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Financial Engineering Education: The Case Study of Financial Modelling Using Games*

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This paper presents the auxiliary learning tools, created as an educational game for a number of topics in financial engineering. The aim of this educational game is primarily to improve the students' understanding of financial engineering using the principle of situational learning. The game is conceived as a simulation of a small scale financial market, where students act as participants in the market transactions, with their own virtual capital. In a defined space of time the students build their portfolios, which contain only shares at the beginning of the game, and they make decisions using the methods and techniques from the given teaching unit. Changing the initial conditions, throughout the game, the students can expand their portfolio acquiring bonds and derivatives such as options, in order to better diversify their portfolios. This educational game has a twofold use, as an auxiliary teaching tool and as a tool for testing the students' achievement. In comparison with completing classic-type tasks, this game can help test the students' competence to apply the acquired knowledge in close-to-reality situations.

Keywords: Financial engineering; education; asset pricing models; games

1. Introduction

This paper presents the auxiliary tools in teaching, created as an educational game for a number of topics in financial engineering (Markowitz's model [1], Capital Asset Pricing Model (CAPM) [2], Vasicek's model [3], and Cox, Ingersoll, Ross (CIR) model [4]). These teaching units are taught at master's academic courses, Investments and Fixed Income courses on the International Masters of Quantitative Finance (IMQF) at the University of Belgrade, Faculty of Economics. The selected teaching units were rated as too demanding from the point of view of the students, hence a large number of them found it difficult to master the subject matter. Given that all the listed teaching units require that students thoroughly understand the manner in which the financial market works, and that the tasks the students are assigned and from which they learn focus upon the above mentioned Asset Pricing Models, the students find it difficult to see the connection between the model and the real market system. Educational games are devised in order that this problem should be overcome to a certain extent.

The game is devised as a simulation of a small scale financial market where students act as participants in the market transactions, with their own virtual capital. In a defined space of time the students build their portfolios and make decisions using the methods and techniques from the given teaching unit. The same platform is used for all the proposed teaching units, as a unique market business simulation, i.e., as basis for the implementation of the proposed models. In a defined space of time the students build their portfolios, which contain only shares at the beginning of the game and make decisions using the methods and techniques from the Markowitz's model. Using the game, the students define one or a number of virtual indices that they use as representative of a simulated market. Using the CAPM model the students come to a decision as to which shares from their portfolio they should keep, and which shares should be sold. In the next stage of the game the students define the value of zero-coupon bonds using CIR model, and the price of call option using Vasicek's model. Changing the initial conditions, throughout the game, the students can expand their portfolio acquiring bonds and derivatives such as options, in order to better diversify their portfolios. The devised game was tested on a group of 20 students. The results have shown that the students easily completed all the tasks from the teaching unit; they even enjoyed learning through playing games. Hence the created educational game improved the students' understanding of the area of financial engineering, making use of the situational learning approach and motivated the students to enrich their knowledge of financial engineering.

However, Radović-Marković points out that Serbia does not have extensive experience in deploying information technology studies. If Serbians or citizens of other nations become more familiar with the techniques, potential learners as well as educators may be able to effectively discern the pros and cons of how information technology enhances and improves education. The author observes that in

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education, information technology (e-learning) is not only changing the way students learn or teachers teach; it rather changes the method in which knowledge is delivered and the place where educators train. She says that it could be expected in the near future that open communication and management approaches will become the driving techniques to enhance learning skills in virtual environments, which will meet new requirements of societies [5].

The game is divided into two levels. The first level aims to introduce students to the classic financial models and the portfolio optimization theory. These are the Markowitz' model [1], and Capital Asset Pricing Model (CAPM by Sharpe) [2]. The second level of the game aims to introduce students to advanced Single-Factor continuous-time term-structure models and numeric simulation techniques. These are CIR (Cox, Ingersoll, Ross) model [4], and Vasicek's model [3]. The teacher may define the game in such a way that a successful completion of the game practically verifies the knowledge students' acquired. In comparison to the classic way of working out problems, the game can serve to check the students' ability to implement the acquired knowledge in close-to-real situations.

The paper is organized as follows. In Section 2 we presented theoretical aspects of educational games. The Markowitz's theory and CAPM are presented in Section 3. In this section we presented games for these two lessons. In Section 4 we presented the CIR and Vasicek models and games for these two lessons. Section 5 contains a conclusion.

