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Statistical Analysis of Academic Results Before and After Four Years of Bologna*

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In this paper, a statistical analysis of the performance of telecommunication engineering students who were majoring in electronic systems in the subject Analysis of Circuit I is conducted. Here, the marks of the students before and after four years of being subject to a student-centered teaching and learning approach, implemented as part of the changeover to the Bologna model, are compared. In this research, the statistical population (i.e., marks of the students in 2005 and 2009) were analyzed in detail in order to identify the specific changes after four years of 'Bologna'. Mann–Whitney and proportion tests were used to analyze the effects of the implementation of the Bologna process, which resulted in an improvement in the academic results of the students. In addition, models for the marks of the years 2005 and 2009 were constructed. These models were very different from each other. A mixture of Gaussians was used to model the 2005 marks, and the 2009 marks followed a Gaussian distribution. Thus, before the start of the educational experiment, in the year 2005, the statistical population was heterogeneous, consisting of two subpopulations, and at the end of it the data resulted in being homogeneous. Therefore, the Bologna process has represented a passing from an unsatisfactory model for the 2005 marks to a more 'reasonable' model for the 2009 marks.

Keywords: non-parametric tests; proportion tests; statistical modeling; Gaussian mixture models

1. INTRODUCTION

THE BOLOGNA PROCESS has changed the way of teaching and learning at higher education level in many European countries. This process has been a turning point that has made tens of countries move toward the European Higher Education Area in few years. The main actors in this are ministries of education, representatives of European universities, higher education institutions, students, quality assurance agencies, and international organizations. However, at its most basic level both students and instructors are the ones who make it possible to change the higher education system for the better.

In this sense, this paper shows a statistical analysis of the academic results of telecommunication engineering students who were majoring in electronic systems in the subject Analysis of Circuits I (AC-I), in the EUIT de Telecomunicación at the Universidad Politécnica de Madrid (EUITT-UPM) during the years 2005 and 2009.

Here, it was decided to analyze the academic results of 2005 and 2009 because that interval of time represents four years of implementation of the educational experiment, carrying out research on continuously improving the results each academic year. Thus, four years after the beginning of this experiment, it can be said that the educational experiment can be modeled as a system in a steady state.

Other statistical analysis in the area of education can be found in [1–3]. However, the main objectives of those papers are very different from those of this paper as they are focused on solving other research problems using other methods.

The statistical analysis conducted in the present paper allowed us to use a magnifying glass to look at the academic results of the students, model the statistical population, and quantify the proportions of students in the low-marks interval (from 0 to 4), the medium-marks interval (from 4 to 6) and the high-marks interval (from 6 to 10). The pass mark is 5 out of 10.

Partial results of this research have been focused on presenting the earliest stages of the educational experiment [4], years 2006 and 2007, the first two years of a holistic experience within the first-year course at the EUITT-UPM, in which six out of nine departments of the school participated actively in it and important decisions on some important issues were made. For instance, determining the student workload and its translation to European Credit Transfer and Accumulation System (ECTS), developing new educational methods that guarantee perfect harmony among all the subjects, promoting tutor session, applying the same evaluation methods in all the subjects,

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setting the standards of using the Virtual Learning Environment, and strengthening the cooperation among all the subjects taught during the academic year.

In addition, in [4] the academic year workload in hours in accordance with the European Higher Education Area was shown, the credits were converted into ECTS credits, and the ECTS credits that each subject has were shown.

In [5], the statistical analysis of the educational experiment carried out in the subject AC-I in the year 2008 was shown. Also, the materials, units of work of AC-I, and common teaching and learning methodologies used in the subject were presented.

Moreover, in [6] a multiple contrast was conducted. There, it was shown that there were differences among all the years under analysis (2004, 2005, 2006, 2007, 2008 and 2009) and the statistical analysis continued with the study of the academic years in which it can be said that there were significant differences. To that end, a pairwise permutation test was applied to all possible pairs of groups and, as a result, for the data under analysis it was found that there were significant differences among the final marks of the students. Finally, in [6] all the years under analysis were grouped into three groups consisting of years that were not significantly different from each other.

In the present paper, it was decided to analyze the statistical population (i.e., marks of the students in 2005 and 2009) in detail in order to see the specific changes after four years of Bologna. To that end, a group of telecommunication students majoring in electronic systems was chosen at random, all the students had similar characteristics. Here, both a comparative analysis and a statistical modeling of the marks of the students in 2005 and 2009 were carried out. The number of students in 2005 was 85 and in 2009 it was 61.

