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Making Engineering Mathematics More Relevant Using a Computer Algebra System*

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One of the uses of Engineering Mathematics is to provide underpinning knowledge which is useful and often essential for Engineering modules later in an undergraduate engineering degree course. It can be the case, however, that students perceive distinct boundaries between an Engineering Mathematics module and Engineering modules and fail to link what is being taught in both. The purpose here is to help alleviate this problem by incorporating a computer algebra system into an Engineering Mathematics module, so making the teaching and learning process more accessible and meaningful, providing students with a more realistic way of how professional engineers tackle problems today, increasing student skills, hopefully increasing their interest and finally breaking down the boundary alluded to above. The inclusion of the computer algebra system gave students a better appreciation of how essential Engineering Mathematics is within Engineering modules in general. However, its inclusion had a detrimental effect on interest. It was noted from the survey that the students liked the hands-on and self-discovery approach.

Keywords: engineering mathematics; computer algebra system; integration

1. Introduction

Engineering Mathematics is usually used to provide underpinning knowledge which is useful and often essential for Engineering modules later in an undergraduate engineering degree course. It can be the case, however, that students perceive a distinct boundary between the Engineering Mathematics module(s) and Engineering modules by failing to link what is being taught in both. Of course, many engineering examples are usually incorporated into the Engineering Mathematics curriculum using the traditional textbook method, but these examples are often quite restrictive in that they do not reflect "real" engineering problems. One way to break out of this mold is to incorporate computer software into the Engineering Mathematics module(s), so providing students with a more realistic way of how professional engineers tackle problems today, increasing student skills, hopefully increasing their interest and finally breaking down the boundary alluded to above.

It is not easy to integrate computer technology into traditional teaching and learning courses [1]. Very commonly the complexity of computer programming and getting students to a suitable standard of programming can be daunting, and often the wider ramifications have been overlooked, underestimated or even denied [1]. Many surveys have been carried out [2] to elicit schemes to start tackling some of these difficulties with an emphasis on the integration of computer algebra systems into university teaching [3]. Even though computer algebra systems have been used for some time in engineering education, there is little in the literature describing their effective integration and evaluation. It is the purpose of the current work to contribute to this by using proper experimental rigour to show advantages and disadvantages of using such a system.

What is integrated here is a computer algebra system *Mathematica* [4, 5]. Although *Mathematica* was chosen here, the current work could have been carried out using many other computer algebra systems, including the well-known Maple [6] and part of the symbolic mathematics toolbox, Matlab [7]. Computer algebra systems are gradually being introduced into traditional mathematical courses in an effort to make the teaching and learning process more meaningful [8]. It is argued here that in addition to making the teaching and learning process more meaningful, computer algebra systems have a wider, though sometimes implicit, use in that they provide essential skills necessary during a student's future career. Technology in general is being widely incorporated into teaching and learning of mathematical subjects, for example [9–11]. Slavit et al. [12] conducted an experimental study with the purpose of identifying the effect of a computer algebra system on students' attitudes and outcomes by using simulations and an instructional approach for teaching differential equations. Godarzi et al. [13] conducted an experimental study concerning the impact of a computer algebra system

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on the teaching of double integrals focusing on their procedural and conceptual understanding. The results showed that the experimental group performed better than the control group. It has been shown that conceptual understanding can be developed through procedural understanding and a learning goal can be achieved if a computer algebra system is properly utilized. This has also been shown to lead to an improvement in students' attitudes towards learning mathematics [14].

2. Integration of computer algebra system

The computer algebra system, *Mathematica* was integrated into the students' Engineering Mathematics curriculum with the primary aims of enhancing cognitive development [15], exposing students to modern professional practice and to break down barriers perceived by students between engineering mathematics and other engineering modules taught in the degree course.

All students, including the students used as the control group, had already been taught the rudiments of the computer algebra system during a 12 week course (1 hour/week) during their previous year of study.

For the course considered here, the engineering mathematics course covered partial differential equations, and vector calculus, was 12 weeks in length, with two one-hour lectures and two two-hour tutorials per week. The computer algebra system was used during one of these two-hour tutorials for one of the two groups tested. For each of these software laboratories, a hand-out consisting of the laboratory objective, a narrative (including relevant theory), the task description and additional necessary comments was distributed to the students.

Six software laboratories were used during the twelve week period entitled:

- Laboratory 1: To investigate the use of Lagrange multipliers in engineering [16, 17].
- Laboratory 2: To investigate the use of Fourier Series as approximations to input functions [16].
- Laboratory 3: Solution of the heat equation by Fourier integrals and transforms [18].
- Laboratory 4: Exploration of using *Mathematica* with the Divergence and Stokes' theorems [16].
- Laboratory 5: Analysis of a single-span Euler-Bernoulli beam [19, 20]
- Laboratory 6: Flow over and downstream of a cylinder [18].

Students were asked to write short reports on each of these laboratories and two long reports on two of the laboratories.