2. Games in education

The notion of education via questions, answers and discussion dates back to the times of the ancient Greece. The method then created set paths for the today's development of education via varied multimedia forms. The manners of education evolved significantly, from the transfer of knowledge through a direct communication between teachers and students or learning from a written text. The knowledge transfer in modern times is performed in a number of methods, such as multimedia contents, distance learning, online lectures and, especially important, education through games. Games have increasingly become a highly valuable method of knowledge transfer, primarily due to their capability of permanently capturing the attention of participants and the fact that they create a feeling of enjoying while learning [6].

Not one sufficiently extensive study has been performed to prove the success in the implementation of games in education, nevertheless, some research points to significant possibilities of implementing games in education [7–9]. Certain forecasts claim that an adequate implementation for educational games will never be found. Nevertheless, numerous experts maintain that these technologies are about to expand tremendously and that their efficiency will be well proven. This is further corroborated by a poll conducted among the academic community, in which as many as 95.24% of respondents gave an affirmative answer to the question: "*Do you maintain that educational games will become a standard part of a curriculum?*" [10, 12].

A conclusion can be drawn that there is a strong connection between different types of games and the improvement of certain skills. Especially interesting are those that affect intellectual skills. Computer games have a potential to facilitate a highly efficient education, for a number of reasons [11]:

- *Scope*. Games often involve a large number of players, in certain cases measured by millions. This means that the contents of the game can reach many more users in comparison with the standard method of education.
- *Any time, any place*. Games do not require that players be present in the classroom, they can be played at any time, and thanks to portable devices, anywhere. Students already spend several hours a day playing games anyway – this is only building on an already existing concept.
- *Interest*. Given that games tend to be as interesting as possible, they (although unconsciously) develop in accordance with the rules of efficient learning.
- *Brain stimulation*. Playing games stimulate the brain to prepare for the learning process.
- This proved to be better than a classic lesson. Understanding of the subject matter rose by 30% when conducted via games [6, 22].

Minović et al. pointed out that continuous advance in technology cause a generation gap between students and teachers to increase. There are constant breaks in communication, misunderstandings and social conflicts arising during the conduct of a course. Today students have grown up using devices like computers, mobile phones, and video consoles for almost any activity; from studies and work to entertainment or communication. These authors found that motivating students with traditional teaching methods such as lectures and written materials proves to be more difficult daily. In order to increase the motivation of students, to better the understanding of the subject matter as well as improve collaboration, new form of teaching was required. That is why digital games are considered to have a promising role in the education process. They conducted a study among university students with the purpose of acquiring empirical evidence to support the claim that game design can be used as an effective form of learning. Their method consisted of monitoring the results of participants in the course of Computer networks. The experimental group of participants experienced a game design as a new learning tool for teaching, while the control group used network programming [12]. Additionally, Minović et al. made an effort to measure the effects of different learning approaches with respect to individual differences in cognitive styles. Initial results provide a good argument for use of game design as a student learning tool. In addition, they reported the influence of cognitive style on the effectiveness of using the game design [12].

Kovačević et al. presented a study conducted among university students with the purpose of acquiring empirical evidence to support the claim that game design can be used as an effective form of learning. They found that learning motivation is another relevant factor of learning performance. The authors decided to observe the effect of different learning contexts both on exam results as the measure of learning outcome and subjectively reported level of motivation. Their initial results provide a good argument for the use of game design as a student learning tool [13].

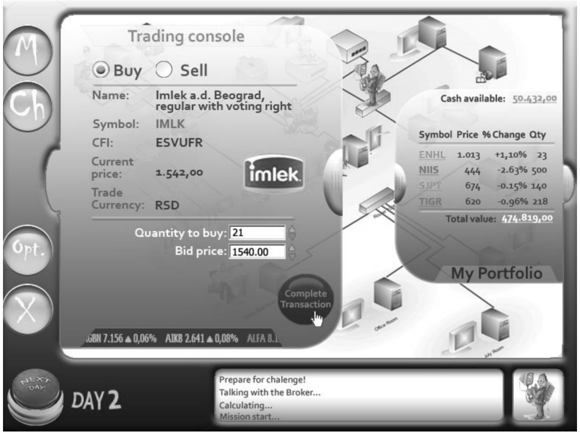
Radović-Marković, Nelson-Porter and Omolaja pointed out that learning using information technology contributes to fast information exchange, more access to the newest knowledge and experiences in this domain and saves the time and money. The author thinks that in this millennium the classic way of education will be slowly substituted by some other forms of education, in which learning from home and office with the help of computers, will come true. Interactive education should provide a completely new dimension of gaining knowledge making learning faster and easier for those who attend certain courses [14].

