has been assessed only based on the positive feedback and students' evaluation at the end of the uncertainty component of the course. However, it is planned and recommended to conduct an exercise-based assessment of the approach. Before teaching the uncertainty component, an exercise should be designed to urge students to make a management decision on a case study based on the results of the deterministic approach. Afterwards, students should be required to repeat the decision analysis process, taking into account the uncertainty-based results. In-class discussions and comparative analysis of the two separate decisions are expected to deepen the understanding and the applicability of the taught concept.

Finally, it should be noted that the probabilistic approach has been used in this paper to address the predictive uncertainty, but it is not the only candidate for the problem. Others have shown the applicability of the Bayesian approach [14] and the fuzzy logic concept [17, 18] for addressing uncertainty-related issues. However, the probabilistic

approach could be the most appropriate one for undergraduate students, since the other approaches are introduced only at the graduate level at many educational institutions.

### CONCLUSIONS

The predictive uncertainty encountered in engineering design and analysis is an important concept for civil engineering students. Developing accurate algorithms and models does not mean that uncertainty can be easily avoidable. The reallife illustrative example used in this paper from the field of water resources engineering demonstrated that simple probabilistic approaches are useful in addressing uncertainty issues. The approach presented in this paper is simple enough to suit senior level undergraduate engineering students and comprehensive enough to address the issue of uncertainty realistically. The example has the potential of clarifying to engineering students that we, as engineers, are better off presenting to decision makers what we are uncertain about. This does not compromise the accuracy of our techniques.

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Amin Elshorbagy is an Associate Professor of water resources engineering at the department of Civil and Geological Engineering, University of Saskatchewan, Canada. He conducts his research through the Centre for Advanced Numerical Simulation (CANSIM) that he established in 2004 to host his research group within the department. He is a licensed professional engineer and professional hydrologist. He has a Certificate in Higher Education Teaching (CHET) from the University of Manitoba. He received his B.Sc. degree in Civil Engineering from Ain Shams University, Cairo, Egypt, in 1989, his M.Sc. degree in Environmental Systems Monitoring and Analysis from ITC, the Netherlands, in 1994, and his Ph.D. in Civil and Geological Engineering from the University of Manitoba, Canada, in 2001.