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Creating the Patterns of Variation with GeoGebra when Teaching Derivative Graphs for First Year Engineering Students*

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The present study investigates how technology assisted and designed teaching influences engineering students' understanding of the connection between the graph of a function and its derivatives. An engineering student group ($n = 27$) was taught with the assistance of GeoGebra while a control group ($n = 20$) was taught in a traditional way. The data of the study consist of the documents and photos of the observation of two lectures and the students' answers to the pre and post tests. In our theoretical framework we discuss the distinction between conceptual and procedural knowledge. When creating the teaching sequences we applied variation theory. In the analysis of the students' pre and post tests results we applied statistical methods. Our experiment revealed that the GeoGebra-assisted teaching design created more opportunities for students to grasp the connection between a function and its derivatives.

Keywords: conceptual knowledge; derivative graphs; technology; variation theory

1. Introduction

This paper is part of a larger project and the previous reports of the technology inspired teaching for engineering students at university level have been presented at ICME 12 [1], CERME 7 [2] and CERME 8 [3] intended as contributions in this area. In this paper we discuss if it is possible to enrich engineering students' learning possibilities of graphical understanding of both functions and their derivatives when applying variation theory in technology-assisted learning environment as well as in the traditional one. Variation theory has been applied in a wide range of teaching and learning studies (see [4–8]), but there are still limited published examples in engineering mathematics education. Apart from our own work [1–3, 9], we find a study by Carstensen et al. [10], who used variation theory to design a coherent set of exercises for electrical engineering students learning Laplace Transforms. Similarly, Fraser et al. [11] applied variation theory to redesign a distillation simulation exercise done by third-year chemical engineering students in order to open up discernment as a way to enhance the possibility of learning of distillation concepts.

Traditionally functions and their derivatives are taught by emphasizing the procedural aspects of the concept. In order to give the students an opportunity to get familiar with conceptual aspects of the notion we decided in our experiment to use GeoGebra due to its dynamical nature. Referring to Hohenwarter et al. [12] by integrating technologies into the teaching practice, teachers can provide creative opportunities for supporting students' learning and fostering the acquisition of mathematical knowledge and skills. In addition, when technological tools are available, students can focus on reflection, reasoning, and problem solving. Technologies also increase a new aspect to the teaching and learning of mathematics by helping students to visualize certain mathematics concept. Hohenwater et al. [12] claimed that the visualization and exploration of mathematical objects and concepts in multimedia environments can support understanding in new ways.

Several changes in higher education such as the weak mathematical preparedness of students entering universities and the emergence of new technologies make new demands on mathematics teaching. The earlier research indicates that the development of mathematical software such as GeoGebra, Maple, Mathematica, Derive, Geometer's Sketchpad and others, have had a positive effect on the student achievements and the teaching and learning of mathematical concepts and ideas [9, 12–22]. Technology is definitively present at teaching of university mathematics but there are still few studies which have examined technology-assisted teaching at university level, even though university mathematics teaching has been changing quickly during the past two decades [15, 22]. The majority of research studies on educational use of technology have been conducted at the compulsory school or upper secondary level (for example, see [13, 23–26]). Results of these studies may be applicable at universities, but because of the substantial differences between the characteristics of school and university

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level teaching, research studies should be conducted at the tertiary level as well. For our purpose in this study we needed dynamic geometry software (DGS). We chose GeoGebra because we were already familiar with this software at our university.

1.1 Students' graphical understanding of the derivative

There are several studies that focus on students' graphical understanding of the derivative [27–29] as well as students' conceptual understanding of the derivative [30–33]. Researchers who investigated students' graphical understanding of derivative reported that students have many kinds of difficulties. For example, Orhun's [29] study including 102 high school students shows that it is common for students to grasp the concept of derivative as algebraic operations or rules, which are used to produce the derivative of the function. The research results also indicated that the students find it difficult to make connections between the graph of derived function and the original function. Students often interpreted the graph of derived function as the graph of function. Graphs convey a lot of information about a function and to acquire a deeper understanding of the concept of derivative, it is important to see the link between the graph of the original function and the derivative graph [29].