Radović-Marković analysed education through e-learning. The author presented an overview of some recent projects with the focus on Serbia and used their results to discuss advantages of using e-learning as an alternative opportunity and support to "face-to-face" education. From the author's viewpoint, it is believed that online learning will not replace face-to-face learning but should still be offered as a style of learning that suits students' needs [5].

3. Markowitz's model and CAPM using games

Markowitz's Portfolio Theory (MPT) serves as basis for the entire area of portfolio management. Also, Markowitz's Portfolio Theory has an important role in assets valuation [15]. In his theory Markowitz presents investor optimization. On the other hand, CAPM analyses the economic equilibrium under the assumption that all investors perform optimizations in a specific manner proposed by Markowitz. The equilibrium concept is a key concept in economics and is important in studying financial markets. Its major task in finance is to provide alternative tools for asset pricing. In the conditions of equilibrium, it is assumed that the assets price is set in such a manner that the demand to own assets equals the total ownership over them [16]. Economy is balanced if prices are such that the total demand equals the total offer in the conditions in which investors act optimally. Hence it is necessary that aggregate expenditure of all the agents in the conditions of equilibrium equals aggregate wealth. Balanced prices depend on investors' utility functions and on their wealth. If the equilibrium with a number of agents is identical to the equilibrium with one agent, we say that the property of aggregation exists, and one agent is called the representative agent. The existence of equilibrium is not always guaranteed and this may prove to be a serious problem. Equilibrium models help us get relations between interest rates of risk-free assets, the risk premium of risky assets, of investors' preferences to risk, and of the total wealth of agents in economics. The most popular equilibrium model in finance is the Capital Asset Pricing Model, CAPM and its extensions [17]. The CAMP suggests that the expected returns on risky assets will be determined by the covariance of their returns with the returns of the market portfolio. Unfortunately, the majority of research and testing is faced with a rather poor support or no support at all from the CAPM [18]. Nevertheless, what is attractive about the CAPM is that it is simple and easy to use. Accordingly, the CAPM has become a benchmark for all asset pricing models and allows for the development of a set of literature in which this model is tested or expanded with anticipations. The CAPM is developed in a static, single-period framework. Its practical implications pose a challenge to numerous intertemporal, multi-period CAPM versions [17]. There are numerous pros and cons as regards the CAPM and they are presented in [19].

Fabozzi, Gupta and Markowitz point out that a combination of Markowitz's portfolio theory and the asset pricing theory makes the basis for the investment risk assessment and the relation between the expected returns on assets and risk. In Markowitz's portfolio theory risk is measured by the volatility of the returns on assets, i.e., the standard deviation of returns on assets. The key concept of this theory is the diversification of the portfolio volatility [20]. This means that it is possible to reduce volatility (achieve diversification) including additional shares into the portfolio. Given that the



Source: Screenshot taken from Educational game produced with development platform [22, 6].

Fig. 1. Educational game—share acquisition.

correlation of securities varies, it is necessary that, for the diversification purposes, the shares with as low as possible correlation (preferably with a negative correlation) should be included into the portfolio. Therefore, unsystematic or specific risk, i.e., individual security risk is possible to eliminate through diversification, whereas systematic or market risk can never be eliminated [15, 6]. Markowitz sets a problem of the selection of an investment portfolio that would minimize the portfolio variance (risk) retaining its anticipated value fixed. The set of solutions to the problem is termed Markowitz's efficient frontier. The efficient frontier is defined in case there are exclusively risky securities on the market, or when risk-free assets exist along with risky ones [21].

3.1 Educational game for the Markowitz model

The game is designed as a simulation of a small-scale financial market where students are participants in the market, with their own virtual capital. It is devised as a 2D adventure, using an educational game editor [22]. Each student assumes his virtual character during the defined space of time. At the beginning of the game, the student receives a certain amount of virtual money to be at his disposal during the game. On the other hand, the situation on the market changes every virtual working day, and so does the optimal portfolio. The goal is to earn as large as possible returns on the portfolio at as low a risk as possible, throughout the game [6].