2.1 Example—laboratory 5

To give an idea of the type of work involved in the laboratories used, extracts of a student's report for Laboratory 5 is now presented.

Cantilever beam analysis

Consider a uniform beam of length L with cross-section area A and second moment of area I with any number of discrete elements (e.g. springs, masses, oscillators) attached. With use of the Euler-Bernoulli theory the governing equation of a uniform beam can be written as:

$$EI\frac{\partial^4 g(x,t)}{\partial x^4} + \rho A\frac{\partial^2 g(x,t)}{\partial t^2} = \mathbf{F}(g(x,t)) \qquad (1)$$

where ρ is the mass density of the beam material, E is Young's modulus, g is the function of deflection and the operator $\mathbf{F}$ depends on the characteristics of the attached discrete systems. For free transverse vibrations, Equation 1 becomes:

$$EI\frac{\partial^4 g(x,t)}{\partial x^4} + \rho A\frac{\partial^2 g(x,t)}{\partial t^2} = 0 \qquad (2)$$

The natural frequencies associated with the beam are harmonic in nature with $g(x,t) = G(x)e^{i\omega t}$ where ω is the natural frequency of the beam. Putting this expression into Equation 2 gives:

$$\frac{d^4 G(x)}{dx^4} - \frac{\rho A}{EI}\omega^2 G(x) = 0 \qquad (3)$$

This is a 4th order ordinary differential equation with the general solution given by:

$$G(x) = C_1 \cos\lambda x + C_2 \sin\lambda x + C_2 \cosh\lambda x + C_4 \sinh\lambda x \qquad (4)$$

where the constants of integration C_i are specified according to the boundary conditions.

Here, for the beam illustrated on Fig. 1, the lowest four eigenfrequencies ω_i together with the corresponding eigenmodes $X_i(x)$ are to be determined.

The material and geometry variables to be specified are: density (ρ), cross-section area (A), Young's modulus (E), second moment of Area (I), the length of the beam (L) and the mass of the weight at the free

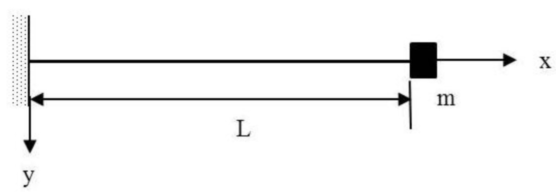

Fig. 1. Cantilever beam with mass on the free end.

end (m). The boundary conditions, considering one end of the beam is clamped, while the other is free can be summarized as:

$$G(0) = 0 \quad \frac{\partial G(0)}{\partial x} = 0 \quad \frac{\partial^2 G(L)}{\partial x^2} = 0$$
$$EI = \frac{\partial^2 G(L)}{\partial x^2} = -m\omega^2 G(L) \tag{5}$$

These boundary conditions together with the general solution (Equation 4) provides a system of linear simultaneous equations and from these a non-trivial solution exists for their vanishing determinant leading to the characteristic equation:

$$\frac{m\psi}{\rho}\left[\frac{\cos\psi\sinh\psi - \cosh\psi\sin\psi}{1+\cos\psi\cosh\psi}\right] + 1 = 0$$
$$\psi = L\cdot\left(\frac{\rho a\omega^2}{EI}\right)^{\frac{1}{4}} \tag{6}$$

This transcendental equation is solved numerically.

With the eigenfrequencies found from Equation 6 and the general solution (Equation 4) it is possible to write the ith eigenmode as:

$$G_i(x) = \frac{\sin\lambda_i + \sinh\lambda_i}{\cos\lambda_i + \cosh\lambda_i}\cos\lambda_i x - \sin\lambda_i x$$
$$-\frac{\sin\lambda_i + \sinh\lambda_i}{\cos\lambda_i + \cosh\lambda_i}\cosh\lambda_i x + \cosh\lambda_i x \tag{7}$$

The characteristic equation (Equation 6) is solved using a graphical method within *Mathematica* as shown below.

The exact coordinates for the point of intersection could be found by simply using the cursor. Once the values of are determined, the eigenfrequencies can be found. Equation 7 plus the eigenfrequency values found in the last subsection were now used to plot the four lowest mode shapes for the beam again using *Mathematica*. For example, for the fourth lowest eigenfrequency the mode shape, calculated using the coding below, was as shown on Fig. 3.

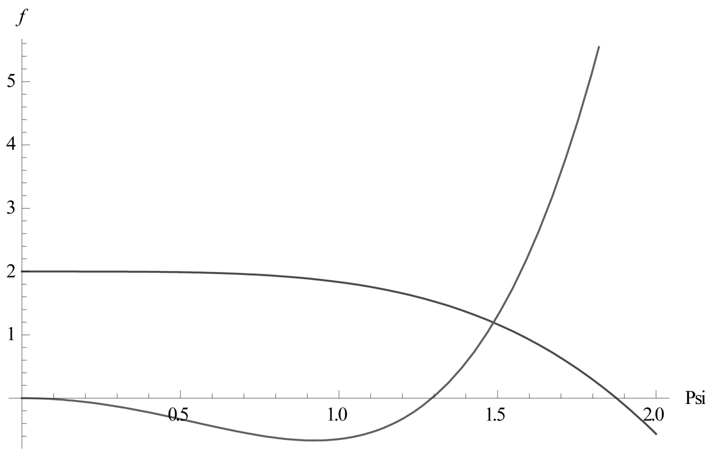

Fig. 2. Graphical solution of the characteristic equation.