In the comparative analysis, Mann–Whitney and proportion tests were used to analyze the presence of effects of the implementation of the Bologna process, which resulted in a shift of location of the marks of AC-I and differences between marks 2005 and marks 2009 were detected.

In the statistical modeling, models for the marks of the years 2005 and 2009 were found. These models were very different from each other. A mixture of Gaussians was used to model the marks of the year 2005 and the marks of the year 2009 followed a Gaussian distribution.

2. COMPARATIVE ANALYSIS BETWEEN MARKS IN 2005 AND MARKS IN 2009

2.1 Exploratory Data Analysis (EDA)

The first step was to conduct an EDA. EDA employs both numerical and graphical methods to both better understand the complexity of the relationship among data and probe the validity of assumptions that are made by formal statistical tests. A combination of numeric and graphical methods leads to boxplots, summary statistics, histogram and density estimate plots, normal probability plots and beyond [7, 8].

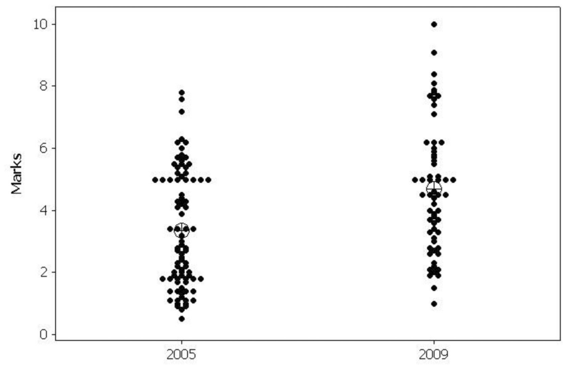

Fig. 1. Individual value plot of Marks2005 and Marks2009.

To that end, the R system for statistical computing and the software Minitab were used [9, 10], the data set was loaded: Marks2005 and Marks2009. Figure 1 shows the individual value plots of Marks2005 and Marks2009.

From Fig. 1, it can be seen that the range of the marks of the years 2005 and 2009 are different from each other. In 2005, it can be observed that there is a concentration of marks below 2, that there are intervals without any mark and the appreciable presence of ties. Also, in Fig. 1 the mean value of both Marks2005 and Marks2009 are highlighted, the mean value of Marks2005 being less than the mean value of Marks2009.

Table 1 shows some summary statistics (minimum, maximum, quartiles, median, mean, standard deviation, skewness and kurtosis).

Despite the fact that the standard deviation of Marks2005 is less than that of Marks2009, the coefficient of variation of Marks2005 is greater than that of Marks2009 (that is, 55.9 % in 2005 and 43.9 % in 2009). Therefore, the relative dispersion in 2005 is greater than that of 2009. Furthermore, the kurtosis indicates heterogeneity in 2005 and the skewness is an index of a stretching of the marks to the right of the mean value.

Figure 2 shows the histogram and density estimate of the marks of the years 2005 and 2009.

From Fig. 2, it can be observed that there is

Table 1. Summary statistics

	Marks2005	**Marks2009**
Min.	0.50	1.00
1st Qu	1.80	3.10
Median	2.80	4.50
Mean	3.34	4.69
3rd Qu	5.00	5.90
Max	7.80	10.00
Sd	1.87	2.06
Sk	0.40	0.45
K	1.97	2.49

