Int. J. Engng Ed. Vol. 24, No. 3, pp. 540–544, 2008
Printed in Great Britain.

0949-149X/91 $3.00+0.00

In-class Paper Demonstrations and Experiments for Solid Mechanics Courses*

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Attending in-class paper demonstrations and performing experiments in solid mechanics courses are very effective ways for students to gain an understanding of the complicated concepts of mechanics. This paper explores a few applications of dogbone tensile tests, stress concentrations and crack kinking or mixed-mode fracture. Furthermore, this handy technique can be extended to other broader areas of mechanics education. Since only simple materials and supplies are used: copy paper, staples, scissors and a paper punching machine, students can repeat these typical mechanics experiments in future in other locations, such as in an office or at home. Therefore, this simple and effective technique can have a remarkable influence on the student's long-term career.

Keywords: mechanics education; mechanics of materials; solid mechanics; experimental mechanics

INTRODUCTION

THE AUTHOR'S first-year teaching assignments at Vanderbilt University included two graduate courses: Advanced Mechanics of Solids, Parts I and II. Part I mainly covers the theory of elasticity, while Part II ranges from plasticity, buckling, and stress waves to fracture mechanics. These highly theoretical courses require a strong mathematical background and the author found that many graduate students felt overwhelmed by the derivation of equations. Therefore, the author decided to add some experimental demonstrations to enable the students to understand complex mechanics phenomena. Before teaching the Rayleigh surface wave, the author showed a vivid movie on stress wave propagation (which can now be seen on the author's Website) based on the author's previous impact mechanics research using high-speed photography [1]. The movie captured the students' interest, hence they concentrated on the long derivation involving the speed relationship between the Rayleigh wave and the shear wave. It was then that the author recalled a famous maxim, "I hear and I forget; I see and I remember; I do and I understand," (Chinese philosopher and educator Lao Tsu, 604–531 BC). Therefore, he decided to develop a few in-class demonstrations, both for the two graduate courses mentioned above, and for two other undergraduate courses: Mechanics of Materials and Statics.

CURRENT CHALLENGE OF SOLID MECHANICS COURSES

Since traditional solid mechanics courses involve the successful applications of applied mathematics [2–3], they are challenging for the engineering students who have less mathematical training. For example, students who have never taken a Partial Differential Equation course would find that it is difficult to understand Boundary Value Problems (BVP, a pure mathematical terminology) in the theory of elasticity [4–5]. Therefore, for modern mechanics educators, the use of too much mathematical derivation without the provision of engaging examples may turn the students' interests to other fields that they perceive to be more fun; this does not help the development of mechanics disciplines. Fortunately, new information technology provides us with excellent tools for dynamic instruction and effective visualization.

The main objective of this paper is to demonstrate that a solid mechanics course is a series of dynamic and fun events, rather than a cluster of boring equations. Indeed, in his series of classical mechanics textbooks, outstanding mechanics researcher and educator, Professor S. Timoshenko, often used experimental stress techniques such as photoelasticity to explain complicated solid mechanics problems [4]. Now, modern multimedia could be used to realize this goal more effectively. Usually, we want to expose students to three areas of mechanics—experimentation, theory and computation [6]. The use of multimedia, such as personal computers and the Internet, can be very helpful in explaining theoretical and computational aspects of mechanics, and can even be used to conduct "virtual experiments" [7]. However, for undergraduate introductory level classes such as the mechanics of materials, the impact of multimedia on teaching fundamental concepts has been less successful [8]. Therefore, effective and original experimental education such as doing tensile tests in mechanics of material courses still cannot be replaced [9–11].