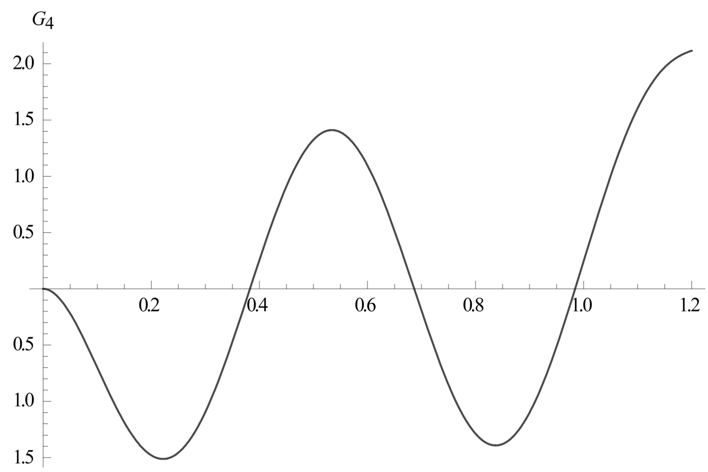

Fig. 3. Fourth lowest mode shape for cantilever beam.

2.2 Comments on the incorporation of mathematica into the solution process

Again using Laboratory 5 as the vehicle for explanation, the purpose of this laboratory was to construct a method to compute solutions for Equation 3, with given boundary conditions. The outline of the stages in the method is shown on Fig. 4. Basically, the code was first written to provide a convenient user-friendly facility for entering data concerning material properties, geometry and to display clearly the boundary conditions imposed. This was followed by some 'manual' work carried out by the student (remembering that a 'black box' solution is not the goal here), where a system of homogeneous equations are formed and a characteristic (transcendental) equation derived. *Mathematica* takes over again, in that it is used to solve for the four lowest eigenfrequencies for the characteristic equation. This is followed by the calculation and plotting of the eigenmodes.

This example shows that it is important to carefully plan for when *Mathematica* can contribute to the conventional mathematical tasks and not construct a laboratory where *Mathematica* is all pervasive. Also the graphical method was chosen to solve the characteristic equation as it was thought more educationally instructive.

3. Evaluation of teaching and learning

The evaluation process was subdivided into two investigations, one in the form of an experiment comparing the knowledge of the group with the computer algebra system in their course with those of a control group using only the conventional text book approach. Participation in the control group was voluntary, and the computer algebra system was not required by the curriculum. The second part of the evaluation is in the form of an online questionnaire eliciting the views of students on the use of the computer algebra system.

3.1 Controlled experiment

To investigate the effectiveness of introducing the software laboratories into the Engineering Mathematics course, a controlled experiment applying a pre-test-post-test control group design was conducted [21, 22]. The students had to undertake two tests, one before the respective course (pre-test) and one after the respective course (post-test) with the introduction of the computer algebra system being evaluated by comparing within-student post-test to pre-test scores, and by comparing the scores between students in the experimental group (A), i.e. those who were taught using the course containing the software laboratories, to those students in the control group (B), i.e. taught using the conventional method of lectures and tutorials only. The various possibilities of the methods of teaching are summarized on Fig. 5. Conventional extra tutorials were given to students of the control group (B) so that approximately similar total time was spent to learn by all students.

To measure the performance of the two groups, four constructs were used with each construct represented by one dependent variable. Each dependent variable has the hypotheses:

1. There is a positive learning effect in both groups (A: experimental group, B: control group). This means post-test scores are significantly higher than pre-test scores for each dependent variable.
2. The learning is more effective for group A than for group B, either with regard to the performance improvement between pre-test and post-test (the relative learning effect), or with regard to post-test performance (absolute learning effect). The absolute learning effect is of interest because it may indicate an upper bound of the possible correct answers depending on the method of teaching.

The design starts with random assignment of students to the experimental group (A) and control

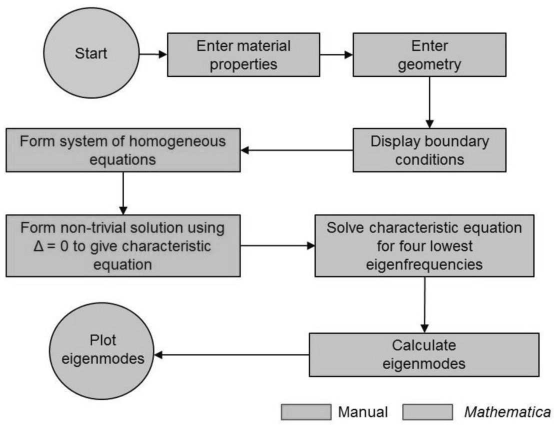

Fig. 4. Flow diagram of the method of solution.