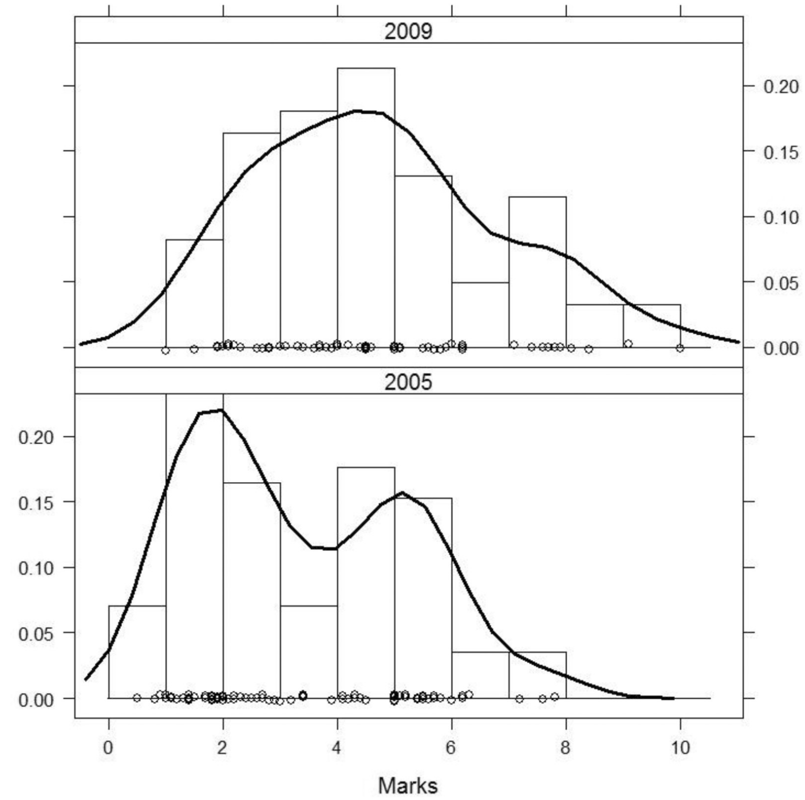

Fig. 2. Histogram and density estimate of the marks of the years 2005 and 2009.

bimodality in Marks2005 and symmetry around the modes. Moreover, it is corroborated that there is a certain asymmetry in Marks2009.

2.1.1 Normality test

In order to check whether the sample data came from a normal population, the Anderson–Darling (AD) and Kolmogorov–Smirnov (KS) tests [11, 12] were conducted. Both of them are empirical cumulative distribution function (ECDF) based tests.

The Normal Probability Plot of Marks2005 and Marks2009 along with the approximate 95% confidence intervals for the percentiles are shown in Figs 3 and 4, respectively.

From Fig. 3, it can be seen that Marks2005 did not follow a normal distribution. Table 2 shows that the results of the AD and KS tests confirmed this statement.

From Fig. 4, it can be seen that Marks2009 did follow a normal distribution. Also, Table 3 shows that the results of the AD and KS tests confirmed this statement.

2.2 Two Sample Location Problem

This subsection was aimed at analyzing the presence of effects of the implementation of the Bologna process that resulted in a shift of location for the marks of AC-I.

Here, a non-parametric test for equality of medians was carried out because Marks2005 did not follow a normal distribution.

First, the equality of variance in Marks2005 and Marks2009 was assessed. To this end, Levene's test was used [13, 14]. This test is used to assess the equality of variance in different samples when the data come from a continuous, but not necessarily normal, distribution.

2.2.1 Levene's test

$H_0 : \sigma_1^2 = \sigma_2^2$
$H_1 : \sigma_1^2 \neq \sigma_2^2$

where σ_1^2 and σ_2^2 stand for variances of Marks2005 and Marks2009, respectively.

Value of the statistic = 0.002 p-value = 0.895.

Table 2. Normality test Marks2005

	AD	KS
Statistic	2.213	0.134
p-value	< 0.005	< 0.010

Table 3. Normality test Marks2009

	AD	KS
Statistic	0.558	0.543
p-value	0.144	> 0.150

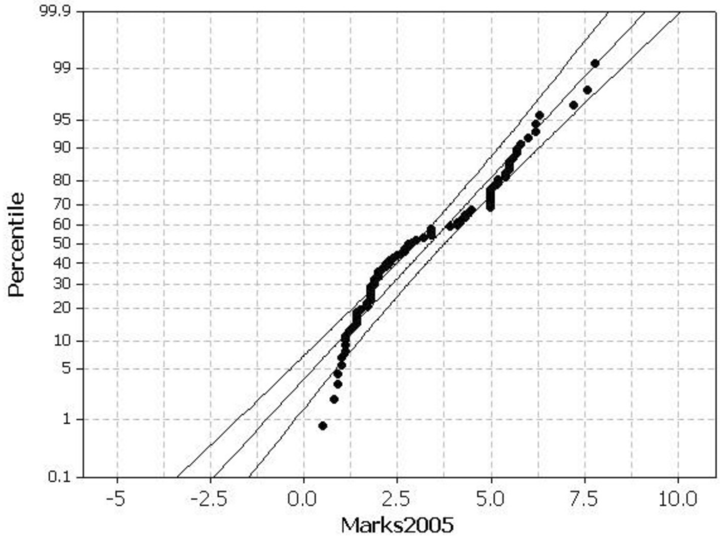

Fig. 3. Normal Probability Plot Marks2005.