One of the most important earlier studies on student understanding of the derivative is Orton's [33] investigation, which involved individual interviews with 110 students. He planned 21 tasks related to differentiation and rate to the students. His study indicated that while the application of the derivative was relatively easy to the participants, the understanding of differentiation and graphical approaches related to rate of change was much more difficult. Students made algebraic errors, symbolic errors and errors concerning limits, in their computation and reasoning. Orton proposed the assistance of calculators, "a more lively approach to the teaching of ratio", and more application to real life situations as a means of helping students come to a more comprehensible understanding.

In order to improve students' conceptions of derivative, mathematics teachers should seriously reflect on their teaching practices with emphasis on conceptual developments in a technological environment [27, 30]. Most of these intervention studies have investigated students' graphical understanding of the derivative in different tasks, but a few have investigated their conceptions in interpreting the graph of a function and constructing its derivative graph [34].

Can technology as a pedagogical tool help students to understand different faces of the mathematical concepts? Technology is becoming increasingly used at teaching of university mathematics but there are still few studies which have examined technology-assisted teaching at the university level, even though university mathematics teaching has been changing quickly during the past two decades [2, 14, 15].

2. Theoretical framework

Knowledge of mathematics can be divided into both conceptual and procedural knowledge [35]. Procedural knowledge refers to computational skills and knowledge of procedures for identifying mathematical components, algorithms and definitions. Procedural knowledge of mathematics has two parts: (a) knowledge of the format and syntax of the symbol representation system and (b) knowledge of the rules and the algorithms, which are useful in mathematical tasks. Conceptual knowledge refers to knowledge of the underlying structure of mathematics. It is characterised as knowledge which is rich in relationships and which includes the understanding of mathematical concepts, definitions and fact knowledge. Both procedural and conceptual knowledge are considered as necessary aspects of mathematical understanding [35]. Thus, teaching of mathematical understanding must include teaching of both procedural and conceptual knowledge [36].

Students' conceptions of the concept of derivative are often dominated by procedural knowledge [33, 37]. The procedural knowledge consists mainly of algebraic methods to determine a function derivative. The procedural knowledge is important for the mathematical work but students find it difficult to relate the algebraic methods to a graphical interpretation [38].Several studies report about teaching experiments in which graphing calculators or computer programs are used in order to introduce the concept of derivative by applying visualizations (for example, see [27, 31]).

These experiments show that student conceptual knowledge can be improved by such methods, but for the students it is laborious and time consuming. It can also reduce students' knowledge on the formal definition [31].

2.1 The purpose of the study

The aim of this study is to investigate if the GeoGebra-assisted teaching design of the concept of derivative compared with traditional way of work at the university level can develop engineering students' graphical understanding of derivative.

The study investigates the following research questions:

1. How can we use GeoGebra to design teaching sequences in order to create to opportunities for

students to grasp the critical faces of the graphs of both functions and their derivatives?

2. Does technology assisted and designed teaching influence engineering students' conceptions of the connection between the graph of a function and its derivatives.

3. Method and design of the study

The study took place during a lecture in mathematics at a Swedish university. A total of 47 engineering students were involved in our study. The students were already divided into two teaching groups (consisting of $n = 27$ respectively $n = 20$ students). The group of $n = 27$ was then randomly selected for learning the concept of derivative with the assistance of GeoGebra while the other group, the control group ($n = 20$), was taught in a traditional way. They were all students at the engineering program, studying the course Real Analysis in one variable. The data were gathered by photos and taking notes of the lectures, doing a pre and post test in both groups and by giving one question regarding the concept of derivative at the final exam for the course. Here, we only discuss the results of the pre and post test and the conducted lectures in the both group. When creating and designing the teaching sequences in the both groups we have applied variation theory. Central in this theory is an assumption that variation is needed to discern aspects of object of learning not previously distinguished by learners. According to this theory the most powerful factor concerning students' learning is how the object of learning is handled in instruction. Marton, Runesson and Tsui [39, p. 16] have identified four patterns of variation in a learning object: contrast, generalization, separation and fusion. They are described as follows:

Contrast: . . . in order to experience something, a person must experience something else to compare it with.

Generalization: In order to fully understand what 'three' is, we must also experience varying appearances of 'three', . . .

Separation: In order to experience a certain aspect of something, and in order to separate this aspect from other aspects, it must vary while other aspects remain invariant.

Fusion: If there are several critical aspects that the learner has to take into consideration at the same time, they must all be experienced simultaneously.

