# The Mathematical Communication Competency in the Engineering Degrees: a Tool to Assess the Starting Point* 

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#### Abstract

In 2006, the authors of this article designed a survey to assess the mathematical profile of students beginning engineering degrees. The study presented here is the result of the idea of validating part of the questionnaire as a tool for measuring skills in the use of mathematical language. Thirteen questions have been considered, and the hypotheses tested were whether any questions were redundant and therefore could have been excluded, and whether correctly answering the questions classified beforehand as requiring a higher level of knowledge was related to correctly answering questions at lower levels.

The chi-square test was used to study the dependence of pairs of related items, and logistic regression was used to determine whether the results for some simple items conclusively explained the results in other more complex items. The study essentially confirmed that eleven of the thirteen questions are a useful instrument for measuring skills in communicating mathematical content among students beginning an engineering degree.


Keywords: assessment, first-year students, mathematical competency in engineering

## 1. INTRODUCTION

In 2006, 2007 and 2008, the authors of this study participated in the e-LKG Platform project, which was financed by the Spanish Government and coordinated by the University of Zaragoza. The project consisted of designing a new platform including knowledge management, group work and e-learning using open source software. It was developed by lecturers from the Applied Mathematics departments of several Spanish universi-ties-Universitat Jaume I (UJI), the Polytechnic University of Madrid (UPM) and the University of Zaragoza (UNIZAR), and software development companies also co-operated. The objective was to create an online course using the MOODLE platform in order to improve mathematical skills among newly admitted university students. More details on this project and on the collaborations

[^0]with foreign groups that came about while it was being undertaken can be found in [10] and [11].

Our work began by defining the curriculum anticipated for between Secondary and University Education (the 'Bachillerato') on a coordinated basis, taking into account the educational objectives of pre-university education and real classroom conditions. We considered the situation carefully, and exchanged ideas and experiences concerning the various levels of the Spanish educational system (in [8], Jennings presents a similar study only for a specific Australian university). This was not only in order to ascertain the basic mathematical content that students who have been recently admitted to a University engineering degree should be aware of, but also aspects related to the knowledge and handling of mathematical language, logical reasoning and deductive skills, reading comprehension, etc. It is obvious that all these aspects are skills that a good user of mathematics requires, and we therefore believe that they are essential for the study of scientific subjects
during a graduate's education. More specific studies can be found in [4], which analyses the influence of the Set Theory on learning algebra, and in [7], which examines the difficulties to connect real world applications of simple engineering concepts to the basic skills necessary to analyse those concepts in pre-engineering studies.

One of the first questions that needed to be answered was: is the curriculum studied by a student who has taken a higher secondary education course the same as the curriculum that would be desirable for a student coming to University for the first time? In other words, has the student covered the curriculum expected by the University in order to be able to make the most of his/her university education? In order to answer this question, we designed a questionnaire, the first version of which was tested at UNIZAR in September 2004. The test was designed to measure the profile of the student based on the following aspects: prior education, psychological aspects as a learner, communication skills in mathematics and application and use of mathematics. Analysis of the results of this test led to an improved questionnaire which was produced in September 2006. Both assessments of the curriculum were undertaken with first-year students on several courses at the universities involved in the project. As mentioned above, the objective was to obtain information on three aspects: the profile of the newly admitted student's attitudes, the learner's characteristics and mathematical profile.

A short description of the objectives of each part of the questionnaire follows:

- Attitudes profile. The traits that define a newly admitted university student's 'attitude' to mathematics. Special consideration was given to the aspects that we thought were most influential on academic performance, and we therefore measured: motivation, vocation and preference for the studies being undertaken; the valuation of mathematical training or instruction; feelings or emotions towards mathematics and the self-image as a mathematics learner.
- The learner's characteristics. The customs, habits and abilities as a learner of students starting university education. University lecturers' expectations for the learners passed on to us concern not only their knowledge, but also their 'maturity' in terms of organising their studies, having initiative, taking decisions, etc. In our assessment, we measured their autonomy as learners, skills in the use of new technologies and experience in teamwork.
- Mathematical profile. The starting point was the curriculum track followed by most students beginning an Engineering or Economics and Business Studies degree. We therefore used the mathematical objectives and content in the Royal Decrees regulating higher secondary education in technological, natural and health and
social sciences courses as a source of information on the mathematics curriculum.