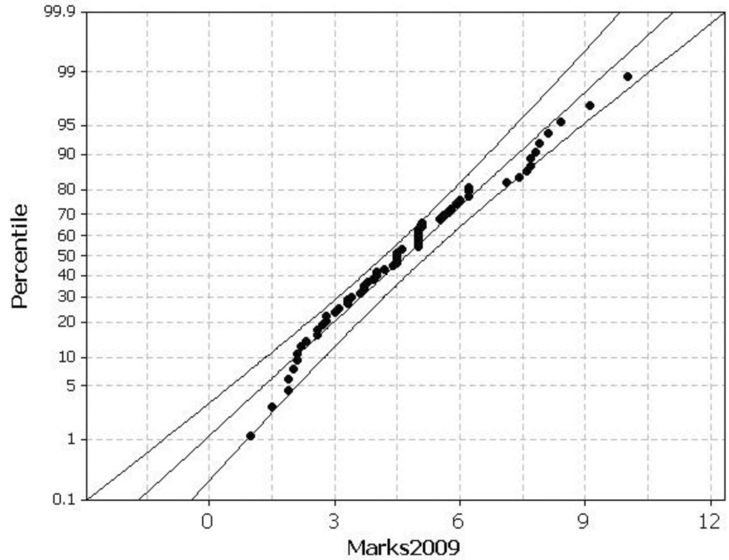

Fig. 4. Normal Probability Plot Marks2009.

Therefore, there was no evidence to reject the null hypothesis. That is, these data did not provide enough evidence to claim that the two populations have unequal variances.
Second, the Mann–Whitney test [11, 15, 16] was conducted in order to determine whether Marks2005 and Marks2009 had the same population median, m.

2.2.2 Two-sample Mann–Whitney test

Let m_1 and m_2 be the medians of Marks2005 and Marks2009, respectively, then

- 2-tailed test

$H_0 : m_2 - m_1 = 0$

$H_1 : m_2 - m_1 \neq 0$

Value of the statistic = 5435.0 p-value = 0.0002.

Therefore, based on the evidence, the null hypothesis was rejected for a significance level $\alpha = 0.05$.

A 95% confidence interval for the difference in the medians was (0.6, 2.1).

Therefore, due to the confidence interval that was obtained (0.6, 2.1) R^+, the next 1-tailed test was carried out:

- 1-tailed test

$H_0 : m_2 - m_1 \leq 0$

$H_1 : m_2 - m_1 > 0.$

p-value = 0.0001.

Therefore, there was evidence to reject the null hypothesis and to accept the alternative hypothesis. The median of Marks2009 was greater than that of Marks2005.

2.3 Testing the Equality of Parameters in Two Bernoulli Populations

This subsection was aimed at carrying out a deeper analysis of the behavior of Marks2005 and Marks2009 that was detected in the previous test. In short, the range of data in which differences could be observed and their respective signs were studied here.

To that end, the scale of marks, which is in the form 0 (the lowest) to 10 (the highest), was coded as follows:

A: Marks in the interval [0, 4]
B: Marks in the interval (4, 6)
C: Marks in the interval [6,10]

Table 4 shows the number of marks in each group.

Next, the proportions of marks of 2005 and 2009 were compared with each other by using tests for proportions [16, 17].

2.3.1 Test for proportions in group A

- 2-tailed test

Let p_1 and p_2 be the proportions of marks in group A in 2005 and 2009, respectively, then:

$H_0 : p_2 - p_1 = 0$

$H_1 : p_2 - p_1 \neq 0.$

Z = –2.10 p-value = 0.036.

Therefore, for a confidence level α = 0.05 there was sufficient evidence to reject the null hypothesis.

A 95% confidence interval for the difference in the proportions was (–0.335781, –0.0117599).

Therefore, due to the confidence interval that was obtained, the next 1-tailed test was carried out:

- 1-tailed test

$H_0 : p_2 - p_1 \geq 0$

$H_1 : p_2 - p_1 < 0.$

p-value = 0.018.

Therefore, there were evidences to accept the alternative hypothesis. The proportion of marks of group A in 2009 was less than that in 2005.

2.3.2 Test for proportions in group B

Let p_1 and p_2 be the proportions of marks in group B in 2005 and 2009, respectively, then:

$H_0 : p_2 - p_1 = 0$

$H_1 : p_2 - p_1 \neq 0.$

Z = 0.13 p-value = 0.896.

Therefore, the data did not support the hypothesis that there was a difference. In group B, both proportions of marks were equal to each other.

Table 4. Number of marks for each group

Group	**Marks2005**	**Marks2009**
A	51	26
B	27	20
C	7	15
Total	85	61

A 95% confidence interval for the difference in the proportions was (–0.143640, 0.164083).

2.3.3 Test for proportions in group C

- 2-tailed test

Let p_1 and p_2 be the proportions of marks in group C in 2005 and 2009, respectively, then:

$H_0 : p_2 - p_1 = 0$

$H_1 : p_2 - p_1 \neq 0.$

Z = 2.61 p-value = 0.009.

Therefore, for a confidence level α = 0.05 the null hypothesis was rejected.