According to Leung [40, 41] who has applied variation theory in dynamic geometry environment these patterns of variation create opportunities for the students to distinguish the underlying formal abstract concept. In order to generate the patterns of variation, we use the dynamical nature of the GeoGebra software, which has the "ability to visually make explicit the implicit dynamism of 'thinking about' mathematical, in particular geometrical, concepts" [40, p. 197–205].

In the analysis of the students' pre and post test results we have used the statistical software Minitab using dependent, two-sided t-test for paired samples at the significance level. We have also compared the students' written responses in order to find any conceptual changes in their conceptions.

3.1 The pre and post test

The test contained three questions, including both conceptual and procedural ones. Students had maximum 30 minutes to do the test. It was not allowed to use any technical facilities. In this paper we focus on these two questions from the test, see Appendix 1: PRE AND POST TEST. The aim of the first question was to investigate how the students grasp the graph of a given function, by asking them to interpret the values of this function and its first and second derivative in the given points. The intention with the second question was to study the students' conceptual understanding of how the shapes of a function and its first and second derivative are related to each other.

4. Results

We used the pre test results as a starting point to design our lectures for both the study and the control group. We used the same lecture plan, including the same examples, in both groups. The lectures were held by two different teachers simultaneously in two classrooms, in order to eliminate any subjective preferences of the teacher to the first or to the second teaching methods.

In the study group we used the dynamic nature of GeoGebra, in order to create patterns of variation of the graph of functions and its derivatives, while in the control group the same patterns of variations were created by using static drawing on the white board. By using this design of the lectures our intention was to encourage students to discern varying aspects of the object of learning,

4.1 Teaching sequences

Teaching sequences were implemented in the study group with a teacher manipulating the computer and students observing the screen. In the first application of GeoGebra (Fig. 1), by moving dynamically the tangent line along the graph of the function and shrinking and moving the neighborhood of the tangent point on the function, we

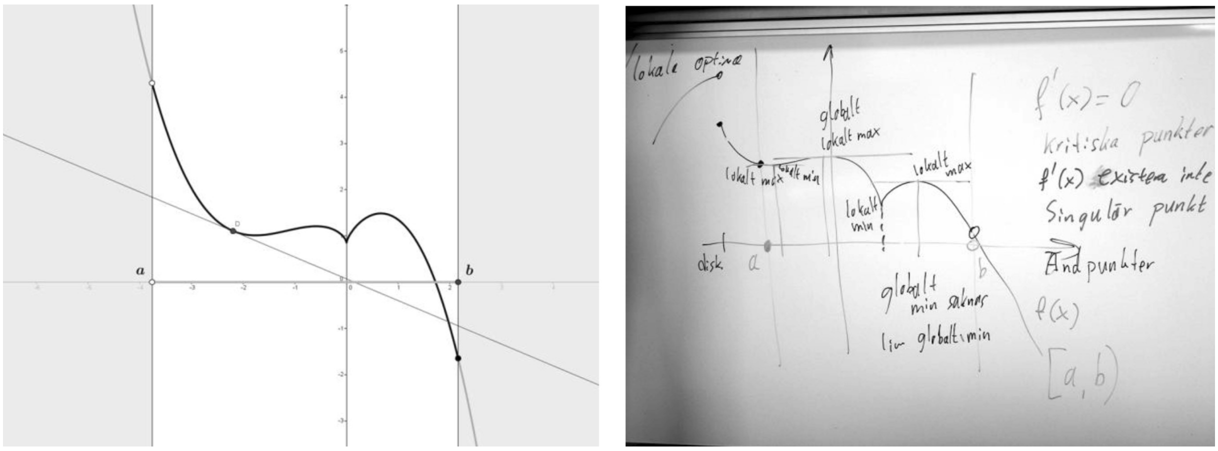

Fig. 1. Two ways (the study group to the left, the control group to the right) to explain local and global extreme points.

visualize what points could be the extreme points of the function.

In this teaching sequence we wanted to explain the concept of local minimum of the function $y = f(x)$ defined on an interval as the point of this interval where $f(a) \leq f(x)$ for all points x in some neighbourhood of this point. In the study group we created the pattern of variation, *generalization*, by moving a point on the function together with its tangent line to different positions, fixing the point and then trying to adjust the neighbourhood to the fixed point in order to achieve the minimum point condition. Further analysing of this teaching sequence we could conclude that another pattern of variation, *fusion* could be distinguished. In the control group the same sequence was conducted by choosing the minimum point and showing that it satisfies the minimum point conditions.