During the game the student gets an insight into the trends of shares on a virtual stock exchange. In the right panel, accessible to the student at all times, the player's current portfolio is presented. The information is also provided as to how much money the player has at disposal for further investments, as well as the current value of his portfolio on the basis of the situation on the market. In the left panel the player can access a virtual trading console, which he can use to buy new shares or sell shares from his portfolio. If a transaction is accomplished, the new state will be updated in the player's portfolio. In the bottom part of the left panel there is a *ticker* that shows the player the current prices of shares in the market. The duration of the virtual day, as well as of the entire trading cycle is the decision of the teacher, so that the game can be implemented during only one class, however, also in a longer period, for example, one week [6].

Figure 1 presents the situation in the game when the player acquires a new share for his portfolio.

Given the described basic premises of the theory, the students are required to select shares for their portfolios. On the basis of the indicators it is necessary that they assess their quality and determine their correlation for the purpose of diversification. A small number of shares is chosen (for example, two shares) so that the students should not have the problem of calculating a large number of covariances i.e., correlation coefficients.

The major goal of the task is the optimization problem. Namely, for a given level of returns it is necessary to minimize the portfolio variance observing the following conditions:

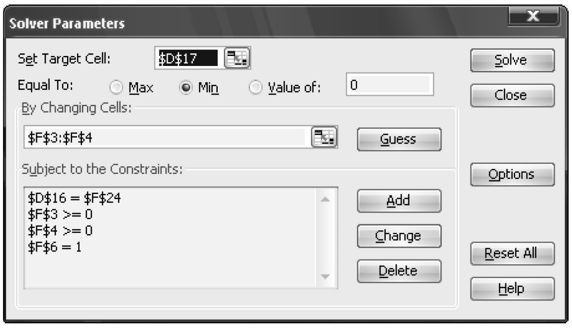
- The expected portfolio returns has to equal a certain desired value of returns.
- The sum of all weights has to equal 1.
- Each weight of the share in the portfolio has to be bigger than or equal to zero [6].

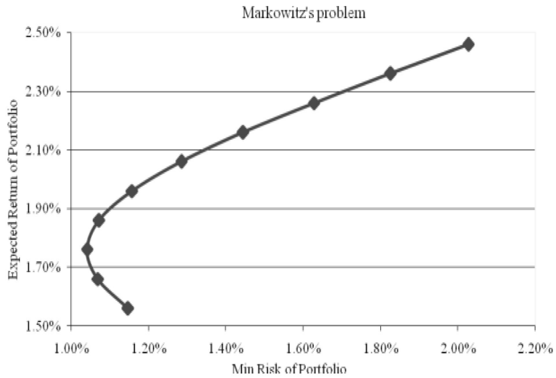
In solving the task students can use Solver, part of the Microsoft Excel package. An example is given in Fig. 2.

Optimization completed, and repeated several times, the risk and return graph is plotted. For the expected value of returns of over 1.76% we have a situation that a chosen portfolio has a minimal variance (MVP), i.e., a minimal standard deviation of 1.04%. An optimization algorithm of Markowitz model devised for these values allows for the share of the first share in the portfolio to be 22%, while the share of the second share should amount to 78% [6].

Efficient frontier is made up of all points from MVP (E(R) = 1.76%) up to the maximum expected return, presented in Fig. 3. This Figure shows that the efficient frontier takes a concave form (in the absence of risk-free assets) [6].

The achieved MVP is not an optimal investment portfolio. The optimal portfolio would, for the acceptable risk level, yield a considerably higher level of expected return in comparison with the MPV. Graphically, the optimal portfolio would be





Source: Screenshot taken from Excel, as a part of Educational game [6]

Fig. 2. Portfolio optimization in MS Excel Solver.

Fig. 3. Expected return and minimal portfolio risk.

found if a tangent were drawn from the risk-free rate of return (assuming that the risk-free rate of return is R_f = 2%) to the efficient frontier. This would probably be a point with the following coordinates: ($E(R)$, σ) = (2.20%, 1.50%) [6].

On completion of the cycle, the teacher receives a report on the achievements of each of the students, as well as individual changes in their portfolios. These data can serve as basis for discussion on the shares of each of the players, on their errors or good moves they made [6].