A 95% confidence interval for the difference in the proportions was (0.0406952, 0.286402).

Therefore, due to the confidence interval that was obtained, the next 1-tailed test was carried out:

- 1-tailed test

$H_0 : p_2 - p_1 \leq 0$

$H_1 : p_2 - p_1 > 0.$

p-value = 0.005.

Therefore, there were evidences to reject the null hypothesis and to accept the alternative one. The median of Marks2009 was greater than the one of Marks2005.

In a complementary way to the previous tests for proportions, a confidence interval for each of the six individual proportions per year and per group was constructed. These confidence intervals are shown in Fig. 5.

Therefore, between the years 2005 and 2009 there was a shift to the right in the marks of the students and this shift occurred without changing the proportion of marks in the medium-marks interval. From 2005 to 2009 the percentage of low-marks decreased and the percentage of high-marks increased.

3. STATISTICAL MODELING OF MARKS2005 AND MARKS2009

This section aims to look for the probability distribution that characterizes best the data under analysis.

The univariate normal distribution suffers from several significant drawbacks when it is used to model data sets. For the case of Marks2005, due to the heterogeneity of the statistical population, the

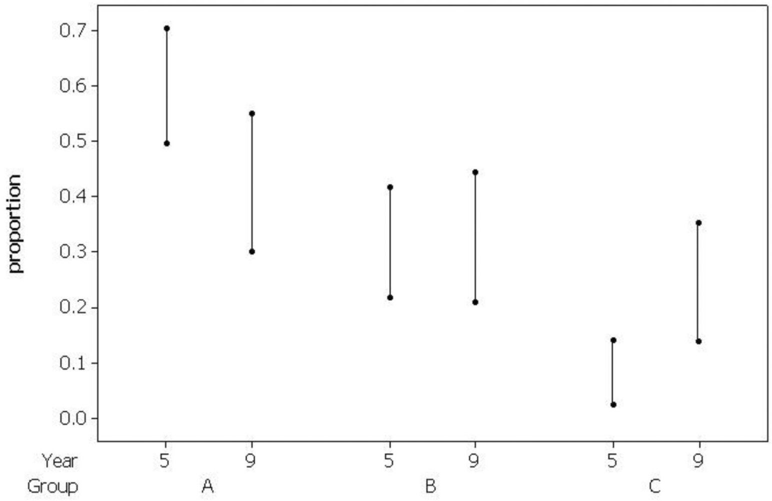

Fig. 5. Confidence interval for individual proportions.

univariate normal distribution cannot give a satisfactory characterization of the data set; however, the use of a linear superposition of two normal populations can give a better characterization of the data. Such a superposition is known as two-Gaussian mixture distribution.

3.1 Statistical Modeling of Marks2005

For Marks2005, given the histogram and density estimate (see Fig. 2), kurtosis $k < 2$ (see Table 1), which indicates heterogeneity [7], and taking into consideration that Marks2005 did not follow a normal distribution (Fig. 3), a distribution to the data was adjusted and, due to the bimodality and symmetry around the modes, a two-Gaussian distribution mixture was used.

Thus, Marks2005, X, was modeled as a mixture of two normal distributions $X_1 \sim N(\mu_1, \sigma_1^2)$ and $X_2 \sim N(\mu_2, \sigma_2^2)$. That is [18]:

$$X = (1 - \Delta)X_1 + \Delta X_2, \tag{1}$$

where $\Delta \in \{0, 1\}$ with $\mathrm{P}(\Delta = 1) = \pi$.

$$f_{X_i}\left({}^{x}/_{\theta_i}\right), i = 1, 2$$

was denoted as the density function of the normal distribution with parameters $\theta_i = (\mu_i, \sigma_i^2)$, $i = 1, 2$. Then, the density of X was

$$f_X\left({}^{x}/_{\Theta}\right) = (1 - \pi), f_{X_1}\left({}^{x}/_{\theta_1}\right) + \pi f_{X_2}\left({}^{x}/_{\theta_2}\right) \tag{2}$$

where $\theta_1 = (\mu_1, \sigma_1^2)$, $\theta_2 = (\mu_2, \sigma_2^2)$ and $\Theta = (\theta_1, \theta_2, \pi)$.

Each normal density $f_{X_i}\left({}^{x}/_{\theta_i}\right)$ was called a component of the mixture.

