Chaos Training Boards: Versatile Pedagogical Tools for Teaching Chaotic Circuits and Systems*

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This paper introduces a versatile pedagogical kit that consists of two Chaos Training Boards and a virtual measurement system for experimentally studying chaos and chaotic dynamics in science and engineering undergraduate programs. The proposed Chaos Training Boards have been designed in a systematic way and have been implemented as educational tools so that science and engineering students can easily investigate autonomous, non-autonomous and mixed-mode chaotic dynamics.

Keywords: chaos; chaotic circuits and systems; education; training board; pedagogical tool

INTRODUCTION

THEORIES AND MATHEMATICAL MODELS for linear electronic circuits have been thoroughly developed and the dynamic behavior of linear circuits has been available since the 1920s. Consequently, the majority of laboratory based courses in Electrical and Electronics [EE] engineering undergraduate education are focused on linear electronic circuits. Laboratory experience plays a very important role in Electrical and Electronics Engineering education. Laboratories for linear electronic circuits are well equipped with a wide variety of laboratory tools at EE university departments all over the world. Traditionally, experimental circuits for laboratory courses are realized using discrete electronic components on an experiments board. However, this makes experiments time consuming, and leads to limited student comprehension of fundamental concepts, and analysis and interpretation of experimental data. To overcome this deficiency, in modern electronic circuits laboratories the experiments are realized using laboratory training boards that have been systematically designed to cover the curriculum of the laboratory course. In addition to these training boards, computer-aided virtual measurement systems are available for measurements, data recording and data analysis [1-4].

In the Electronic Circuits Laboratory, of the firstsemester junior EE laboratory course at Erciyes University, Turkey, a similar series of training boards are used. A sample board is shown in Fig. 1. By using this board, which consists of several preconstructed circuits, students can easily carry out the laboratory experiments related to operational amplifier-based applications such as an integrator and half-wave precision rectifier, but also analyze as well as interpreting the results within an adequate time frame.

In contrast to theoretical and experimental courses in linear electronic circuits, the theoretical and experimental course background is usually insufficient for nonlinear circuits and systems, especially chaotic circuits. Although there are several courses related to nonlinear circuits and systems in the curricula of EE education programs, these courses lack a systematic laboratory as is common for courses based on linear electronics circuits and systems. However, along with lectures and course topics on nonlinear circuits, especially chaotic circuits, various simulation and experimental demonstration studies have been introduced into student laboratories [5-10]. In these laboratories, while simulation-based studies have been generally realized by circuit analysis and mathematical/ numerical simulation tools, experimental studies generally include laboratory reports on a specific chaotic system that is modeled electronically. As mentioned above, although these theoretical and practical studies on chaotic circuits and systems are very useful, laboratory tools designed in a systematic way are required for introducing chaotic circuits and systems in graduate and undergraduate research and education programs. The purpose of this paper is to describe the pedagogical tools that have been designed in a systematic way and implemented as electronic-based training boards. In addition, we present a virtual measurement system for these chaos training boards experiments.

NONLINEAR CIRCUITS AND SYSTEMS COURSE AT ERCIYES UNIVERSITY: COURSE OUTLINE

Although our students had the necessary skills to study, analyze and design linear circuits, they



Fig 1. Linear electronics training board used in electronic circuit laboratory course at Erciyes University.

were not familiar with nonlinear dynamics, especially chaotic circuits. From the aspect of modern engineering and education, our EE undergraduate students need to learn not only the features of chaotic dynamics, i.e. how to control and avoid chaos, but also how to design chaotic circuits and implement possible engineering applications. To meet these requirements, five years ago we offered an elective course entitled 'Nonlinear Circuits and Systems'. The most important goals of the course were to introduce nonlinear circuit dynamics, which exhibit very complex and strange behavior as compared with linear circuit dynamics, and to demonstrate possible engineering applications of nonlinear circuits. At first, the course was not supported by a laboratory experiments program: the course contents were purely theoretical. However, we used some simulation tools for demonstration nonlinear circuit behaviors. We noticed that, without a laboratory, it was very difficult to attain the goals of our course. We tried to overcome this problem by adding some discrete laboratory experiments to the course. Although this provided a partial improvement, it did not provide a solution to enable us to realize our goals. Finally, we decided to design and implement a systematic laboratory for training chaotic circuits. We achieved this in one year by constructing two Chaos Training Boards. We started to use the boards in the 14-weeks course by arranging the course so that the first 10-weeks were devoted to theoretical lectures and the last 4weeks to laboratory studies. The revised course content is summarized in Table 1.