The educational legislation currently in force mentions the desirability of developing skills in using mathematical language among higher secondary education students. We included the following aspects of this skill in the questionnaire: understanding and reading common mathematical symbols; converting everyday language into mathematical language and vice versa; understanding a phrase written with mathematical symbols and understanding common ways of expressing mathematics. Another aim was to observe the skills acquired by the students in terms of the use and recognition of some aspects of mathematics. The decision regarding what these aspects were going to be was only subject to a restriction imposed by the population that underwent the assessment: newly admitted students to Engineering degrees. The dispersed nature of the population being studied meant that it was necessary to select simple questions, which were basic from the mathematical point of view but truly important in their future education
The questionnaire consisted of 44 questions, and was completed in September 2006 by 335 newly admitted students on several degrees at the UJI, UNIZAR and UPM universities (more details can be found in [2]). The first six questions ascertained the student's academic profile prior to university. There were 16 questions numbered from 7 from 22 in the questionnaire, which were to ascertain the attitude and characteristics of the newly-admitted student. Finally, 21 questions (numbered from 23 to 44) assessed the mathematical profile. In specific terms, the objective of questions 23 to 35 was to ascertain the mathematical communication skills that students possess, to measure various levels of knowledge and use of the most well-known mathematical symbols in mathematical communication. This set of questions will hereinafter be referred to as the communication block (see Appendix). Questions 36 to 44 covered knowledge, understanding and skills in using some 'simple' mathematical ideas. The questions generally had four possible answers, only one of which was correct. Only questions 23 to 29 had an open response and differed from the general structure.

A complete study of the results obtained can be found in [3]. This study contains a refinement of the data obtained and a statistical analysis of the items. In specific terms, we present the analysis of the relations between the questions in the communication block in Sections 2 and 3. This is in order to ascertain the dependence between them shown by the sample, and as a consequence, to decide whether they measure different levels of knowledge or whether they are redundant questions, meaning that some can therefore be dispensed with. This procedure could help to complete the study of results in more general works such as the analysis of the concepts 'compe-
tence' and 'assessment' and their mutual relationship made by Højgaard in [6]. Finally, the conclusions drawn are presented in Sections 4 and 5.

## 2. STUDY OF DEPENDENCE BETWEEN THE ITEMS IN THE COMMUNICATION BLOCK

The mathematical communication block consisted of questions 23 to 35 . Questions 23 to 29 were open questions in which students were asked to translate seven mathematical symbols commonly used to communicate mathematical concepts into everyday language. The percentages of correct answers obtained are shown in Fig. 1.

The aim of questions 30 to 35 was to measure more in-depth knowledge of these symbols, i.e. whether the student was able to use them to understand the meaning of a phrase written in symbolic language, and to transcribe a phrase from everyday language to symbolic language. The symbols which are covered in questions 23 to 29 subsequently appear in some of questions 30 to 35 , as shown below:
A. Question 27 (symbol $\in$ ) $\rightarrow$ Question 30 (a phrase in everyday language which must be transcribed in symbolic language using only the symbol $\in$ ).
B. Question 29 (symbol $\varnothing$ ) $\rightarrow$ Question 34 (phrase describing the significance of the empty set).
C. Question 30 (use of the symbol $\in$ ) $\rightarrow$ Question 35 (use of several symbols, one of which is $\in$ ).

In the following study, we use $Q N$ to denote the dichotomy variable:
$Q N=\left\{\begin{array}{l}1, \text { if question } N \text { has been answered correctly, } \\ 0, \text { if question } N \text { has been answered incorrectly }\end{array}\right.$

$$
N=23,24, \ldots, 35
$$

The study of the relationship or lack thereof between some of these qualitative or categorical variables is carried out by analyzing contingency tables. The simplest situation that we present is the comparison between two qualitative variables with only two possible options as a response (zero or one), meaning that, in this case, the contingency tables are limited to two by two matrices.