There were two possible models:

$$M_1 : f_X\left({}^{x}/_{\Theta}\right) = (1 - \pi) f_{X_1}\left({}^{x}/_{\theta_1}\right) + \pi f_{X_1}\left({}^{x}/_{\theta_2}\right), \quad \sigma_1 \neq \sigma_2 \tag{3}$$

$$M_2 : f_X\left({}^{x}/_{\Theta}\right) = (1 - \pi) f_{X_1}\left({}^{x}/_{\theta_1}\right) + \pi f_{X_1}\left({}^{x}/_{\theta_2}\right), \quad \sigma_1 = \sigma_2 \tag{4}$$

For model M_1, for maximum likelihood, both the parameters $\mu_1, \mu_2, \sigma_1, \sigma_2$ of the distributions that form the mixture and the proportion of each distribution in the mixture $1 - \pi$ and π were estimated. To that end, the expectation-maximization (EM) algorithm was used [18–20].

Table 5 shows the results for $M1$.

The quality of fit was assessed by applying the KS test [11, 12] (p-value of Table 5).

For model M_2, for maximum likelihood, both the parameters μ_1, μ_2, σ of the distributions that form the mixture and the proportion of each distribution in the mixture $1 - \pi$ and π were estimated. To that end, the expectation-maximization (EM) algorithm was used.

Table 6 shows the results for M_2.

The quality of fit was assessed by applying the KS test (p-value of Table 6).

Next, the best model between M_1 and M_2 was chosen. To that end, the Bayesian Information Criterion (BIC) [18] was used:

$$BIC_i = BIC(M_i) = -2 \log L(M_i) + p(M_i) \log n, i = 1, 2 \tag{5}$$

where $L(M_i)$ represents the likelihood function for parameters in M_i evaluated at the maximum like-

Table 5. Normal mixture with unequal variances

$\hat{\mu}_1$	$\hat{\mu}_2$	$\hat{\sigma}_1^2$	$\hat{\sigma}_2^2$	$\hat{\pi}$	KS
1.78	4.99	0.42	1.39	0.48	0.70

Table 6. Normal mixture with equal variances

$\hat{\mu}_1$	$\hat{\mu}_2$	$\hat{\sigma}^2$	$\hat{\pi}$	KS
1.95	5.30	0.74	0.41	0.91

Table 7. BIC mixture normals

BIC(M_1)	**BIC(M_2)**
338.18	337.96

lihood estimators, and $p(M_i)$ represent the number of parameters of the model M_i. Table 7 shows the BIC for models M_1 and M_2.

Therefore, the model M_2 with parameters $\mu_1 = 1.95$, $\mu_2 = 5.30$, $\sigma^2 = 0.74$, $\pi = 0.41$ was chosen, because it had the smallest BIC [18].

Figure 6 shows the mixture and the components of model M_2.

Afterwards, the *a posteriori* probabilities of the models were estimated by means of [18]:

$$\hat{P}\left({}^{M_i}/_{D} \right) = \frac{e^{-\frac{1}{2}BIC_i}}{\sum_{i=1}^{2} e^{-\frac{1}{2}BIC_i}}, \qquad (6)$$

where D represents the data (i.e., Marks2005).

Then, the relative merits of each model were assessed. Table 8 shows the *a posteriori* probabilities of both models, M_1 and M_2.

Remark 1 (Gaussian mixture model classification): The *a posteriori* probabilities were also calculated by using Bayes' theorem [18, 19]. For the case under analysis, Bayes' theorem was applied as follows:

The estimated *a posteriori* probability that the i-

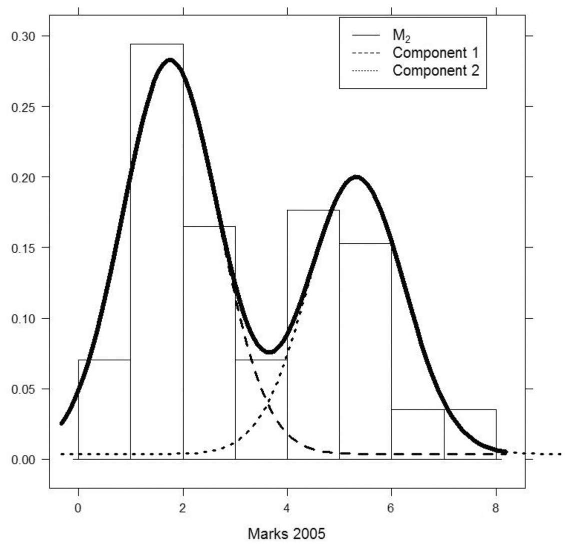

Fig. 6. Mixture and the components of model M_2.