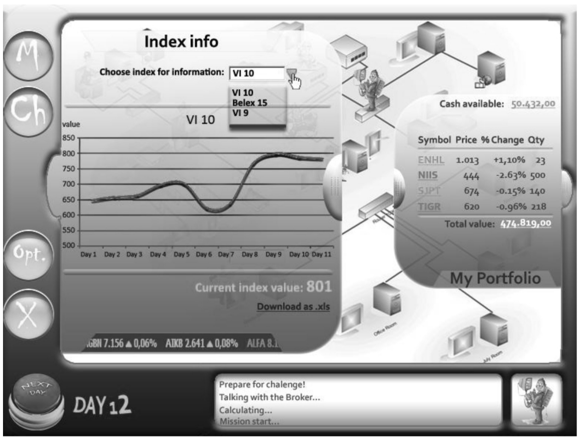
3.2 Educational game for CAPM

The game presented in subchapter 3.2, a stock exchange trading simulation, is used to cover the teaching unit about CAPM, too. The educator may define one or more virtual indices that students will use as representative of simulated market. Hence the educator sets the initial conditions for work on the concrete CAPM task. The student uses the presentation of the trends of one of the indices offered from the moment the game begins. Using the CAPM method he comes to a suggestion as to which securities he should keep in his portfolio and which ones he might sell, as well as which securities he may need to expand his portfolio [6].

Figure 4 presents a situation in which the student gains an insight into the flow of a virtual index VI 10 set by the educator. He can download the data in the .xls format.

The student's task is to choose one share, find returns on this share, find the return of the market index and finally find the risk-free return rate. The next task is to determine the characteristic determination for the chosen share, that is, to read the segment alpha and the gradient beta using the regression analysis. Using the obtained determination coefficient, R^2, from the regression analysis, the student has to find out what percentage of the total risk belongs to the systematic risk. Accordingly, he has to decide which percentage of the variance (total risk) for the chosen share refers to the specific risk of the company. On completing the regression analysis, the student has to decide whether the chosen security can be ranked as offensive or as defensive. Besides, the student is expected to examine the performances of the chosen share using the Jensen index [6].

In dealing with the task the students use the regression analysis which is part of the Microsoft Excel package. Since the CAPM model is a one-factor model, and it has no time dimension, it is called unconditional CAPM. Hence it is necessary that an assumption should be introduced that refers to the time behaviours of the return series and assess the model in time in that manner. The assumption is that returns are independent identically distributed (iid) over time and that their distribution is a multi-dimensional normal. The usual method for parameter estimation in the model is the method of ordinary least squares [19]. Statistically speaking,



Source: Screenshot taken from Educational game produced with development platform [22, 6]

Fig. 4. Educational game—review of indices.

CAPM is a one-index model expressed by the following equation:

$$E(R_i) - R_f = \alpha_i + \beta_{iM}\left[E(R_M) - R_f\right] + \varepsilon_i \quad (1)$$

The estimation of parameters α i β is done by the regression analysis, ϵ_i is the component linked with unexpected events that affect only an individual (firm specific) security. Empirical tests of CAPM are focused upon three implications of equation (1):

- α_i denotes to what extent the individual security return rate is higher in comparison with the market return.
- β_{iM} denotes the sensitivity of the security to the changes in the market index return.
- Market risk premium is positive ($[E(R_M) - R_f] > 0$) [19].

If the value of the Jensen index (α coefficient) is positive, it means that the security has better performances than anticipated by CAPM and may be underpriced. Similarly, a negative value of the Jensen index means that the security can be overpriced. The problem with the Jensen index as performance indicator is that it does not indicate the real level of the security risk. For example, if we assume that two securities bear equal values of the Jensen index, and their beta coefficients are different, then the security with a lower beta is more attractive, because it offers an equal level of performance above equilibrium on average, however, at a lower risk [17]. The higher the β, the higher the (market) risk of securities, as well as the total variance (risk). The β equals 1 for the entire market. If $\beta > 1$ for a security, such a security ranks as offensive, whereas if $\beta < 1$, the security is defensive.

On completing the trading, the teacher gets a report on the achievement of each of the students, as well as individual changes in their portfolios. On the basis of his documentation and his projections, the teacher then assesses the success of the students' activities and continues to discuss the concrete activities of each student.