* Accepted 9th January 2007

IN-CLASS PAPER DEMONSTRATIONS AND EXPERIMENTS

For experimental demonstrations, not every educator has sufficient laboratory facilities or experience to conduct educational experiments such as the classical photoelasticity technique [9]. In order to develop a handy education technique, the author has explored an in-class paper (A4 copy paper, etc.) experiment, which can be employed not only in classroom teaching, but also for outreach presentations. Since the mechanics of paper is a typical plane stress problem, the author has extensively used A4 copy paper in classroom lectures for mechanics of materials (dogbone tensile tests), theory of elasticity (stress concentration around a central hole), plates and shells (Kirchhoff's assumptions) and fracture mechanics (crack kinking and fracture modes). For example, a classical mixed-mode fracture experiment [12] can easily be conducted and takes only a few minutes. For a laboratory experiment to demonstrate the same mixed-mode fracture phenomenon, the process may take at least several days, from specimen preparation to the final experiment. Moreover, the cost would be much higher than that of our paper experiment. In the following sections, we will describe in detail the steps of a few typical in-class experiments from specimen preparations to final experiments, and give the related theoretical background.

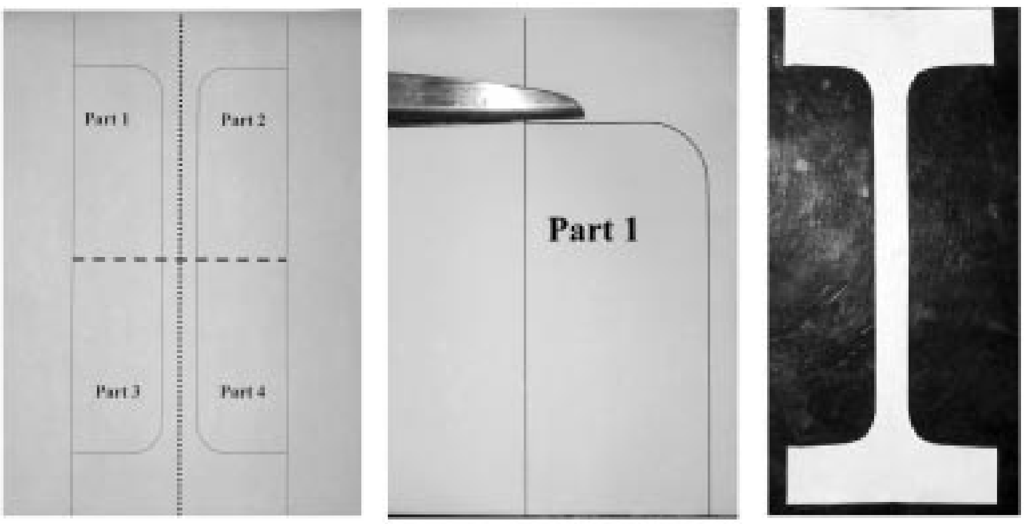
Applications in basic mechanics courses—dogbone tensile tests and stress concentrations

Traditional mechanics of materials courses mainly cover the mechanical response of a metal bar subjected to tension and other loadings. Therefore, the corresponding laboratory class is focused on the stress and strain measurements of a steel or aluminum cylinder specimen. With the extensive applications of new polymers and composite materials, as well as thin films [13], dogbone specimens have become quite popular for use in tensile experiments. Figure 1 shows the preparation process of a paper dogbone specimen. The first step is to print out symmetrical axes and specimen edges using computer software such as Microsoft Word. The second step is to fold the whole paper into a quarter so that only "Part 1" can be seen. Then, using scissors, the curved edges are carefully cut. Now a dogbone specimen is ready to be tested as seen in Fig. 1(c). For comparison, a rectangular paper specimen (a strip) is also cut. Students can pull the ends of these two kinds of tensile specimens and compare their failure patterns. The final fracture of the dogbone specimen is often at the specimen center (a valid tensile test). However, the location of the final fracture of the rectangular paper specimen is random, i.e., at the specimen center or the grip location, etc.