Table 8. *A posteriori* probabilities

$\hat{P}\left({}^{M_1}/_{D} \right)$	$\hat{P}\left({}^{M_2}/_{D} \right)$
0.4726	0.5273

th observation belongs to the m-th component 1 or 2 is given by

$$\hat{P}\left({}^{m}/_{x_i} \right) = \frac{\hat{\pi}_m f_{X_m}\left({}^{x_i}/_{\hat{\mu}_m, \hat{\sigma}} \right)}{\sum_{i=1}^{2} \hat{\pi}_m f_{X_i}\left({}^{x_i}/_{\hat{\mu}_m, \hat{\sigma}} \right)}, \qquad (7)$$

where x_i is the i-th observation of Marks2005 and $\hat{\pi}_1 = 1 - \hat{\pi}, \hat{\pi}_2 = \pi$.

Once the marks of 2005 were adjusted to a mixture of normals (model M_2), the observations were classified into two groups taking into consideration their probabilities of membership of each one of the two components [18]. Thus, the threshold that divided the components was found. Given the value x of X, the threshold allowed one to decide to which group an element belonged.

Here, both the *a posteriori* maximum probability criteria and Bayes' theorem were used to determine the threshold [21, 22]. The threshold was

$$u = \frac{\hat{\mu}_1 + \hat{\mu}_2}{2} + \frac{\hat{\sigma}^2}{\hat{\mu}_1 - \hat{\mu}_2} \log \frac{\hat{\pi}}{1 - \hat{\pi}} = 3.71.$$

Also, the probability of error of the classification [21, 22] was

$$P(error) = 0.02.$$

Finally, it was important, when classifying, to find the qualitative characteristics that distinguished the students of the groups that were obtained.

Taking into consideration the previous analysis for Marks2005, it can be said that there were some factors that were significantly affecting the academic results of the students and originating heterogeneity. That is to say, there were some factors that were producing a higher variability than the rest and, as a result, these factors were segmenting the population.

Here, it is important to point out that for the case under analysis the only information available for the year 2005 was the marks of the students, which was not enough to identify the qualitative factors that differentiated the students of the two groups. However, it was suspected that two important factors, among others, were first, the fact that new first year students had problems with the Mathematical tools that we use in Analysis of Circuits, and second, the fact that the traditional teaching and learning system had to change for the better, it was obsolete.

For more than four years during the development of the educational experiment the research team worked on solving the above problems. The results of this work are shown in the next section.

3.2 Statistical Modeling of Marks2009

For Marks2009, taking into account the normality (Fig. 4), a Gaussian distribution, Y, was adjusted to model the marks of the students as

Table 9. Normal univariate

$\hat{\mu}$	σ	KS
4.69	2.06	0.18

$$Y \sim N(\mu, \sigma^2) \tag{8}$$

The density function of the normal distribution with parameters $\theta = (\mu, \sigma)$ was denoted by $f_Y(y/\theta)$. Then, for Marks2009 the following model was used:

$$M : f_Y(y/\theta)$$

The parameters μ and σ were estimated for maximum likelihood. The results of such an estimation are shown in Table 9.

The quality of fit was assessed by applying the KS test (p-value of Table 9).

Figure 7 shows the graph of the adjusted distribution along with the histogram. From this figure, it can be seen that there is no factor that produces a significant variability.

This satisfactory result is in part due to the fact that the instructors involved in the educational experiment have been working on a student-centered teaching and learning approach [4–6] for more than four years.

In addition, instructors of the departments of Applied Mathematics and Circuits and Systems have worked intensively together to solve the problems that students have with Mathematical tools for Analysis of Circuits. As a result of such a collaboration, a book was written for telecommunication engineering students who are interested in either learning or improving their knowledge of Math for Analysis of Circuits [23].

Figure 7 shows that despite the fact that there is plenty of room for improvement, the work on solving the main factors that were suspected of dividing the population in 2005 has paid off.

3.3 Estimated Probabilities Obtained from the Models M_2 and M

The estimated probabilities of Marks2005 for the chosen model M_2 are shown in Table 10, and the estimated probabilities of Marks2009 for the chosen model M are shown in Table 11.

Finally, Fig. 8 shows the confidence interval for the proportions shown in Fig. 5 along with the estimated probabilities obtained from the models M_2 and M.