Contingency tables provide very superficial information on the dependence of variables or lack thereof. In order to obtain a more reliable


Fig. 1. Percentage of correct answers to questions 23 to 29.
quantification of the statistical dependence between variables, it needs to be considered in more depth, and there are various analytical procedures that are more detailed than the tables in this respect. In our study, we use the chi-square test ( $\chi^{2}$ test) (see [5]) to decide on the dependence of the variables in the cases reviewed here. It is also possible to use the 'relative risk' $(R R)$ and 'odds ratio' $(O R)$ measures of association (see [1]).

### 2.1. Study of the relationship between items 27 and 30

The contingency table for the dichotomic variables: a correct or incorrect answer to question 27 and to question 30 is shown in Table 1.

We considered the relative frequencies in Table 1 as point estimators of the probabilities concerned. We found that the probability of a correct answer to question $27(Q 27=1)$ after an incorrect answer to question $30(Q 30=0)$ is

$$
\begin{equation*}
\operatorname{Prob}(Q 27=1 / Q 30=0)=0.5497, \tag{1}
\end{equation*}
$$

while the probability of a correct answer to question $27(Q 27=1)$ after a correct answer to question $30(Q 30=1)$ is

$$
\begin{equation*}
\operatorname{Prob}(Q 27=1 / Q 30=1)=0.8114 . \tag{2}
\end{equation*}
$$

As the latter probability is greater, it 'seems' that the probability of a correct answer to question 27 'depends' on a correct answer to question 30.

This dependence can also be quantified using the association measures $R R$ and $O R$ :

$$
\begin{equation*}
R R=\frac{\operatorname{Prob}(Q 27=1 / Q 30=1)}{\operatorname{Prob}(Q 27=1 / Q 30=0)}=1.48 \tag{3}
\end{equation*}
$$

As $R R$ is greater than 1 , the probability of a

Table 1. Contingency table for the variables $Q 27$ and $Q 30$

| $Q 30$ | 0 | 1 | Marginal $Q 27$ |
| :--- | :--- | :--- | :--- |
| $Q 27$ |  |  |  |
| 0 | $41(12.24 \%)$ | $46(13.73 \%)$ | $87(25.97 \%)$ |
| 1 | $50(14.93 \%)$ | $198(59.10 \%)$ | $248(74.03 \%)$ |
| Marginal $Q 30$ | $91(27.16 \%)$ | $244(72.84 \%)$ | 335 |

correct answer to question 27 is associated with the answer to question 30 , and it is 1.48 times more probable in $Q 27$ if a correct answer is given to $Q 30$ than an incorrect one.

$$
\begin{equation*}
O R=\frac{\operatorname{odds}(Q 27 / Q 30=1)}{\operatorname{odds}(Q 27 / Q 30=0)}=3.53 \tag{4}
\end{equation*}
$$

where,

$$
\begin{equation*}
\operatorname{odds}(Q 27 / Q 30=1)=\frac{\operatorname{Prob}(Q 27=1 / Q 30=1)}{\operatorname{Prob}(Q 27=0 / Q 30=1)} \tag{5}
\end{equation*}
$$

and

$$
\begin{equation*}
\operatorname{odds}(Q 27 / Q 30=0)=\frac{\operatorname{Prob}(Q 27=1 / Q 30=0)}{\operatorname{Prob}(Q 27=0 / Q 30=0)} \tag{6}
\end{equation*}
$$

In other words, the relationship between correct/ incorrect answers to $Q 27$ is 3.53 times more probable after a correct answer to $Q 30$ than after an incorrect one.

We used the $\chi^{2}$ test in order to confirm our suspicions, and to decide whether the dependence observed in the sample is statistically significant. Statgraphics software provides a $p$-value $<0.01$, and we can therefore state that the two variables are dependent with a reliability of over $99 \%$.

These data show that the result in question 27 depends on the result obtained in question 30 and, in short, that question 27 adds no extra information to that obtained with question 30. As a consequence, the sample supports the decision to remove question 27 from future questionnaires.