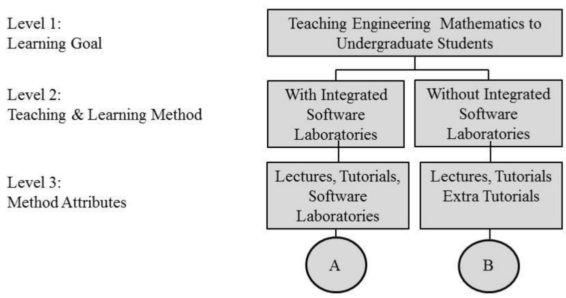

Fig. 5. Scheme outlining the two methods of teaching.

Table 1. Personal characteristics

Characteristics	
Average age	20.8 years
Percentage female	52%
Preferred learning style(s)	
Reading with exercise	19%
Lecture	10%
Tutorial	27%
Laboratory	20%
Working in groups (with peers)	24%
Opinion of most effective learning style(s)	
Reading with exercise	14%
Lecture	12%
Tutorial	28%
Laboratory	21%
Working in groups	25%

Table 2. Experimental variables

Dependent variables	
J.1	Interest in Engineering Mathematics ('Interest')
J.2	General knowledge of Engineering Mathematics ('Understand general')
J.3	Understanding of 'simple' Engineering Mathematics ('Understand simple')
J.4	Understanding of 'difficult' Engineering Mathematics ('Understanding difficult')
Subjective perceptions	
S.1	Available time budget versus time need ('Time pressure')
S.2	Course evaluation (useful, engaging, easy, clear)

group (B) with the members of both groups completing a pre-test and post-test. The pre-test measured the performance of the two groups before the courses and the post-test measured the performance of the two groups after the courses. The students did not know that the post-test and pre-test questions were identical and neither were they allowed to retain the pre-test questions with the correct answers only given to the students after the experiment.

The students were in the penultimate year of an engineering course with the number of students in group A, $N_A = 68$, and in group B, $N_B = 68$. The personal characteristics of the students are summarized in Table 1.

The initial testing was conducted after a short introduction as to the purpose of the experiment and general organizational issues. The pre-test was then carried out with the data for the dependent variables collected. Following the pre-test, the students were placed in either the control group or the experimental group and all students participated in both the pre-test and post-test. After completing their courses, both groups of students performed the post-test using the same questions as during the pre-test, thus providing data on the dependent variables for the second time. In addition the students were asked to answer questions about subjective perceptions.

Data for two types of variables were collected, the dependent variables (J.1, . . . , J.4) and the subjective perception variables (S.1, S.2). These variables are listed in Table 2. The dependent variables are constructs used to capture aspects of learning provided by the courses and each was measured using 5 questions. Selected examples of questions used are shown in Table 3.

The results for the dependent variable J.1 were found by applying a five-point Likert-type scale [23] with each answer mapped to the value range R= [0, 1].

The values for variables J.2–J.4 are average scores derived from five questions for each. Missing answers were marked as incorrect. The data for the subjective perception variables was collected after the post-test. The values for variable S.1 are normalized averages reflecting the time needed for understanding and doing the tasks associated with Weeks 2–12.

The descriptive statistics for the experiment are summarized in Table 4. The columns 'Pre-test scores' and 'Post-test scores' show the calculated values for mean, median (m) and standard deviation of the raw data collected, and the column 'Difference scores' shows the difference between the post-test and pre-test scores.

Table 5 shows the calculated values for mean, median and standard deviation of the raw data collected on subjective perceptions.

The students in control group (B) expressed less need of additional time than those of the experimental group (A), while students of both groups were fairly equal in their perception of their respective course usefulness, engagement, difficulty and clarity.

Standard significance testing was used to investigate the effect of the treatments on the dependent variables J.1 to J.4. The null hypotheses were:

$H_{0,1}$: There is no difference between pre-test scores and post-test scores within group (A) and control group (B).

$H_{0,2a}$: There is no difference in relative learning effectiveness between group (A) and control group (B).

$H_{0,2b}$: There is no difference in absolute learning effectiveness between group (A) and control group (B).

For $H_{0,1}$ and focusing on the experimental group (A), Table 6 shows the results using a one-tailed t-test for dependent samples. Column one specifies the variable, column two represents the Cohen effect size, d, [24, 25], column three the degrees of freedom, column four the t-value of the study, column five the critical value for the significance value $\alpha = 0.10$ and column six lists the associated p-value.

It can be seen from Table 6 that all dependent variables achieve a statistically and practically significant result.