The new course supported by training boards seems to have been successful in its objectives and has received a very positive feedback from the students. Students stated that the training boards' experiments were very useful and helped them understand the circuit structure and complex dynamics of chaotic circuits. And most of the students stated that they would have liked to spend more time on the topics of chaos and chaotic circuits.

Table 1. Course content of nonlinear circuits and systems course at Erciyes University

Topics	Period
Linear and Nonlinear System Concepts	1 week
Linear and Nonlinear Circuit Elements	1 week
Linear and Nonlinear Oscillator Structures	1 week
Concepts of Chaos, Bifurcation and Fractal Geometry and some Applications	2 weeks
Nonlinear Resistor Structures and Chua's Diode	1 week
Chaotic Oscillator Structures	2 weeks
Control and Synchronization Applications of Chaotic Circuits	2 weeks
Lab 1_Plotting of DC characteristic of Voltage and Current mode nonlinear resistor structures	1 week
Lab 2_Experimental studies with chaos Training Board-I: The investigation of autonomous and non-autonomous chaotic dynamics.	1 week
Lab 3_ Experimental studies with chaos Training Board-I: The investigation of switched chaotic system and mixed- mode chaotic dynamics.	1 week
Lab 4_Experimental studies with Chaos Training Board-II: The investigation of different chaotic oscillator structures.	1 week



Fig. 2. Chaos Training Board-I and II.

Table 2.	Chaos	Training	Board-I	configura	ations
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Jumper adjustment on the board	Positions of the switches on the board	Chaotic circuit type formed by jumper adjustment	Oscillation mode
J3	Static switching S1-OFF/S2-ON	Chua's sircuit	Autonomous
J2	Static switching S1-ON/S2-OFF	MLC sircuit	Non- autonomous
J1	Dynamic switching	MMCC	Mixed-mode

After giving an outline of the course, we will describe the proposed Chaos Training Boards and sample laboratory experiments using these boards.

DESCRIPTION OF THE CHAOS TRAINING BOARDS

Photographs of the Chaos Training Board-I and II are shown in Fig. 2. These boards are the laboratory trainers that enable science and engineering students to investigate chaotic dynamics.

Chaos Training Board-I

Chaos Training Board-I consists of eight preconstructed circuit blocks. The numbered blocks on the training board are as follows:

- 1. Mixed-Mode Chaotic Circuit (MMCC) Part
- Switching Signal Unit—A square wave generator
- 3. Switching and Control Unit
- 4. Wien-bridge Oscillator
- 5. Current Feedback Operational Amplifier (CFOA)-Based Floating Inductance Simulator
- Current Feedback Operational Amplifier (CFOA)-Based Grounded Inductance Simulator
- 7. Voltage mode Operational Amplifier (VOA)-Based Nonlinear Resistor
- 8. CFOA-Based Nonlinear Resistor

The core of Training Board-I is the Mixed-Mode Chaotic Circuit (MMCC) [11] block shown in Fig.

3(a). The MMCC Circuit operates either in the chaotic regime determined by the autonomous circuit part, or in the chaotic regime determined by the non-autonomous circuit part, depending on the static switching. It is capable of operating in mixed-mode, which includes both autonomous and non-autonomous regimes, depending on dynamic switching. Because it possesses these versatile features, the MMCC circuit offers an excellent educational circuit model for studying and practical experimenting in chaos and chaotic dynamics.

As a result of the flexible and versatile design of the training board, the user can configure the main chaotic circuit block in two ways. First, the user can configure the circuit in a conventional way by placing discrete inductor elements in related sockets in the training board. As an alternative, the user can configure the chaotic circuit in an inductorless form by using CFOA-based grounded and floating inductance simulators and a Wien-bridge Oscillator block located to the left side of the board. In this inductorless configuration, the user can also use the Wien-bridge oscillator block instead of the LC resonator part on the training board. The switching control of the MMCC circuit is provided by a Switching and Control Unit on the training board. A 4016 IC was used as the switching element. A common static and dynamic (S/D) switching point, VQ, can be connected to three positions via J1, J2 and J3 jumper adjustments on the board. These jumper adjustments are summarized in Table 2.