### 2.2 Study of the relationship between items 29 and 34

In this case, we have the following contingency table, referred to as Table 2. And the values of the association measures $R R$ and $O R$ are:

$$
\begin{equation*}
R R=\frac{\operatorname{Prob}(Q 29=1 / Q 34=1)}{\operatorname{Prob}(Q 29=1 / Q 34=0)}=2.67 \tag{7}
\end{equation*}
$$

A correct answer to question 29 is therefore 2.67 times more probable after a correct answer to question 34 than after an incorrect answer.

$$
\begin{equation*}
O R=\frac{o d d s(Q 29 / Q 34=1)}{o d d s(Q 29 / Q 34=0)}=4.59 \tag{8}
\end{equation*}
$$

The relationship between correct and incorrect answers to Q29 is therefore 4.59 times greater after a correct answer to Q34 than after an incorrect answer.
Using the $\chi^{2}$ test and the software, we obtained a $p$-value $<0.01$, and we can therefore state that the two variables are dependent with a reliability of over $99 \%$.

As a consequence, these calculations support the decision to remove question 34 or 29 from future questionnaires. In this case, it is advisable to eliminate question 29, as the answers confirm that many students find the symbol $\varnothing$ confusing. Indeed, the symbol $\varnothing$ is used with various meanings depending on the context. In fact it is used to denote the diameter of a circumference, the number 0 and the empty set.

### 2.3 Study of the relationship between items 30 and 35

The contingency table for this case is given as Table 3. The values of the association measures are:

$$
R R=1.12 \text { and } O R=1.54
$$

The result of the $\chi^{2}$ test for this case gives a $p$ value $>0.10$. This value shows that the hypothesis of the variables' independence cannot be ruled out.

The results do not appear to be clear enough to accept the dependence of the variables, as the values of the association measures are not much greater than 1 . In specific terms, the probability of a correct answer to question 35 after a correct answer to question 30 is almost the same (1.12) as after an incorrect answer. Furthermore, the $\chi^{2}$ test does not involve a decision. As a consequence, we cannot eliminate either of the two questions. In view of the correct answers to question 30 ( $72.84 \%$ )

Table 2. Contingency table for the variables $Q 29$ and $Q 34$

| $Q 34$ | 0 | 1 | Marginal $Q 29$ |
| :--- | :---: | :--- | :--- |
| $Q 29$ |  |  |  |
| 0 | $84(25.07 \%)$ | $107(31.94 \%)$ | $191(57.01 \%)$ |
| 1 | $21(6.27 \%)$ | $123(36.72 \%)$ | $144(42.99 \%)$ |
| Marginal $Q 34$ | $105(31.34 \%)$ | $230(68.66 \%)$ | 335 |

Table 3. Contingency table for the variables $Q 30$ and $Q 35$

| Q35 | 0 | 1 | Marginal $Q 30$ |
| :--- | ---: | :--- | :--- |
| $Q 30$ |  |  |  |
| 0 | $56(16.72 \%)$ | $35(10.45 \%)$ | $91(27.16 \%)$ |
| 1 | $124(37.01 \%)$ | $120(35.82 \%)$ | $244(72.84 \%)$ |
| Marginal $Q 35$ | $180(53.73 \%)$ | $155(46.27 \%)$ | 335 |

and the correct answers to question 35 (46.27\%) we can deduce that question 35 is more difficult to understand than question 30 , but dependence should not be accepted.

## 3. LOGISTIC REGRESSION ANALYSIS OF THE COMMUNICATION BLOCK ITEMS

The aim of questions 30 to 35 was to measure more in-depth knowledge of these symbols, i.e. whether the student was able to use them both to understand the meaning of a phrase written in a symbolic language, and to transcribe a phrase from everyday language to symbolic language. We will specifically analyse some dependences regarding questions 31 and 35 :
A. Questions $23(\forall)+24(\exists)+27(\in) \rightarrow$ Question 31 (a phrase written in symbolic language using the three symbols shown which must be interpreted to be converted into everyday language).
B. Questions $23(\forall)+25(\Rightarrow)+27(\in) \rightarrow$ Question 35 (a written phrase in everyday language which must be translated into the symbolic language using the three symbols shown).