Table 3. Example questions (pre-test, post-test, subjective perceptions)

J.1 example question
I consider it very important for engineering students to know as much as possible about engineering mathematics.
(1 = fully agree/5 = fully disagree) Circle number below.

Agree 1 2 3 4 5 Disagree

J.2 example question
(a) Let $f(x, y) = 2x^3 - 24xy + 16y^3$. Determine the nature of the critical points of f.
(b) Use the Lagrange Multiplier technique to find the point on the circle

$$(x-1)^2 + (y-2)^2 = 4$$

that is closest to the point (2,4).

J.3 example question
A function is defined in the interval $[0, k]$ as $f(y) = 1$. Sketch the function in the interval $[-3k, 3k]$ and show its Fourier Sine series expansion is:

$$f(y) = \frac{4}{\pi}\sum_{m=1}^{\infty}\frac{1}{2m-1}\sin\left(\frac{(2m-1)\pi y}{k}\right)$$

J.4 example question
A surface is represented by the following equation

$$x^2 + y^2 - 4x - 2y - z + 9 = 0$$

(a) Complete the squares and identify the surface.
(b) Let $z = k =$ *constant* and determine the standard form of the contours.
(c) The surface intersects the plane $z - 2y = 4$. Parametrically define the line of the intersection.
(d) Consider the point (3, 2, 6) on the above surface.
(i) What direction produces the greatest increase in z.
(ii) What direction is normal to the surface.

S.1 example question
I did not have enough time to:
– complete the tutorials
– complete the laboratory/extra tutorial sessions
– write-up the laboratory/extra tutorial reports
– complete the post-test

S.2 example question
I consider the explanations/information provided for laboratory/extra tutorial sessions

1 2 3 4 5

Useful	Useless
Boring	Engaging
Difficult	Easy
Clear	Confusing

Table 4. Scores of dependent variables

	Pre-test scores				Post-test scores				Difference scores			
	J.1	**J.2**	**J.3**	**J.4**	**J.1**	**J.2**	**J.3**	**J.4**	**J.1**	**J.2**	**J.3**	**J.4**
Group A												
($\bar{x}$)	0.77	0.70	0.39	0.35	0.79	0.86	0.48	0.54	0.02	0.16	0.09	0.19
(m)	0.81	0.66	0.40	0.32	0.84	0.78	0.50	0.41	0.03	0.12	0.1	0.09
(σ)	0.12	0.30	0.27	0.24	0.11	0.24	0.16	0.21	0.11	0.19	0.24	0.20
Group B												
($\bar{x}$)	0.86	0.61	0.46	0.29	0.87	0.67	0.48	0.41	0.01	0.06	0.02	0.12
(m)	0.84	0.63	0.43	0.26	0.86	0.65	0.45	0.31	0.02	0.02	0.02	0.05
(σ)	0.13	0.17	0.19	0.11	0.21	0.19	0.17	0.14	0.09	0.18	0.28	0.17

Table 7 shows the results of testing hypothesis $H_{0,1}$ for the control group (B) using a one-tailed t-test for dependent samples. The structure of the table is the same as that of Table 7.

It can be seen from Table 7 that the control group (B) achieved statistically and practically significant results for dependent variables J.2 and J.4. For J.1 and J.3 no significant results can be found.

For $H_{0,2a}$ which states that the difference between post-test and pre-test scores of group A is not

Table 5. Scores of subjective perceptions

	S.1	S.2
Group A		
($\bar{x}$)	0.64	0.58
(m)	0.62	0.57
(σ)	0.13	0.08
Group B		
($\bar{x}$)	0.59	0.59
(m)	0.57	0.56
(σ)	0.11	0.10

Table 6. Results for 'post-test' versus 'pre-test' for group A

Variable	d	df	t-Value	Crit.$t_{0.90}$	p-Value
J.1	0.1738	67	1.423	1.294	0.080
J.2	0.5890	67	4.820	1.294	0.000
J.3	0.3605	67	2.951	1.294	0.002
J.4	0.8426	67	6.897	1.294	0.000

Table 7. Results for 'post-test' versus 'pre-test' for group B

Variable	d	df	t-Value	Crit.$t_{0.90}$	p-Value
J.1	0.0573	67	0.469	1.294	0.320
J.2	0.3328	67	2.724	1.294	0.004
J.3	0.1109	67	0.908	1.294	0.184
J.4	0.9532	67	7.802	1.294	0.000

Table 8. Results for 'performance improvement' (Group A versus Group B)

Variable	d	df	t-Value	Crit.$t_{0.90}$	p-Value
J.1	0.0995	134	1.152	1.289	0.126
J.2	0.5404	134	6.256	1.289	0.000
J.3	0.2301	134	2.664	1.289	0.004
J.4	0.3772	134	4.366	1.289	0.000

Table 9. Results for 'post-test improvement' (Group A versus Group B)

Variable	d	df	t-Value	Crit.$t_{0.90}$	p-Value
J.1	*–0.477*	134	*–5.522*	1.289	1.000
J.2	0.877	134	10.152	1.289	0.000
J.3	0.000	134	0.000	1.289	0.500
J.4	0.728	134	8.427	1.289	0.000

significantly larger than the one for group B. Table 8 shows for each dependent variable separately the results of testing hypothesis $H_{0,2a}$ using a one-tailed t-test for independent samples.