Fig. 3. (a) MMCC circuit, (b) *i-v* characteristic of nonlinear resistor in MMCC circuit.

Chaos Training Board-II

Chaos Training Board-II consists of six preconstructed chaotic circuit blocks and two preconstructed inductance simulator blocks. The numbered blocks on the training board are as follows:

- 1. Chaotic Colpitts Oscillator
- 2. Transistor-based Non-autonomous Chaotic oscillator
- 3. Chaotic RLD Circuit
- 4. Chaotic Wien-bridge Oscillator
- 5. Integrator Based Chaotic Oscillator
- 6. (4-D) Chaotic Oscillator
- 7. CFOA-Based Floating Inductance Simulator
- 8. CFOA-Based Grounded Inductance Simulator

Chaos Training Board-II presents a collection of six chaotic circuits. These circuits are selected to illustrate the variety of ways in which chaos can arise in simple analog oscillator structures containing active elements, specifically BJT and Op-amp. The first chaotic circuit example on Training Board-II, shown in Fig. 4, is the chaotic Colpitts oscillator [12].

Undergraduate students are familiar with this oscillator circuit; this circuit is introduced as a sinusoidal oscillator circuit in analog electronic courses, but yet it can be driven to chaos. In a classical Colpitts oscillator circuit, a transistor plays the role of both the active amplifying device and the nonlinear element. So, the sinusoidal oscillator circuit in Fig. 4 is able to exhibit



Fig. 4. Chaotic Colpitts oscillator on Board-II.



Fig. 5. Transistor-based non-autonomous chaotic oscillator on Board-II.



Fig. 6. RLD chaotic circuit on Board-II.



Fig 7. Wien bridge-based chaotic oscillator on Board-II.



Fig 8. Four-dimensional chaotic circuit on Board-II.

nonlinear chaotic dynamics according to the parameter setting in the circuit structure.

The second example on Training Board-II, shown in Fig. 5, is the transistor-based non-autonomous chaotic circuit [13] coupling of an oscillator and a transistor. This is a very useful example to show that a coupling mechanism, consisting of two electronic circuits which are not in harmony, can exhibit chaotic dynamics.

The mechanism behind this circuit is based on a capacitor charging process by means of 'forward–reverse fighting' of the transistor.

Another example on the board is a non-autonomous chaotic circuit referred to as an RLD chaotic circuit [7]. As shown in Fig. 6, the RLD circuit consists of an AC voltage source, a linear resistor, a linear inductor and a diode that provides the nonlinearity in the circuit. One can observe in this circuit that the current can be chaotic while the input AC-voltage is a linear oscillator.

The other important circuit example on the board is the chaotic Wien-bridge oscillator [14] shown in Fig. 7.

Since introducing chaotic behavior in the Colpitts oscillator in [12], there has been increasing interest in investigating the possible chaotic behavior of the classical sinusoidal oscillators. In particular several Wien-bridge-based chaotic oscillators have been designed and studied in the literature. The chaotic Wien-bridge-based oscillator circuit on the board is one of the circuits proposed in literature. This chaos generator is formed by two circuit blocks using VOAs as active elements as shown in Fig. 7. The first block is a Wien bridge oscillator with the gain

$$K_1 = 1 + \frac{R_3}{R_4}$$

and the second block is a Negative Impedance Converter (NIC). In this design [11], by including a diode in the positive feedback loop of A2, the NIC is only activated when

$$V_{C3}(K_2 - 1) > V_D$$

where

$$K_2 = \left(1 + \frac{R_7}{R_8}\right)$$

is the gain of A2 and V_D is the forward voltage drop of the diode.

We configured this oscillator block on the board so that students can investigate this circuit in two oscillation modes: the linear sinusoidal mode and the nonlinear chaotic oscillation mode. This operation is controlled by a two-positioned jumper adjustment.