4. CIR'S and Vasicek's model using games

The main advantage of these one-factor CIR and Vasicek models is their simplicity as the entire yield curve is a function of just one state variable. Moreover, this state variable is observable—at least in principle (in practice, we use a short-term interest rate as a proxy). However, there are several problems with one-factor models. First, the model assumes that changes in the yield curve, and hence bond returns, are perfectly correlated across maturities, and, not surprisingly, this assumption is easily contradicted by the empirical evidence. Apart from that, the assumption of perfect correlation is highly problematic for several "practical" purposes, for example Value-at-Risk calculations, and pricing derivatives on interest-rate spreads. Second, the shape of the yield curve is severely restricted. Specifically, the Vasicek and CIR models can only accommodate yield curves that are monotonically increasing or decreasing and humped. Moreover, with time-invariant parameters, one-factor models tend to provide a very poor fit to the actual yield curves observed in the market [23]. An important topic in financial econometrics studies of the yield curve is to learn something about risk premium.
Clearly, this cannot be done with models that take the initial yield curve as given and use relative-pricing techniques (valuation under the risk-neutral measure) to price derivatives [23]. In the present paper, we will make the game for equilibrium models with a single factor.

In practice, single-factor models are calibrated in such a way that the models' parameters are obtained as solutions to a minimization program of the difference (i.e., the squared spread) between market prices of reference bonds and theoretical values generated from the model.

In this section, the students' task will be to learn the single-factor continuous-time term-structure models. Contrary to the previous models where the students worked in discrete time, compiled their portfolios only from shares and used the technique of regression analysis, here the students will work in a continual time, master the techniques of numeric simulations and learn how to calculate the value of bonds and options. Thus their portfolio can be expanded by bonds and financial derivative tools such as options.

Towards the end of the twentieth century, tremendous efforts and progress have been made in valuing interest rate sensitive derivative securities. Broadly speaking, two different approaches have been used. Some authors have modelled interest rates in an equilibrium setting and derived bond prices and other interest rate derivative securities prices based on the equilibrium movements of the underlying interest rates. Examples include Vasicek [3], and CIR (Cox, Ingersoll, and Ross [4]) [24].

Before we introduce the next teaching units, we make the following assumptions:

- Standard economic assumptions for a continuous-time model: Frictionless bond markets with continuous trading, no distorting taxes, no short-sale restrictions, and no divisibility problems. Investors always prefer more wealth to less, i.e. the marginal utility of wealth is positive at all levels of wealth.

- The short rate (instantaneous interest rate) follows the general stochastic differential equation (SDE):

$$dr_t = \mu(r_t)dt + \sigma(r_t)dW_t \tag{2}$$

  where $\mu(r_t)$ and $\sigma(r_t)$ are the drift and volatility functions, respectively, and W_t a Brownian motion (Wiener) process.
- The market price of risk for the term structure, $\lambda(.)$, only depends on the short rate r_t.
- All bond prices, i.e. $B(t,T)$ for all $T > t$, are functions of a single state variable, the short rate r_t (in addition to t and T) [23].

There are two ways to represent the bond price, $B(t,T)$:

$$B(t, T) = e^{-R(t,T)(T-t)} \tag{3}$$

$$B(t, T) = e^{-\int_t^T f(t,s)ds} \tag{4}$$

where $B(t,T)$ is the price, at time t, of a zero-coupon bond maturing at time T (the maturity date). The time to maturity of this bond is $\tau = T - t$. It is important to note the distinction between the maturity date and the time to maturity – they are only identical when $t = 0$. In general, we assume that $B(t,T)$ exists for all $T > t$. $R(t,T)$ is the yield to maturity with continuous compounding at time t, for a zero-coupon bond maturing at time T, and $f(t,T)$ is the instantaneous forward rate at time t, for a zero-coupon bond maturing at time T [23].