It can be seen that the hypothesis $H_{0,2a}$ can be rejected for the variables J.2, J.3 and J.4.

Table 9 shows for each dependent variable separately the results of testing $H_{0,2b}$ using a one-tailed t-test for independent samples.

The two variables which show statistically significant results are J.2 and J.4 and hence $H_{0,2b}$ can be rejected for these variables. The variables J.1 and J.3 indicated that the students thought Engineering Mathematics more important to their engineering studies than those who used the computer algebra system in their course whilst students got equal results for J.3 irrespective of whether they used the software or not.

3.2 Online questionnaire

An anonymous online survey was conducted after students obtained their grades for the laboratory reports to aid formative evaluation of the introduction of the computer algebra system. Only students in the experimental group (A) were surveyed. A questionnaire using 10 statements as listed in Table 10 was designed for this survey. Students were requested to respond to each item in the questionnaire using a five-point scale: strongly agree, agree, neutral, disagree and strongly disagree plus a column for no opinion. An opportunity was also provided for students to comment on their experience at the end of the questionnaire to collect qualitative feedback on their experience so far with the computer algebra system.

Generally, student feedback surveys have a very low response rate [26, 27]. However the response rate here was high (>75%) and overall, the results from the survey were positive. The responses to the survey are shown on Fig. 6 and indicate that students felt that they benefited from their exposure to the computer algebra system.

From Fig. 6 it can be seen that for the first six statements there was generally a strong positive

Table 10. A list of questions/statements used in the survey for students' feedback

No.	Statement
1	I found the software easy to use.
2	The use of software laboratories enhances my understanding of the theory course.
3	The combination of software and traditional Engineering Mathematics helps me concentrate on mathematical concepts better.
4	The use of this software would help with those engineering concepts I have already encountered in other modules.
5	I like the 'hands-on' and 'self-discovery' approach when using the software.
6	By using the software, I now feel Engineering Mathematics is a more important part of Engineering than I did before.
7	I now have a knowledge of programming skills sufficient for me to work on without help.
8	The software has added to my skills needed for future professional projects.
9	I will attempt to make use of this software in future engineering modules.
10	The skills provided by this software will make me more confident in my future career.

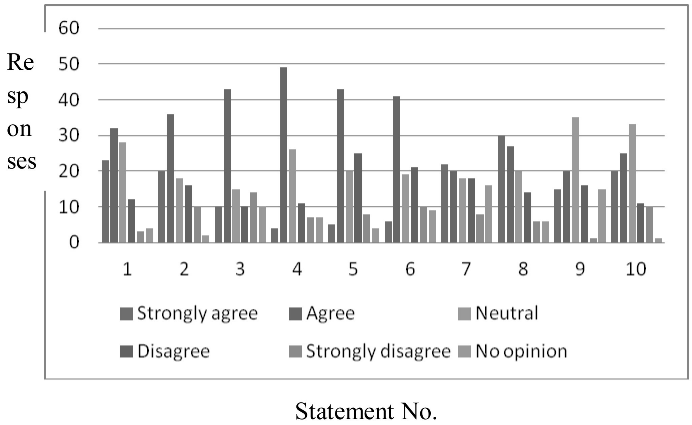

Fig. 6. Chart showing survey results.

response. This included the important opinion that the students did indeed feel that Engineering Mathematics was now viewed as a more important part of general engineering than before using the software package (Statement 6). For statements 7 and 8 the results were more balanced but the students were still reasonably positive about the skills imparted to them. For statements 9 and 10 there was a distinct reserve in the students' responses with no opinion either way resulting as the mode.

In additional comments, most of the students expressed the view that the amount of material introduced was correct, although some felt that they were not yet comfortable with the mechanics of programming within the computer algebra system and that they took a long time to complete correctly. Students were particularly appreciative that they could visualize their results very easily which they felt helped with their understanding of the problem/topic at hand. Students generally agreed that the combination of traditional teaching and software laboratories led to better understanding of engineering mathematics and of how professional engineers work in industry and research. Students showed some enthusiasm for learning more about the software package.

In addition to the questionnaire of Table 10, the students were asked "Would you recommend that the computer algebra system remains as part of the Engineering Mathematics course in the future?" To this 73% said yes so showing that the majority thought the software package as having a positive impact on their studies.