The fifth chaotic circuit example on the board, shown in Fig. 8, is a four-dimensional chaotic circuit [15], which consists of four energy-storage elements, a VOA-based NIC and a diode. This is a simple circuit structure for higher-dimensional chaotic circuits. Chaotic circuits in literature are generally based on a three-dimensional circuit structure. Chaotic phenomena in more than three- dimensional electrical circuits have been



Fig. 9. Integrator-based chaotic circuit on Board-II.



Fig. 10. Virtual measurement system used for chaos training boards experiments.

studied with great interest. In order to introduce such a higher dimensional chaotic circuit structure and related interesting phenomena, we placed this four-dimensional chaotic on circuit board-II.

The last chaotic circuit example on the board, shown in Fig. 9, is an integrator-based chaotic circuit [16]. This circuit is an inductorless chaotic circuit; it has an interesting circuit structure consisting of three integrator blocks with cascade connection and a feedback path. Integrator blocks have been constructed with CFOAs. As in the first training board, for configuring the chaotic circuits including an inductor element in inductorless form on the board, we placed CFOA-based grounded and floating inductance simulators to the right side of the second board.

EXPERIMENTAL STUDIES WITH THE WORK-BOARDS

Researchers interested in nonlinear science and chaos can use these work-boards for investigating autonomous, non-autonomous and mixed-mode chaotic dynamics. The configurations of the proposed work-boards are very simple and versatile. In this section, in order to show the versatility and efficiency of the boards, we give some experiment examples with the proposed boards. For monitoring chaotic dynamics in the time and frequency domain, and X-Y mode, various oscilloscope and spectrum analyzer models can be used in laboratory experiments. In our laboratory experiments, we used the virtual measurement system shown in Fig. 10.

This system is a PC-compatible virtual measurement system using a PC oscilloscope module [17]. The PC oscilloscope module incorporates a software program that turns into an oscilloscope and spectrum analyzer. This system is flexible, easy to use and has many advantages over conventional instruments, including multiple views of the same signal, and on-screen display of voltage and time. The features of the PC oscilloscope module used in



Fig. 11. Experimental measurements of MMCC circuit on the Board-I with autonomous mode configuration.

the virtual measurement system in our experiments are listed below:

Bandwidth
Sampling rate
Channels
Oscilloscope time bases
Spectrum ranges
200 MHz
10 GS/s
2 + Ext. trigger/signal gen.
1 ns/div to 50 s/div
0 to 100 MHz

1 MB

USB2.0

- Record length
- PC connection
- (USD1 1 commotible)
- (USB1.1 compatible)

EXPERIMENTS WITH CHAOS TRAINING BOARD-I

By applying the jumper adjustments defined in Table 2, the users can easily realize laboratory experiments on relevant chaotic circuits on Chaos Training Board-I.

By configuring the J3 jumper adjustment on the board according to Table 2, an autonomous Chua's circuit [18] configuration is obtained and the user can investigate the autonomous chaotic dynamics on this configuration. By adjusting the





(b)

Fig. 12. Experimental measurements of MMCC circuit on the board with non-autonomous mode configuration: (a) period-2 and (b) chaotic behavior.

 R_2 potentiometer in the MMCC circuit, the user can observe a Hopf-like bifurcation from DC equilibrium, a sequence of period-doubling bifurcations to double band Chua chaotic attractor and a boundary limit. Some experimental observations via the virtual measurement system in Fig. 10 for the autonomous mode of the board are shown in Fig. 11. These results illustrate a chaotic time series of voltage across of C_1 and C_2 in the MMCC circuit, the frequency response of V_{C1} and the chaotic attractor measured in V_{C1} – V_{C2} plane.

By configuring the J2 jumper adjustment on the board according to Table 2, the non-autonomous MLC circuit [19] configuration is obtained and the user can investigate non-autonomous chaotic dynamics on this circuit configuration. The V_{AC} sinusoidal signal is taken from the sine-wave output of an external function generator. Its amplitude and frequency are determined as $V_{\rm rms}$ = 100 mV and f = 8890 Hz, respectively. By adjusting the amplitude of the AC signal source and/or the R₁ potentiometer located in the nonautonomous part of the MMCC circuit, the user can easily observe the complex dynamics of bifurcation and the chaos phenomenon. Some experimental observations via the virtual measurement system in Fig. 10 for this non-autonomous mode of the board have been illustrated in Fig. 12.