In this section, we aim to analyse the dependence of one group of variables on another, such as in cases A and B mentioned above, using Logistic Regression Theory (see [9]).

### 3.1 Study of the relationship between items 23, 24 and 27 and item 31

Let us consider the dependent variable $Y=Q 31$ and let us suppose as a hypothesis that correct answers to questions 23,24 and 27 all have the same influence on a correct answer to question 31. As a consequence, we use the variable $X=Q 23+$ Q24+ Q27, the result of adding the number of correct answers to questions 23, 24 and 27 as the variable predicting the probability of a correct answer to question 31 .

Logistic regression theory provides us with a mathematical function in the shape of an S (also called a logistic curve) given by the following expression:

$$
\begin{equation*}
\operatorname{Prob}(Y=1)=\frac{1}{1+\mathrm{e}^{-(A+B X)}}=\frac{\mathrm{e}^{A+B X}}{1+\mathrm{e}^{A+B X}} . \tag{9}
\end{equation*}
$$

Using Statgraphics software, we obtain

$$
\begin{equation*}
\operatorname{Prob}(Y=1)=\frac{\mathrm{e}^{-1.00379+0.374377 X}}{1+\mathrm{e}^{-1.00379+0.374377 X}} \tag{10}
\end{equation*}
$$

Figure 2 represents the resulting logistic curve. With this function, we can calculate that the probability of a correct answer to question 31 ( $Y$ $=1$ ) is 0.2682 if none of the three other questions has been answered correctly ( $X=0$ ), 0.3476 if one question has been answered correctly ( $X=1$ ), 0.4366 if two questions have been answered


Fig. 2. Logistic model given by Equation (10). $\operatorname{Prob}(Y=1)$ is the probability of correct answer for $Q 31$ and $X$ is the number of correct answers obtained in response to $Q 23, Q 24$ and $Q 27$.
correctly $(X=2)$ and 0.5298 if all three questions have been answered correctly ( $X=3$ ). The points represent the values of $\operatorname{Prob}(Y=1 / X=0)$, $\operatorname{Prob}(Y=1 / X=1), \operatorname{Prob}(Y=1 / X=2)$ and $\operatorname{Prob}(Y=1 / X=3)$, which in this case are 0.2553 , $0.3962,0.4070$ and 0.5410 respectively.

Obviously, a correct answer to questions 23, 24 and 27 influences the probability of a correct answer to question 31, and as the number of correct answers increases, this probability also increases.
The coefficient 0.374377 given by logistic regression (see Equation (10)) is associated to the discrete variable $X$, the possible values of which are $0,1,2$ and 3 , and shows that the relationship between a correct and an incorrect answer in question 35 , i.e.

$$
\begin{equation*}
o d d s=\frac{p}{1-p} \tag{11}
\end{equation*}
$$

varies when $X$ is changed as follows:

$$
\begin{equation*}
O R=\mathrm{e}^{0.374377\left(X_{i+1}-X_{i}\right)}, i=0,1,2,3 . \tag{12}
\end{equation*}
$$

In this case, $X_{i}=i, i=0,1,2,3$.
As a result, the value $\mathrm{e}^{0.374377}=1.454$ measures the increase in the proportion of correct/wrong answers to question 31 if the number of correct answers to three possible questions ( 23,24 and 27) changes by 1 . In the event of correct answers to the three questions 23,25 and 27 , i.e. $X=3$, the odds of question 31 estimated by the model obtained is 1.23. Under these circumstances, the probability of a correct answer to question 31 is therefore 1.13 times greater than that of an incorrect answer.

As a consequence, assuming a dependence of the variables does not seem to be acceptable.