4. Discussion

4.1 Controlled experiment

When considering the positive learning effect within the experimental group (A), a statistically significant positive change of scores was found from the pre-test to post-test for the dependent variables J.1 to J.4, showing that the course which included the computer algebra system had raised understanding and interest in learning aspects of Engineering Mathematics. The results for the control group (B) were much less positive where variables J.2 and J.4 showed improvement but variables J.1 and J.3 did not. The reasons for this disturbing lack of growth of interest in the subject could be many ranging from 'students do not really see Engineering Mathematics as an important part of the overall engineering programme' to 'the fact that Engineering Mathematics can be in the eyes of some students highly demanding with good attention to detail needed'.

With the exception of the variable J.1 testing the performance of relative learning effectiveness between groups (hypothesis $H_{0,2a}$) showed that the experimental group (A) yielded significantly better scores for relative effectiveness for all dependent variables. For absolute learning effectiveness the results were more mixed. There was a definite absolute increase in interest of Engineering Mathematics amongst the control group (B) and it was found that the inclusion of the computer algebra system made no difference to the understanding of 'simple' Engineering Mathematics. More reassuringly it was found that the inclusion of the computer algebra system did make a significant difference in the understanding of 'difficult' Engineering Mathematics.

It should be noted that the time needed to conduct the six laboratories, which generated additional time pressure on the subjects in the experimental group (A) did not seem to have any negative effect on the scores of variables J.2, J.3 and J.4.

4.2 Questionnaire

From the results shown on Fig. 2 and in Table 11, the overall response by the students in the experiment group (A) was positive to the inclusion of the computer algebra system within the Engineering Mathematics course. It did become clear however that students need to be better 'primed' regarding the use and programming of *Mathematica* in that frustration can lead to concentration on the mechanics of getting a solution rather than understanding and exploring the concepts behind the given problem. In particular it became obvious that the input of numerical using the current method inside lines of coding was particularly frustrating and prone to error, and therefore a better interface needs to be generated for this function. It was found however, that the interface design provided by the computer algebra system and in particular, the ease of using the provided 'Help', which had lots of clear examples, built in, does provide students with hands-on experience, gained through an interactive and reasonably user-friendly environment, and encourages student self-learning. This type of approach is quite normal in today's modern professional environment [3]. It was noted from the survey that the students liked the hands-on and self-discovery approach.

4.3 Mathematica as a cognitive tool

The computer algebra system used here allowed students to concentrate on difficult concepts without focusing on routine skills and procedures. Why was this so? Using an axial coding scheme, interrelationship of categories [28] was explored. The categories which contributed to cognitive demand based on students' similar ideas and/or comments emerged from interviews (formal and informal), observations and transcripts, and, were refined to *recollection*, *cooperation*, *construction*, *Mathematica* and *frustration*. There were subcategories of the category *Mathematica* referring to *calculation*, *organization* and *visualisation*. The relationships deduced between the categories are shown on Fig. 7. As the computer algebra system 'took care' of these three subcategories in a supportive and structured way, this allowed and encouraged the students to explore concepts and think at a higher level. According to Day [29], the emergence of categories *frustration* and *recollection* are indicators of higher levels of cognitive demand.

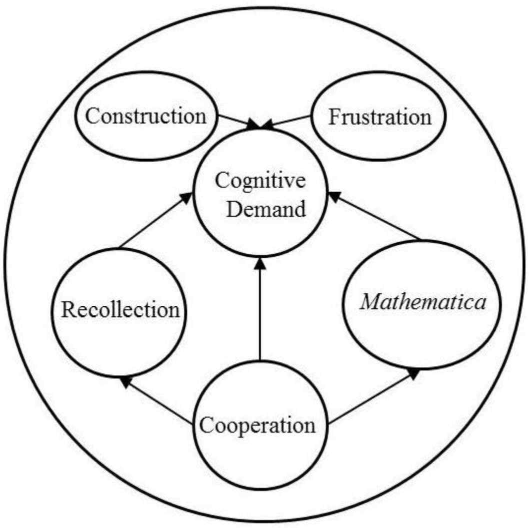

Fig. 7. Deduced categories which contribute to cognitive demand.

The most unexpected result from the above analysis was the degree to which the category cooperation played a significant role. For example assistance was frequently offered when one member of the students had difficulty with the computer algebra system, and many valuable discussions concerning mathematics broke out. There was also evidence of cooperation between students to recall already learned concepts and apply them in a new context. Because the students were not so burdened with routine skills and procedures, time was available for students to share mathematical meanings.

The importance of visualization can hardly be overestimated in general cognitive skill acquisition and problem solving processes [30, 31]. Pictures activate mental processes such as the perception of spatial relationships, intuitive comprehension of complex processes, or the observation of patterns and therefore, aid the process of understanding. It was observed here that the computer algebra system did indeed provide the necessary algorithms needed to compute visualizations. It was appreciated that much less effort was needed to produce quality representations than with classical approaches, especially in 3-dimensions. It may be, after noting the interest and satisfaction of students when achieving a particular plot, that in future, a good starting point in teaching the basic rudiments of *Mathematica* would be to include as much visualization as possible.