By configuring the J1 jumper adjustment on the board according to Table 2, the dynamic and continuous switching operation is provided, and the mixed-mode chaotic phenomenon that includes both the autonomous and non-autonomous chaotic dynamics is observed. In this mode the switching time, which determines the durations of autonomous and non-autonomous chaotic oscillations, can easily be adjusted via the R_3 potentiometer located in the square-wave generator block on the board. By adjustments of the R_1 and R_2 potentiometers in the MMCC circuit, a variety of mixed-mode chaotic dynamics are observed. Some experimental observations via the virtual measurement system in Fig. 10 for this mixed-mode of the board are shown in Fig. 13.

EXPERIMENTS WITH CHAOS TRAINING BOARD-II

Since all chaotic circuit models on Training Board-II are mounted as pre-constructed circuit blocks, laboratory experiments related to these



Fig. 13. Experimental measurements of MMCC circuit on the board with the mixed-mode configuration



Fig. 14. Experimental measurements of Colpitts oscillator on the Board-II.



Fig. 15. Experimental measurements of transistor-based chaotic circuit on the Board-II: (a) period-2 and (b) chaotic behavior.

circuits are easily performed using the virtual measurement system. We will describe three sample experiments. Figure 14 shows measurements on the Colpitts oscillator.

These results illustrate the time series of voltage across of C_1 and C_2 in a Colpitts oscillator circuit, the frequency response of V_{C1} and chaotic attractor measured in V_{C1} - V_{C2} plane. The parameters of

the Colpitts oscillator have been adjusted so that the circuit oscillates in chaos mode.

In the transistor-based chaotic circuit model in Fig. 5, by varying the amplitude of the oscillator, some interesting complex dynamic series from periodic behaviors to chaotic behavior can be observed. Figure 15(a) shows measurement results illustrating the bifurcation phenomena and Fig. 15(b) shows measurement results illustrating the chaotic phenomena.

In the Wien-bridge-based chaotic circuit model, there are two oscillation modes for investigation. By configuring the J1 jumper adjustment on Board-II, a classical linear Wien-bridge oscillator configuration is obtained and the user can investigate linear sinusoidal dynamics in this configuration as shown in Fig. 16(a). By configuring the J2 jumper adjustment on Board-II, a nonlinear circuit block is coupled to a classical linear Wien-bridge oscillator configuration and this nonlinear oscillator oscillates chaotically. Figure 16(b) shows measurement results illustrating the chaotic phenomena.

SUMMARY

We have presented an educational set including two chaos training boards and a virtual measurement system for investigating chaotic dynamics in science and engineering undergraduate programs. A laboratory program devised with the proposed chaos training boards can easily be accompanied by nonlinear courses in science and electrical and electronics engineering departments. The proposed training boards with the virtual measurement system offer several advantages:

- Undergraduate students can investigate and study the autonomous, non-autonomous and mixed-mode chaotic dynamics by using the proposed boards.
- Students can compare alternative design and implementation ideas for nonlinear circuits and systems on the boards.
- There are grounded and floating inductance simulator blocks [20] on the boards. These blocks can be used for alternative inductorless realization of the chaotic circuits on the boards.



(b)

Fig. 16. Experimental measurements of Wien-bridge-based chaotic circuit on Board-II: (a)linear oscillation mode behavior; (b) chaotic behavior.

- Experimental configurations and board adjustments are very easy. Students can configure chaotic circuits on the board by using the connection wires and jumpers.
- Students can obtain time domain, frequency domain and *X*-*Y* projection measurements at the same time via a virtual measurement system and they easily manage these measurements on a computer screen.

We hope these boards and measurement configuration will be very useful as examples of implementing laboratory apparatus for nonlinear courses in science and engineering education programs.

Acknowledgement—The author would like to thank Professor Charles Yokomoto and Professor Maher Rizkalla from Purdue School of Engineering and Technology for their helpful discussions.

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