### 3.2 Study of the relationship between items 23, 25 and 27 and item 35 <br> In this case, $Y=Q 35$ is considered a dependent variable and $X=Q 23+Q 25+Q 27$ as an



Fig. 3. Logistic model given by Equation (13). $\operatorname{Prob}(Y=1)$ is the probability of correct answer for $Q 35$ and $X$ is the number of correct answers obtained in response to $Q 23, Q 25$ and $Q 27$.
independent variable and a predictor for the probability of a correct answer in question 35 .

Logistic regression theory gives us the following logistic curves:

$$
\begin{equation*}
\operatorname{Prob}(Y=1)=\frac{\mathrm{e}^{-0.856675+0.353528 X}}{1+\mathrm{e}^{-0.856675+0.353528 X}} \tag{13}
\end{equation*}
$$

Figure 3 shows the resulting logistic curve. Proceeding as in the previous section, we find that the probability of a correct answer to question 35 is 0.2980 if none of the three other questions has been answered correctly, 0.3768 if one of the three questions has been answered correctly, 0.4627 if two of the three questions have been answered correctly and 0.5508 if all three questions have been answered correctly. The points represent the values of $\operatorname{Prob}(Y=1 / X=0), \operatorname{Prob}(Y=1 / X=$ 1), $\operatorname{Prob}(Y=1 / X=2)$ and $\operatorname{Prob}(Y=1 / X=3)$, which in this case are $0.2632,0.3714,0.5447$ and 0.6800 respectively.

A correct answer to questions 23,25 and 27 therefore influences the probability of a correct answer to question 35 and once again, as the number of correct answers increases, so does the probability of a correct answer to question 35 .
The coefficient 0.353528 given by Equation (13) is associated with the discrete variable $X$, the possible values of which are $0,1,2$ and 3 , and shows that the relationship between a correct and an incorrect answer in question 35 , i.e. odds (see Equation (11)) varies when $X$ is changed as follows:

$$
\begin{equation*}
O R=\mathrm{e}^{0.353528\left(X_{i+1}-X_{i}\right)}, i=0,1,2,3 \tag{14}
\end{equation*}
$$

As a result, the value $\mathrm{e}^{0.353528}=1.424$ measures the increase in the proportion of correct/wrong answers to question 35 if the number of correct answers to three possible questions (23, 25 and 27) changes by 1 .

In the event of a correct answer to the three questions 23,25 and 27, i.e. $X=3$, the odds for
question 35 estimated by the model obtained is 1.23. Under these circumstances, the probability of a correct answer to question 31 is therefore 1.13 times greater than that of an incorrect answer.

As a consequence, assuming a dependence of the variables does not appear to be acceptable in this case either.

## 4. DISCUSSION OF RESULTS

The group containing questions 23 to 35 in the questionnaire we designed-a group we called a communication block-aims to estimate the development of skills in mathematical communication among young people joining Engineering and Economics degrees at the UJI, UPM and UNIZAR. The design used included seven questions, numbered from 23 to 29 , in which students were only asked to write the meaning of seven mathematical symbols, and which can therefore be classified as involving low levels of knowledge. As well as these questions, another six were formulated, numbered from 30 to 35 , covering the use or application of these symbols in phrases of varying complexity, and answering them correctly therefore required a higher level of knowledge.

The answers given to the questions in the communication block in September 2006 enabled a profile to be constructed of an average student joining the degree courses mentioned above, in terms of their skills in communicating mathematics, which was published in [3]. In the sections above, we tested the questions in the mathematical communication block in order to ascertain whether any of them are redundant and could therefore be removed without this reducing the information provided by the block of questions as it had initially been designed.

There is extensive literature on studies similar to the one analysed here, not only in education, such as multiple-choice examinations and others, but also in other areas such as health surveys, interviews in human resources departments in companies, in government, etc.