Table 10. Estimated probabilities. Model M_2

$\hat{P}(X \leq 4)$	$\hat{P}(4 < X < 6)$	$\hat{P}(X \geq 6)$
0.6088	0.3038	0.0873

Table 11. Estimated probabilities. Model M

$\hat{P}(X \leq 4)$	$\hat{P}(4 < X < 6)$	$\hat{P}(X \geq 6)$
0.3675	0.3720	0.2603

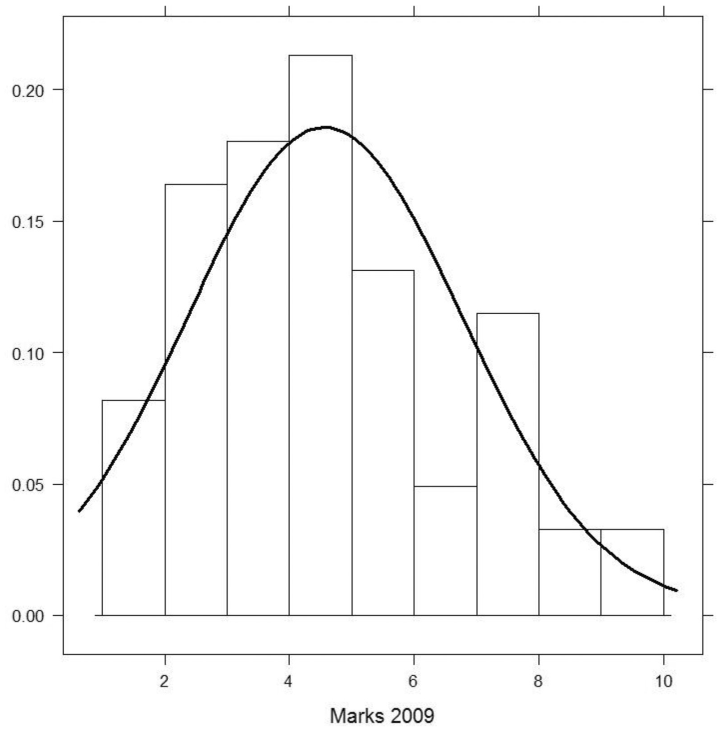

Fig. 7. Histogram and adjusted density of Marks2009.

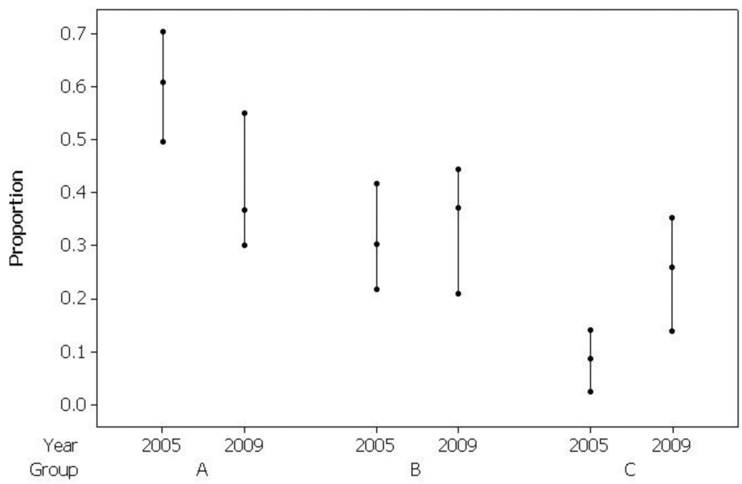

Fig. 8. Confidence interval for proportions and estimated probabilities obtained from the models M_2 and M.

4. CONCLUSIONS

In this paper, it was shown that between the years 2005 and 2009 there was a shift to the right of the marks of the students and that this shift took place without changing the proportion of marks in the medium-marks interval. The percentage of marks of group A (low-marks interval) in 2009 was less than the one in 2005 and the percentage of marks of group C (high-marks interval) in 2009 was greater than the one in 2005.

The change of the model from a heterogeneous one for 2005 to a homogeneous one for 2009 has a remarkable qualitative importance. Now, it can be said that the factors that were dividing the population in 2005 (i.e., Marks2005) into two groups have lost their importance in 2009 (i.e., Marks2009).

For the statistical population under analysis, it has been shown that before Bologna in 2005 there was a subpopulation consisting of the 59% of the students that had mean value (i.e., average mark) equal to 1.95 and another subpopulation consisting of the 41% of the students that had mean value equal to 5.3. Also both subpopulations had standard deviation equal to 0.86. However, after four years of Bologna in 2009 there was only one population, which was modeled by using a Gaussian distribution that had mean value equal to 4.68 and standard deviation equal to 2.06.

Moreover, before Bologna the estimated probability of a student belonging to the low-marks interval was equal to 0.6088 and after Bologna it was equal to 0.3675. Furthermore, before Bologna the estimated probability of a student belonging to the high-marks interval was equal to 0.0873 and after Bologna it was equal to 0.2603.

To sum up, the reality is that, for the years under analysis, the Bologna process has represented both a significant improvement in the academic results of the students and has passed from an unsatisfactory model for the 2005 marks to a more 'reasonable' model for the 2009 marks.

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