In addition to greatly helping with visualization, it was found that this particular computer algebra system was efficient in handling the input of data. It has often been the case that the method of entering data involved students writing lines of code leading to mistakes and frustration. The *Manipulate* environment provided by *Mathematica* can be used, but this takes some effort and expertise, so the function *CreateDialog* was chosen to provide a quick and easy method of entering data. For example, for Example 5: Analysis of a single-span Euler-Bernoulli beam, as mentioned above, a few lines of code produces the Dialog box for input of data as shown on Fig. 8.

Finally, in this subsection, the managing of intrinsic cognitive demand is discussed. Intrinsic cognitive demand is the demand placed on the

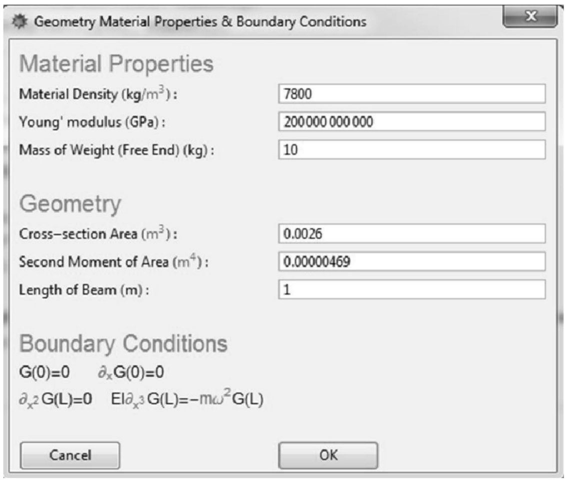

Fig. 8. Dialog input box created by the function *CreateDialog*.

learner by the nature of the materials being learned. Ayers [32] notes that "as expertise develops in a domain, the intrinsic load caused by a specific task decreases as the interactions become learned and incorporated into schemas". This was taken advantage of when use was made of *Mathematica* in learning Engineering Mathematics. With any mathematical problem there are sub-problems to be solved which can easily be done using a computer algebra system, so alleviating some of the intrinsic demand from the learner. Care must be taken however that unless the design of the computer algebra system is done carefully additional extraneous cognitive demand could be imposed. Possible sources of extraneous cognitive load could be the "split-attention" effect, when the syntax of the computer algebra system and the normal mathematical syntax do not match, and, when there is a need to recall what work has been already done. The split-attention effect has been discussed by Sweller [33] and occurs when a student needs to integrate information from several sources when studying a particular problem. When using *Mathematica,* this effect became evident when there was a sequence of input and output statements. Another hurdle to overcome was that the mathematical notation and the syntax of the computer algebra system did not always match. This was particularly evident when trying to understand both the mathematics and the *Mathematica* syntax at the same time. The last of the sources of extraneous cognitive loads, i.e. trying to keep a trace of what had been done before, was to some extent alleviated by the cooperation of other students.

4.4 Validity

The validity of the present work is now discussed. It is recognized that interest in a topic and evaluation of a teaching session are difficult to measure, and to alleviate this problem the instruments for measuring variables J.1 and S.2 were derived from measurement instruments that have already been successfully applied in similar kinds of studies [21, 34].

To alleviate selection threats when dividing the students into two groups, a randomization procedure was used. This, together with the student characteristics of similar age, level of experience of mathematics (especially in calculus) and general level of education when starting the respective Engineering Mathematics courses gave reasonable assurance of minimum bias. Also, there was no change in teaching staff throughout the courses, so reducing any 'selection history effect', and, as none of the subjects left their respective group, there was no 'dropout interaction effect'. Students in both groups were asked not to discuss their course with members of the other group to try to reduce 'diffusion or contamination' effect. Also, it was mentioned to all students that each course was a legitimate method of acquiring mathematical skills and knowledge necessary for later engineering modules, and one course was not necessarily better than the other. This was an effort to minimize any 'rivalry or resentment' threats.

After selection any differences in the ability of the groups was captured by collecting pre-test scores.

Regarding the use in general of a computer algebra system, although most authors concentrate on their advantages, there were early reports on continuing controversies [35] such as gadgetry over intellect and proof-abuse. Although there have been rebuttals to each of the concerns expressed, it is still the case today that computer algebra systems are still not in universal use for Engineering Mathematics instruction.

5. Conclusions

This paper has described the use and efficacy of integrating a computer algebra system into a traditional engineering mathematics course. It has been shown that the incorporation of the computer algebra system has enhanced cognitive development in that an improvement was seen in relative and absolute results from a controlled experiment. However, the inclusion of the computer algebra system had a detrimental effect on interest when compared to the control group possibly due to the fact that the programming needed can be a tedious business even though the results can be very satisfying. The students were also exposed to what they might expect in modern professional practice in that it is now expected they should efficiently and thoughtfully integrate technology when appropriate. They should not however succumb to using technology as a "black box". With appropriate choice of laboratory content it was possible to include within the engineering mathematics more complex and relevant material found in later engineering modules of the degree course so helping to break down any perceived barrier.

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