The second teaching sequence (Fig. 2) should help the students to understand how to plot a function in the same coordinate system as the graphs of its first and second derivative are plotted.

In order to plot a function when having its first and second derivatives plotted in the same coordinate system we choose a fixed point and estimate the slope and decide if the function is concave up or down, by approximating the value of the first and the sign of the second derivatives in the fixed point. The intersection points of the derivatives and the x-axes are among the fixed points mentioned previously.

In the study group we used the dynamical nature of GeoGebra by moving a vertical line across the graphs of the first and second derivatives and in this way creating the opportunities for the students to experience the overall impression of how the function should look like. In this way, the pattern of variation, *generalization*, was created. In the control group, except the intersection points with the x-axes, only few static points were investigated.

4.2 *Quantitative and qualitative differences distinguished in pre and post test results*

We analysed the results of the pre and post test of both the study and the control group with the Minitab software using dependent, two-sided t-test for paired samples at the significance level $p < 0.050$.

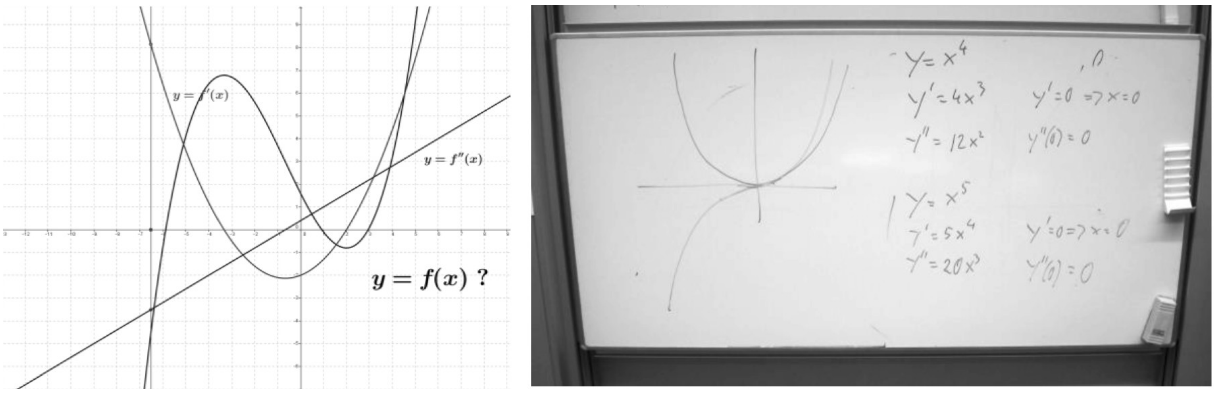

Fig. 2. Two ways (the study group to the left, the control group to the right) of plotting the function $f(x)$ by exploring the graphs of its first and second derivatives.

Table 1. Pre and post test results for the study and the control group

	Study group			Control group			
	Mean	**St Dev**	**p**	**Mean**	**St Dev**	**p**	**Max scores**
Pre test question 1	10.23	4.54	0.066	11.50	3.00	0.053	18
Post test question 1	11.00	5.15		12.68	3.52		18
Pre test question 2	3.08	2.40	0.307	3.00	2.41	0.783	6
Post test question 2	3.63	2.56		2.73	2.39		6
Pre test total	13.31	6.32	0.030*	14.50	5.00	0.061	24
Post test total	14.63	5.74		15.41	5.01		24

*$p < 0.050$.

As we can see from the Table 1 that the only statistically significant improvement can be noticed in the total results for the study group.

In order to find qualitative differences between pre and post test results we reflected on some students' responses to the question 2. When we analysed the students' responses in the pre and post tests we noticed some differences in their conceptions.

The second question in our test focused on the students' conceptual understanding of how the shapes of a function and its first and second derivative are related to each other. One of the students made following responses:

Student 1 pre test: In the second column, $f''(x)$ of $f'(x)$ is constant because the slope of the function is always constant.

Student 1 post test: In the first column my starting points are the extreme values. There the first derivative must be zero. It $(f'(x))$ is first negative ($f(x)$ decreasing) and then up, then down again.