4.1 Game for CIR model

The game presented in chapter 3, the simulation of stock exchange trading (only with shares), is now used to learn the units that deal with equilibrium determining the risk-free rate and bond prices, using the numeric simulation technique. Particularly, this was done to illustrate the workings of a general equilibrium model suggested by the authors in [4]. CIR adopts an equilibrium approach to endogenously determine the risk-free rate. This is an example of square root process where the evolution of the spot rate, r_t, is modelled by the SDE. The CIR model [4] has the specification of the term structure:

$$dr_t = \kappa(\mu - r_t)dt + \sigma\sqrt{r_t}dW_t,\ r(0) \geq 0 \tag{5}$$

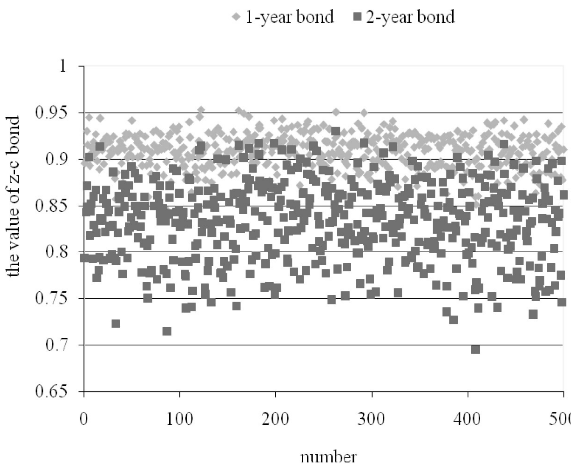
where κ, μ, and σ, are positive constants, representing reversion rate, "long-term value", and volatility parameter, respectively. In this model the spot rate is "elastically" pulled towards the long term value. Although the SDE (5) has a path wise unique solution, a closed form representation has not been available to date [25]. The bond-pricing formula takes the exponential-affine defined in [23].

Concretely, the goal in the game is to determine bond pricing with different maturities using the CIR model. The educator can define the following parameter values: Δ, κ, μ, σ and r_0 (for example: Δ = 0.01, κ = 0.2, μ = 7%, σ = 12%, the initial interest rate is r_0 = 10%, and the market price of risk, λ, is zero). In this way the educator sets the initial conditions for solving the concrete CIR task. The student uses simulation to generate B paths of the interest rates with Δ = 0.01 (where Δ is the time between t and t+1), and for each path computes the value of zero-coupon bonds with face value F (for example of $1) and different maturities (for example of 1-year and 2-years) since the moment the game begins. Then, the student estimates the value of these zero-coupon bonds. Playing the game in which a number of 500 numerical simulations is set for the CIR model, the student comes to a decision as to which bond price and with which maturity is the most favourable to be included into his prospective portfolio. Initial parameters can change so that the student can choose his bond in dependence of the values of the set parameters.

The price of zero-coupon bond with maturity of 1-year is 0.9085, and the price of zero-coupon bond with maturity 2-years is 0.8313 for the initial parameters set.

4.2 Game for Vasicek's model

The goal of this game is to price options as derivative securities. Hence the student will consider pricing a call option on a zero-coupon bond. An option is a contract in which the seller (writer) grants the buyer the right to purchase from, or sell to, the seller an underlying asset (here a bond) at a specified price within a specified period of time. The seller grants this right to the buyer in exchange for a certain sum of money called the option price or



Source: Authors' estimation

Fig. 5. CIR model Simulation in Excel.

option premium. The price at which the instrument may be bought or sold is called the exercise or strike price. The date after which an option is void is called the expiration date. An American option may be exercised any time up to and including the expiration date, and a European option may be exercised only on the expiration date [26]. Factors that influence option prices are: Current price of underlying security[1], Strike price[2], Time to expiration[3], Short-term risk-free interest rate[4], and Expected volatility of yields (or prices)[5].

The Vasicek's [3] model: the short rate follows the mean-reverting Gaussian process (sometimes called the Ornstein-Uhlenbeck process) as follows:

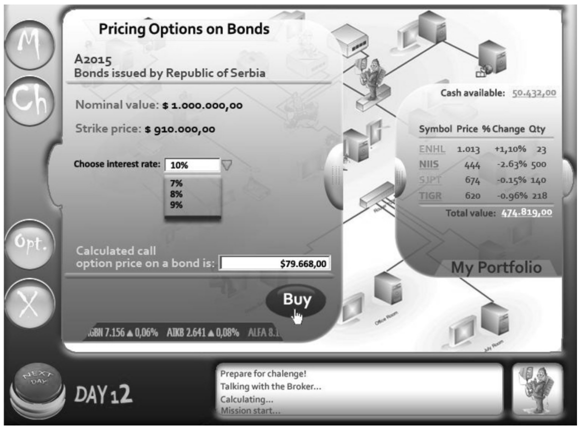
$$dr_t = \kappa(\mu - r_t)dt + \sigma dW_t, \tag{6}$$

where κ measures the speed of the mean reversion (the larger κ, the faster the speed of mean reversion), μ is the unconditional mean, and σ is the instantaneous volatility of the short rate. The Vasicek's model is the continuous-time equivalent of a first-order autoregressive process, or AR(1) model [23]. In the devised game the student is encouraged to investigate how different parameter values for κ, μ, σ, and Δ affect the shape of the term structure.