The dogbone specimen can be used to demonstrate another important solid mechanics phenomenon: stress concentration. As shown in Fig. 2(a), a paper punch is used to punch a hole in the dogbone specimen. According to classical elasticity solutions, for an infinite plate with a circular hole of radius, a, subjected to remote uniaxial tension, σ, the stress components along the hole edge expressed in a polar coordinate system are [4]

$$\begin{aligned}
\sigma_{rr} &= \frac{\sigma}{2}\left(1-\frac{a^2}{r^2}\right)+\frac{\sigma}{2}\left(1+\frac{3a^4}{r^4}-\frac{4a^2}{r^2}\right)\cos(2\theta) \\
\sigma_{\theta\theta} &= \frac{\sigma}{2}\left(1+\frac{a^2}{r^2}\right)-\frac{\sigma}{2}\left(1+\frac{3a^4}{r^4}\right)\cos(2\theta) \qquad (1)\\
\sigma_{r\theta} &= -\frac{\sigma}{2}\left(1-\frac{3a^4}{r^4}+\frac{2a^2}{r^2}\right)\sin(2\theta).
\end{aligned}$$

For $\theta = \pi/2$, $3\pi/2$, $\sigma_{\theta\theta}$ attains its maximum value 3σ. Here, "3" is called the stress concentration



(a) (b) (c)

Fig. 1. Dogbone tension specimen preparation using A4 copy paper: (a) mark fold lines on A4 paper using computer software such as Microsoft Word; (b) fold the whole paper into one quarter and cut paper using scissors; (c) a complete dogbone specimen.

factor. For a finite dogbone specimen with a central hole, the stress concentration factor is less than 3. By conducting tensile tests of the dogbone specimens with and without central holes, students will be able to notice that the dogbone specimen with a central hole often fails at the hole edge due to the high stress concentration. Because these elasticity equations are usually beyond the undergraduate student curriculum, the use of the stress flow concept is more effective to explain the stress concentration phenomenon. As with fluid visualization, the tensile stress of the dogbone specimen at the far field is quite uniform as shown in Fig. 2(b). However, the stress flow becomes "very crowded" when it encounters the central hole, since the stress flow cannot pass across the hole. Therefore, a stress concentration occurs at the location with a significant flow concentration as seen in Figure 2(b). This is a clear explanation of the stress concentration for undergraduate students (even for graduate students). The stress flow concept could be more effective if students have taken the fluid mechanics course, and know how to apply the continuity or mass equation.

Applications in advanced mechanics courses—fracture and fiber-reinforced composites

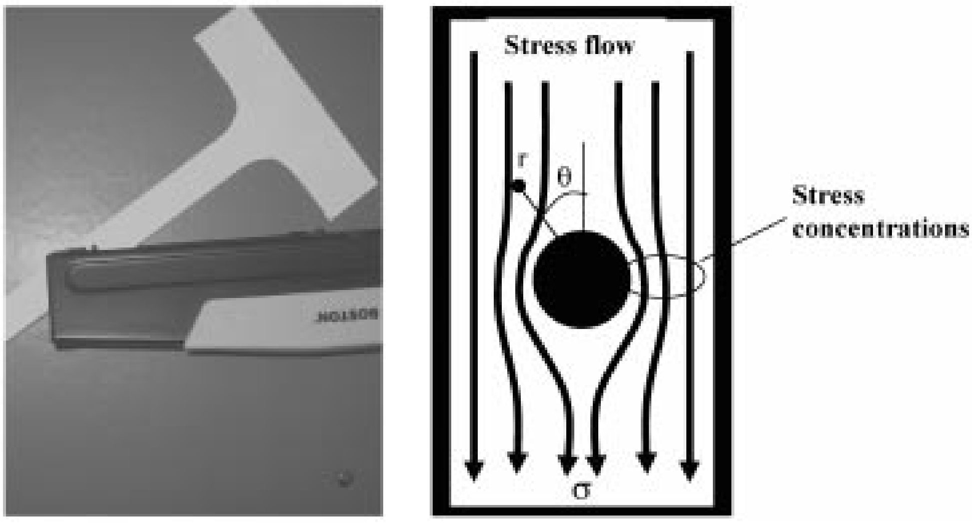
For advanced solid mechanics courses, fracture mechanics is one of the most important subjects. After the theory of elasticity and plasticity, the major challenge for graduate students is to understand the concepts of the singular stress field at a crack tip (theoretical stress values are infinity, based on the elasticity solutions), and the stress intensity factor of a crack in Linear Elastic Fracture Mechanics. In experimental mechanics, the singular stress field will result in a high stress concentration at the crack tip or bi-material corner [14]. Of course, a fracture experiment using photoelasticity or other techniques may lead to a much clearer understanding of fracture mechanics. However, many instructors do not have sufficient time and laboratory facilities to prepare these demonstrations. The author has developed a convenient paper experiment to demonstrate the concepts of the stress intensity factor and mixed-mode fracture, which are among the most important topics in fracture mechanics courses. As shown in Fig. 3, first we fold a piece of A4 copy paper and cut an initial 'crack', then we load the initial 'crack' by pulling on the paper. When the pulling load is sufficiently large, sudden crack kinking occurs, due to the mixed-mode load, as shown in Fig. 3(c). This experiment is easy to conduct and one can change the fold angle of the initial crack, and obtain different crack kinking patterns. Indeed, the crack kinking phenomenon involves an interesting theoretical background.

A crack will propagate away from its original path due to any change in local fracture mode mixity (a mechanics factor) or fracture toughness (a material factor). This phenomenon is often called "crack kinking or deflection" [15–18]. For a two-dimensional elastic solid, the full-field stress tensor of a mixed-mode main crack as seen in Fig. 3(b), can be expressed in a polar coordinate system as in Williams [19] and Anderson [20].

$$\sigma_{ij}^{m}(r,\theta) = \frac{K_I^m}{\sqrt{2\pi r}}\sum\nolimits_{ij}^{I}(\theta) + T\delta_{i1}\delta_{j1} \quad (2)$$

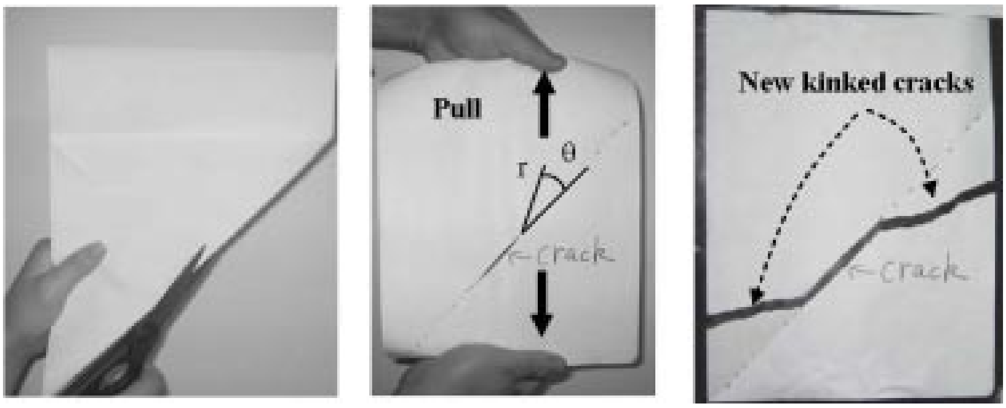
$$+\frac{K_{II}^m}{\sqrt{2\pi r}}\sum\nolimits_{ij}^{II}(\theta) + O(r^{\frac{1}{2}}) \qquad (i,j = 1,2)$$

where m and k denotes "main crack" and "kinked crack" respectively. K_{I} and K_{II} are mode I and mode II stress intensity factors; T is a nonsingular term; $O(r^{\frac{1}{2}})$ represents higher order terms of the length scale r and will be dropped if the kinked crack length l is very small; and known functions $\Sigma_{ij}^{I}(\theta), \Sigma_{ij}^{II}(\theta)$ represent the angular variations of



(a) (b)

Fig. 2. Stress concentrations during tension experiments: (a) punch a hole in a dogbone specimen using a paper punching machine; (b) stress flow around the circular hole.



(a) (b) (c)

Fig. 3. Paper experiment to demonstrate the crack kinking phenomenon: (a) folding paper and cut an initial 'crack'; (b) loading the initial 'crack' by pulling paper; (c) crack kinking due to mixed-mode loading.

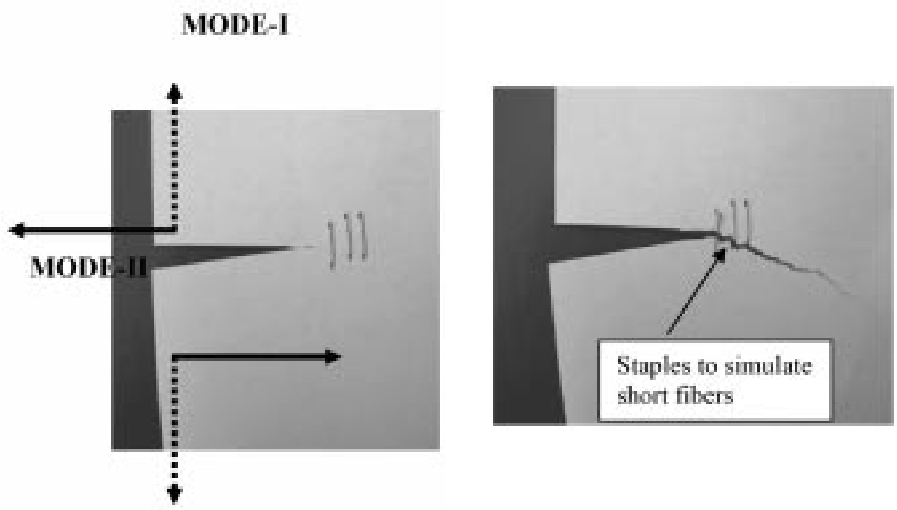
stress components. Previous research on crack kinking reveals that there is a relation between the stress intensity factors before and after crack kinking, which can be expressed in a combined way as [16, 21]:

$$K_I^k = c_{11}K_I^m + c_{12}K_{II}^m + b_1 T\sqrt{l} \qquad (3)$$

$$K_{II}^k = c_{21}K_I^m + c_{22}K_{II}^m + b_2 T\sqrt{l}. \qquad (4)$$

Numerous results have been reported to determine the coefficients c_{ij} and b_i $(i,j = 1,2)$. From the above relations it is obvious that the stress intensity factors of the kinked crack depend on the stress intensity factors of the initial main crack. Furthermore, we can use stress or energy criteria to predict the crack kinking initiation load and its angle of kink [12, 22]. Therefore, by using different kinds of paper materials and differently inclined angles of the main cracks, a course project on mixed-mode fracture allows students to obtain a deep understanding of the concepts of fracture mechanics. In addition to demonstrating the mixed-mode fracture phenomenon, paper with an edge crack as shown in Fig. 4(a) can be used to demonstrate mode I-opening, mode-II in-plane shear and mode-III out-of-plane shear, by holding the two edges and applying different kinks of load (in-plane tension; tearing, etc).

The edge-cracked paper could be slightly modified to demonstrate the mechanics of fiber-reinforced composites. As seen in Fig. 4(a), several staples fixed in front of a main crack are used to increase the local fracture resistance of the paper material. These stiff and strong staples are analogous to short micro/nano fibers used in composites [23–24]. In Fig. 4(b), after we pull the paper with an edge crack, the main crack kinks along the ends of the short staples, since the local fracture toughness



(a) (b)

Fig. 4. Demonstration of fracture modes and fiber-reinforced composites using paper and staples: (a) mode-I opening and mode-II in-plane shear modes; (b) short fibers can provide high fracture resistance.

of the paper is significantly increased by these strong staples. This experiment demonstrates a fracture mechanics conclusion: a crack tends to choose a path of low fracture resistance (toughness).

These paper experiments and demonstrations used in the classroom may have a long-term influence on the student's future career. Since we need only simple materials and supplies: copy paper, staples, scissors and paper punching machines, students can repeat these experiments elsewhere in the future, such as in an office or their homes. Therefore, the general public can easily understand the important role of solid mechanics in scientific research.

CONCLUSIONS

In-class paper experiments for solid mechanics courses are an effective way for students to learn to understand complicated mechanics concepts. This paper explores a few such applications in dogbone tensile tests, stress concentrations and crack kinking or mixed-mode fracture. This handy technique of solid mechanics education can be used to demonstrate the impact of solid mechanics research to the general public.

Acknowledgement—The author gratefully acknowledges the support from the National Science Foundation and the Office of Naval Research.

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