## 5. CONCLUSIONS

The results obtained and set out in the sections above lead to the following conclusions:

1. The study of independence, using the $\chi^{2}$ test, between the question pairs $(Q 27, Q 30),(Q 29$, $Q 34$ ) and ( $Q 30, Q 35$ ), shows that independence is statistically rejected in the first two pairs with a reliability of over $99 \%$, but not in the third. The quantification of the dependence between the variables in the first two questions using the odds ratio leads us to decide that questions 27 and 29 could be excluded from the questionnaire. The relationship between correct and incorrect answers to $Q 27$ is therefore 3.53
times greater after a correct answer to Q30 than after an incorrect answer (see Equation (4)) and 4.59 times to the pair $(Q 29, Q 34)$ (see Equation (8)).
2. Meanwhile, the logistic regression applied to questions 23,24 and 27 as variables predicting question 31 and also applied to questions 23, 25 and 27 with regard to question 35 produces a good fit in both cases. It is also clear in both cases that if a student gives a correct answer to all three questions, the probability of a correct answer to question 31 is almost the same as that of an incorrect answer (the probability of a correct answer is only 1.13 times higher) and the same is true of question 35 (the probability of a correct answer is only 1.23 times higher). As a consequence, none of the questions involved should be excluded.
3. The suitability of open questions, like those numbered from 23 to 29 , must be considered for future questionnaires measuring understanding of the concepts dealt with here. In these cases, automatic quantification is difficult.

In consequence, we consider that the eleven questions ( 23 to 35 except for 27 and 29) are a good instrument for measuring skill in using symbolic and normal language, and the communication of mathematical content among students beginning university studies.

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## APPENDIX: COMMUNICATION BLOCK ITEMS

23. Meaning of symbol $\forall$.
24. Meaning of symbol $\exists$.
25. Meaning of symbol $\Rightarrow$.
26. Meaning of symbol $\Leftrightarrow$.
27. Meaning of symbol $\in$.
28. Meaning of symbol $\subset$.
29. Meaning of symbol $\varnothing$.
30. Indicate which of the following mathematical expressions symbolises the phrase ' $x$ is an element of set A ': (a) $x \subset \mathrm{~A}$, (b) $x \in \mathrm{~A}$, (c) $x \Rightarrow \mathrm{~A}$, (d) $x \Leftrightarrow \mathrm{~A}$.
31. Which of the following sentences show the correct meaning of the expression: ${ }^{\bullet} \forall x \in, \mathrm{R}, \exists n \in \mathrm{~N}$ such that $n>x$ ?
(a) There are real numbers with natural numbers greater than it.
(b) For every $x$ belonging to real numbers, there is $n$ which belongs to natural numbers such that $n$ is greater or equal to $x$.
(c) For every $x$, R exists and N exists such that $n$ is greater or equal to $x$.
(d) Every real number has a natural number greater than it.
32. Accept that the following statement is true: 'If a streetlight is more than 7 metres tall, then it is blue'. Indicate which of the following phrases is also correct:
(a) If a streetlight is blue, then it is more than 7 metres tall.
(b) If a streetlight is less than 7 metres tall, then it is not blue.
(c) If a streetlight is not blue, then it is 7 metres tall or less.
(d) If a streetlight is not blue, then it is more than 7 metres tall.
33. Given the equality $(2 x-1)+2 x+(2 x+1)=246$, where $x$ is a natural number. Which of the following sentences expresses its meaning in everyday language?
(a) There is an even number which, when added to the previous number and the subsequent number, makes 246.
(b) There is an odd number which, when added to the previous number and the subsequent number, gives 246.
(c) There is an even number which, when added to the two following numbers, gives 246 .
(d) The sum of three consecutive numbers is always 246 .
34. Indicate the correct way of continuing the phrase starting: The empty set . . . .
(a) is number 0 , (b) only contains 0 , (c) has all elements, (d) has no elements.
35. Let us consider a bag of fruit and call all pears A and all green fruit B. To indicate that pears are green, we will write:
(a) $x \in \mathrm{~A}$ given that $x \in \mathrm{~B}$.
(b) $\exists x \in \mathrm{~B}$ given that $x \in \mathrm{~A}$.
(c) $\forall x \in \mathrm{~A} \Rightarrow x \in \mathrm{~B}$.
(d) $\forall x \in \mathrm{~B} \Rightarrow x \in \mathrm{~A}$.

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[^0]:    * Accepted 16 August 2010.