Call option price on a zero-coupon bond is given and calculated in our case by the equation:

$$P_t = P_{t+1} e{-}\Delta r_t. \tag{7}$$

The goal of this game is to price options as derivative securities. Hence the student will consider pricing a call option on a zero-coupon bond with a set nominal value (for example of \$1m), and strike price (for example of \$0.91m). The educator can define the following parameter values: Δ, κ, μ, σ, and r_0 (for example: $\Delta = 0.01$, $\kappa = 0.2$, $\mu = 7\%$, $\sigma = 12\%$, initial interest rate is $r_0 = 10\%$, and market price of risk, λ, is zero). In this way the educator sets the initial conditions for solving the task using simulation. Namely, for each path he will compute the payoff of the option at date one and discount it back to time zero. He will average the values of these discounted payoffs to estimate the value of the bond. Playing the game using the Vasicek's model



Source: Screenshot taken from Educational game produced with development platform [22]

Fig. 6. Educational game—Pricing Options on Bonds.

the student finds prices of a call option on a zero-coupon bond. Setting a different value on the current interest rate r_0 (for example, 7%) or other initial parameters, the student will restart the simulation to get a different price for a call option. Thus he will be able to find out through a game which call option on a zero-coupon bond and with which current interest rate r_0 is the most favourable to be included into his prospective portfolio.

For nominal value of \$1,000,000, and strike price of \$910,000, and under assumption that the initial interest rate is 10%, we have that the call option price on a zero-coupon bond is \$79,668. For the same nominal value and strike price as the previous, but under the assumption that the initial interest rate is 7% we have that the call option price on a zero-coupon bond is \$84,897.

5. Conclusion

The aim of the educational game is to improve students' understanding of the area of financial engineering using the situational learning approach. This paper presents an educational game created as an auxiliary teaching aid, for a number of teaching units dealing with Asset Pricing models, and these are: Markowitz's model, Capital Asset Pricing Model (CAPM), Cox, Ingersoll, Ross (CIR) model, and Vasicek's model. All the above mentioned units require that the students understand the manner in which the finance market works and students find it difficult to find a connection between the model and the real market system. The educational game is devised with the aim to overcome this problem to a certain extent. The game is divided into two levels. The first level aims to introduce students to the classic finance models and the portfolio optimization theory, whereas the second level introduces students to the advanced Single-Factor con-

[1] As the price of the underlying bond increases, the value of a call option rises and the value of a put option falls.
[2] Call (put) options become more (less) valuable as the exercise price decreases.
[3] For American options, the longer the time to expiration, the higher the option price because all exercise opportunities open to the holder of the short-life option are also open to the holder of the long-life option.
[4] Price of call option on bond increases and price of put option on bond decreases as short-term interest rate rises (through impact on bond price).
[5] As the expected volatility of yields over the life of the option increases, the price of the option will also increase.

tinuous-time term-structure models and numerical simulation techniques. The game was tested on a group of 20 students. It turned out that the students easily mastered all the assigned theme units; they even enjoyed learning by playing games. The educational game has therefore improved the students' understanding of the finance engineering area using the situational learning approach and motivated the students to expand their knowledge in the area of financial engineering. Namely, if knowledge is acquired through close-to-realistic problem solving, the understanding is improved, and so is the likelihood that this knowledge will be used in the future, in new situations. Besides, the use of games in education raises the students' motivation. The entertainment factor also contributes to the fact that students will devote more time to learning the given matter. The presented educational game can be used in two ways, as an auxiliary teaching tool, and as a tool to test the students' achievement. Namely, the teacher may define the game in such a way that a successful completion of the game practically verifies the students' achievement. In comparison with completing classic-type tasks, games can help test the students' competence to apply the acquired knowledge in close-to-reality situations.

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