In the second column, once again, I start with the point zero, at $x = 2$ the derivative cuts the y-axis and the original function ought to have an extreme value. $f''(x)$ negative (constant) because $f'(x)$ is a decreasing straight line.

I start from the zero points in the column 3. At -1 and 1 it is 0, hence the derivative have extreme values (2).

This student belonged to the study group and the student received 2 of 6 points in the pre test, and 6 of 6 points in the post test. The student response indicates a change in the conceptual understanding of the connection between a function and its derivatives.

5. Discussion

When analyzing the instruction in both groups we could find that the dynamical nature of GeoGebra software created possibilities to present some patterns of variation, *generalization* and *fusion*, in a more powerful way in the study group compared with the control group. According to Leung [40] these patterns of variation create opportunities for students to understand the underlying abstract concepts. Our statistically processed results indicate that the GeoGebra, with its dynamical nature, is a more powerful tool than the traditional way of teaching, when creating opportunities for the students to grasp the overall picture of the connection between a function and its derivatives (cf. [31]). Our results show that the only statistically significant difference was found in total result of the study group. Comparing the pre and post test response of the student we could also notice that the reasoning moved from the procedural to the conceptual one [35]. This student belonged to the study group which also demonstrates the power of the GeoGebra.

The results in this study could be interpreted as that the simple drawing of mathematical objects and figures as it is usually done in traditional mathematics teaching is not good enough for building conceptual understanding of basic mathematical concepts. In contrast, by using the dynamical nature of GeoGebra it is easier to create varying appearances of mathematical concepts, move fort and back, move from global to local perspective and back, and create cognitive conflicts.

6. Conclusion

The purpose of this study was to investigate how the technology assisted teaching of functions and their derivatives compared with the traditional one can support the teaching and learning of mathematical concepts and ideas in engineering education. When designing the teaching sequences in both groups we applied variation theory, in order to create patterns of variation of graphs and their derivatives that could encourage students to discern varying aspects of the object of learning.

Our study showed that the integration of technology such as GeoGebra within variation theory into university mathematics teaching made it possible for us to actively explore the mathematical structures through systematically varying one aspect of the object of learning while at the same time keeping fixed another one. This approach gave quick feedback and helped students visualize and discern simultaneously varying aspects of the object of learning.

As our findings indicated, the use of GeoGebra within variation theory is a potentially effective tool for developing and designing the teaching and teaching sequences in mathematics. This approach provided also a useful tool for increasing the teachers' awareness of the critical aspects of students' learning and enhancing the learning of mathematics.

We also want to note that further work need to be undertaken to identify which other factors than the integration of technology in teaching and learning of mathematics can be of benefit to both teachers and students in engineering education.

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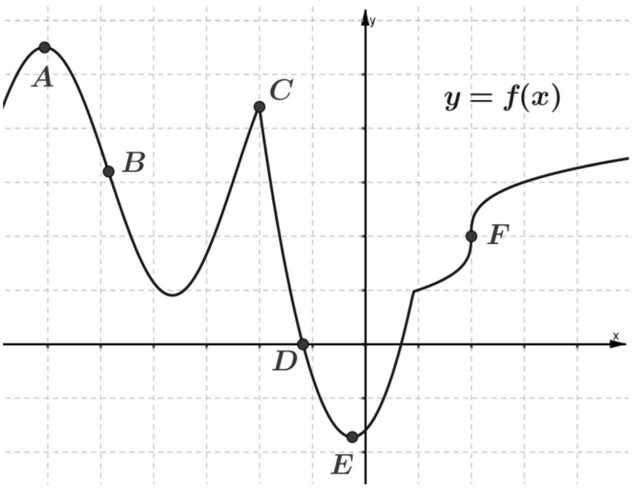
Appendix 1: Pre and Post Test

Question 1: Specify which value the following functions $f(x)$, $f'(x)$, $f''(x)$ assume in each point, A to F. Indicate in the table whether the value is positive $(+)$, negative $(-)$, zero (0) or not defined (x). Motivate with your own words one/some of your responses:



	Functions		
Point	$f(x)$	$f'(x)$	$f''(x)$
A			
B			
C			
D			
E			
F			

Question 2: Fill in the empty squares with one of the numbers (1) to (6), depending on what you believe the correct placement of the proposed graphs to the right is. Motivate with your own words one/some of your responses: