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A Review of Mathematical Modeling in Engineering Education*

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Mathematical modeling is both an important tool to professional engineers and instructional method in engineering education. This paper reviews the literature around mathematical modeling activities in the undergraduate engineering classroom. Twenty-seven journal articles were selected through a literature review process based on decided inclusion and exclusion criteria. From there, multiple themes arose surrounding the topics of student strategies to mathematical modeling, instructional implementation of mathematical modeling, and assessment of these activities. The result of the study is a clearer picture of what is most effective in implementation of mathematical modeling methods, as well as the effects of modeling activities on the learning of undergraduate engineering students. In addition, considerations around assessment of mathematical modeling activities are discussed. This study also identifies gaps in the literature that need bridged to obtain a holistic view of mathematical modeling activities and their true utility to the education of our engineering workforce.

Keywords: mathematical modeling; engineering education; undergraduate

1. Introduction

Analytical problem solving skills are core to both engineering and engineering education [1]. More specifically, mathematical modeling skills are critical problem solving skills that engineers commonly use in the workplace [2]. Given that mathematical modeling requires students to solve difficult and often ambiguous problems, the goal of this study is to understand techniques both students and instructors employ in mathematical modeling activities.

If the modeling process truly is fundamental to the engineering toolbox, its instruction requires evidence-based research in creating efficient learning experiences for engineering students, as well as the effectiveness of those experiences on student learning. Creating realistic learning experiences that truly engage students in complex problem solving, requires the integration of ideas and procedures drawn from multiple disciplines [3]. This process of designing complex problem-solving experiences entails extending beyond the traditional methods for teaching of engineering science and design [4]. Thus, mathematical modeling, much like engineering, is a cross-disciplinary exercise that exceeds the scope of any one individual course or field. Mathematical modeling may be a useful tool for educators to address the wide variety of education goals needed in the engineering classroom as it can address many of the complex and ill-defined systems required by engineers [5]. Hence, the engineering field finds itself positioned to teach a much broader knowledge base than ever before using mathematical modeling techniques.

In addition to the adequacy of teaching mathematical modeling concepts, however, is the frequency by which mathematical modeling activities are becoming critical pieces to the engineering education classroom. Problems that are more realistic and open-ended are showing up much more frequently across all disciplines and levels within the university [6]. Modeling problems are often situated and benefit from being authentic and realistic open-ended problems [4, 7]. If these problems and activities are showing up more commonly, students' approach and learning as well as instructors' assessment of these materials are becoming important criteria in determining the success of these exercises. As seen previously, assessment of these activities becomes more challenging given the broad scope of many of these modeling problems. Because the assessment becomes more broad, the more interpretation is required from the instructor, which leads to the instructor needing to give more open-ended feedback or inquisitive feedback to students [8]. By nature, open-ended problems, such as modeling problems, require interpretation of the student solution because of the infinite responses on which a student could land. This creates multiple implications, the biggest of which is that in some open-ended problems there may be a high degree of variability in assessment of these exercises. In turn, one can then ask the question as to how instructors should appropriately assess these activities that are both feasible

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from an instructional level, and constructive to the overall learning of the engineering student.

The above topics lead to two questions of which the engineering education community needs answered. Given that mathematical modeling is important to both education and professional practice, and the frequency by which we see these activities is on the rise, an understanding of how both instructors and students interact with these mathematical modeling activities would prove beneficial. In addition, because of the nature of these problems as open-ended constructs, understanding how instructors can better assess these modeling activities is important to avoiding the subjective nature of grading that can result. Considering these ideas, this paper synthesizes research through a literature review on the use of mathematical modeling in the engineering classroom. Our goal was to find out how mathematical modeling differentiates learning in the undergraduate engineering classroom, the strategies to modeling used by undergraduate engineering students, and assessment of student solutions to mathematical modeling problems. The review lends itself to help educators better use these tools at the undergraduate level so that schools and universities produce engineers that are more capable of solving complex and open-ended problems with the important engineering practice of mathematical modeling.

2. Scope and Research Questions

The first part of a literature review identifies scope and research questions that will be investigated throughout the process of the review [9]. The research questions for this paper were:

(1) What strategies do students use when solving mathematical modeling problems?
(2) How can mathematical modeling activities be structured to promote learning in the undergraduate engineering classroom?
(3) What techniques do instructors use for assessing the work from these open-ended modeling problems?

From the research questions, there were limits that clearly defined what the entirety of the scope would be for the review. The first shows that the review should focus on the learning process associated with mathematical modeling and modeling activities. In addition, question one limited the scope to the undergraduate engineering classrooms, giving a clear population for the articles selected to address. Questions two and three limited the scope to aspects involved in the processes of learning and teaching, focusing on pedagogy and assessment of the process in question. It is important to note that there are many types of modeling outside of mathematical modeling itself. For this review, mathematical modeling was defined as mathematical representation of physical phenomena [10]. This stands apart from physical modeling, conceptual modeling, as well as other types of engineering modeling. Albeit that these skills are important to the engineering curriculum, the scope of the research questions above is limited to only mathematical modeling activities. However, these activities can be carried out in a host of different formats and it is not the purpose of this paper to limit the format by which these activities can manifest.

3. Methods

This paper uses elements of a systematic review such as; (1) identifying scope and research questions, (2) defining inclusion and exclusion criteria, and (3) finding and cataloging sources from databases and from applied inclusion and exclusion criteria [9]. Given that the scope and research questions are listed above, this section sets out to look at the inclusion and exclusion criteria, the finding and the cataloging of the sources, and the selecting of the final set of papers to be reviewed.

3.1 Keywords

The keywords used for the review were pulled directly from the research questions listed previously. Table 1 outlines each of the keywords used, along with justification as well as synonyms used throughout the search process.

Some of the synonyms listed in Table 1 are not exact synonyms for the keyword but were used in the context of such in the search strings. "MEA" or "model-eliciting activity" was used as a synonym in place for mathematical modeling. It has been shown in previous research that model-eliciting activities can be used as an appropriate method to bring mathematical modeling into the engineering classroom [4, 11]. Because of the prominence of the word in the engineering field, it was important to bring the term into the context of this review. In addition, words such as college and first-year are not direct synonyms for the central keyword "undergraduate", however given the frequency by which these terms were seen by the authors in initial searches they were added as synonyms to ensure quality of results from the selected databases.

3.2 Databases

For the literature review process, selection of databases was taken to ensure that adequate breadth of search was included, but elimination of unnecessary information was also considered in the search. For literature reviews, the primary

Table 1. The keywords, synonyms, and the relevance of the keywords to the research questions

Keyword	**Relation to research questions**	**Synonyms**
Mathematical Modeling	Related to all three research questions as the central investigation of the paper is the effects of mathematical modeling on the undergraduate engineering education.	Modeling, MEA, MEAs, model-eliciting activities, model-eliciting activity, model-development
Education	The scope of the research questions is limiting to the uses of mathematical modeling in educational undergraduate spaces.	–
Engineering	The scope of the research questions is limiting to the uses of mathematical modeling by engineering students.	–
Undergraduate	The population relating to the research questions is limited to undergraduate engineering students.	College First-year

source of literature should be subject specific databases [9]. Because the research topic included both education aspects as well as engineering aspects, both engineering and education subject specific databases were used for the search. Three search engines were identified for the purposes of this review. (1) *ERIC: Education Resource Information Center*, which is provided by the Institute of Education Sciences, produced many articles that were relevant to the use of modeling in the engineering classroom. (2) *Education Source*, provided additional education journals that were not included in the other education databases. There were some duplicates between database (1) and (2), however they each provided enough unique articles that both were important to consider for the purposes of this review of the literature. And finally (3) *Compendex* was included as it gave many studies from noneducation researchers in the engineering field. Given the interdisciplinary aspects of the field of engineering education, the results showed that there were many engineering education studies that were published in more traditional engineering journals, and *Compendex* could capture many of these studies, whereas the two education focused search engines could not.

3.3 Search Strings

Search strings were finalized after multiple iterations. Table 2 lists the exact search strings used throughout the process to generate the results obtained. It is important to note that each mention of a keyword includes not only itself, but all synonyms as well. The search structure was as follows:

(1) Search the title for the keyword “mathematical modeling.”

Table 2. Exact search strings and options used for each database

Database	**Search String**	**Additional Options**
ERIC	title:(“Mathematical Modeling” or “Modeling” or “mea” or “Model-eliciting activities” OR “meas” OR “Model-eliciting activity” or “Model-development”) and (title:(“Education”) or abstract:(“Education”) or descriptor:(“Education”)) and (descriptor:(“Engineering”) or abstract:(“Engineering”) or title:(“Engineering”)) and (title:(“Undergraduate” or “College” or “First-year”) or abstract:(“Undergraduate” or “College” or “First-year”) or descriptor:(“Undergraduate” or “College” or “First-year”))	“Peer-Reviewed” box checked “Journal Articles” criterion selected
EBSCO: Education Source	*TI(“Mathematical Modeling” or “Modeling” or “MEA” or “MEAs” or “Model-eliciting activities“ or “Model-eliciting activity” or “Model-development”) and (TI(“Education”) or AB(“Education”) or SU(“Education”)) and (AB(“Engineering”) or TI(“Engineering”) or SU(“Engineering”)) and (TI(“Undergraduate” or “College“ or “First-year”) or AB(“Undergraduate” or “College“ or “First-year”) or SU(“Undergraduate” or “College“ or “First-year”))*	“Scholarly (peer-reviewed) Journals” box checked
Engineering Village: Compendex	((((((“Mathematical Modeling” OR “Modeling” OR “MEA” OR “MEAs” OR “Modeling-eliciting activities” OR “Model-eliciting activity” OR “Model-development”) WN TI) AND ((“Engineering”) WN KY)) AND ((“Education”) WN KY)) AND ((“Undergraduate” OR “College” OR “First-year”) WN KY)))	“Journal Articles” box checked

(2) Search the title, abstract, and subject for the keyword "education."
(3) Search the title, abstract, and subject for the keyword "engineering."
(4) Search the title, abstract, and subject for the keyword "undergraduate."

The central topic of the paper is mathematical modeling, and as such, results needed that keyword, or synonym, in the title to be included in the results. Given the research questions presented, mathematical modeling must be central to the paper to yield relevant results. The rest of the keywords were qualifiers to the central topic, and as such, the title, abstract, and subject were all searched for mentions of these keywords. All search strings for all three selected databases were of the same format, using the same words, with the same logical statements.

In addition to the pure search strings, there were additional options given by the search engines that were added. All three databases had an option to limit the search field to journal articles that was used for the search. In addition, two of the three databases had an option that limited the search results to peer-reviewed documents. *Engineering Village* did not have an explicit filter for peer-reviewed documents, as a result, criteria for those papers were accomplished through the manual inclusion and exclusion criterion limiting process.

3.4 *Inclusion/exclusion criteria*

Inclusion and exclusion criteria must be determined to take the search results and narrow them to only the articles and materials that were directly related to the subject of the study [9]. Table 3 lists all such criteria that were established to carefully select the articles that would be synthesized for the literature review. It is important to know that some inclusion and exclusion criteria were applied using the databases themselves, while others were needing to be applied manually by the lead author himself.

The inclusion criteria for this study were fourfold. (1) Studies included were peer-reviewed journal articles. For this study, grey literature and other materials were not included as to keep all sources peer-reviewed and of high quality. (2) The study must focus on the use of mathematical modeling in the classroom or educational setting. Papers focused on computational modeling, which is often needed to solve higher order mathematical models, were included unless the process seemed to not have students or the program strongly use mathematics. (3) The study must focus on the learning, teaching, or the effects of use of mathematical modeling. There were results that had a modeling activity or component to them, but it was merely peripheral to the paper, where the research question investigated had nothing to do with learning or teaching modeling in an education context itself. Curricular innovation in modeling methods is important but falls outside the scope of this paper. (4) The study must contain elements of mathematical modeling such as equation modeling, programming modeling, procedural mathematical modeling, or model-eliciting activities. There were many studies in the results that focused on modeling other than mathematical modeling (physical modeling, solid modeling, or purely simulation). This is likely due to the multiple uses of the term "model". It is not the intent of this paper to imply that these other modeling activities are not important to the engineer's education or of equal importance to mathematical modeling.

In addition, four exclusion criteria were used to narrow the search results. (1) The study focused on another population other than undergraduate engineering or engineering education students. However, students who had exited high school, were

Table 3. Inclusion and exclusion criteria for the search results

Inclusion Criteria	**Exclusion Criteria**
The study is a peer reviewed journal article.	The study focused on another population other than undergraduate engineering or engineering education students (includes summer bridge). This would exclude other STEM disciplines such as computer science, technology, or strictly mathematics.
The study must be on the use of mathematical modeling techniques in the classroom or educational settings.	The study only focuses on the development of new modeling tools, techniques, or laboratories for specific/disciplinary problems without students data/feedback or does not draw significantly from educational theories/literature.
The study must focus on the learning, teaching, or the effects of use of mathematical modeling.	The study focused solely on physical modeling, solid/3D modeling, or other types of modeling.
The study must contain elements of mathematical modeling such as equation modeling, computational modeling, or model-eliciting activities.	The study's main focus is on student learning through another topic (such as distance learning or a flipped classroom), where modeling is a minor component to the exercise.

going into an undergraduate engineering or engineering education program, and were participating in a summer bridge program were considered as in scope for this process. If the primary focus of the paper was not one of these two populations it was excluded. If papers equally focused on another population (such as graduate engineering students, or high school mathematics) and engineering undergraduate students it was included. Computer science/software engineering, engineering technology, and information systems students were excluded. If the authors were not reasonably clear on what students were undergoing an intervention or for who it was intended, it was excluded. (2) The study only focused on the development of new modeling tools/techniques/laboratories for specific or disciplinary problems without students data/ feedback or did not draw significantly from educational literature. Some results were articles that were solely on a new modeling technique, without implications for the new method's implementation, learning, assessment, or teaching. If a discipline/ topic specific novel method or tool was introduced it was excluded unless it had specific data around the incorporation of the exercise. However others like Diefes-Dux, Bowman, Zawojewski, and Hjalmarson [12] were included as they gave a general mathematical modeling method (MEA) and its connections to broader models and modeling educational theory/literature by proposing a framework, even though they had no student data or feedback in the paper. (3) The study focused on physical modeling or solid modeling. Studies that focused on types of modeling such as surface modeling, geometric/3D modeling, relational modeling, functional modeling, and software system modeling were excluded. Techniques such as building information modeling commonly used by construction engineering and management disciplines were excluded. (4) The study's focus is on student learning through another topic, where modeling is a minor component to the exercise (such as distance learning). Given that mathematical modeling is the main interest of the research questions, the articles should focus on mathematical modeling as the vehicle of learning for this exercise.

Initially, there were 97 results from all three search engines. However, once duplicates were removed from the paper set, 86 individual papers were the result. These 86 papers are referred to by the authors as the *initial set.* These papers were then subject to the series of inclusion and exclusion criteria stated above. Initially, the titles and abstracts of papers were read with the inclusion and exclusion criteria applied. After this process, the paper set had dwindled down to 56 results. In the 56 results however, there were multiple papers that could not be determined to fit the listed inclusion and exclusion criteria from the abstract alone. From there, the introduction and discussion/conclusion of each paper were read with the inclusion and exclusion criteria applied to make the final paper set, which included 27 studies. These 27 papers are referred to by the authors as the *final set.* These 27 papers were then read for ideas that related to each of the guiding research questions for the study, with the ideas being organized into themes. In addition, key characteristics of each of the papers such as the methodology used, engineering discipline addressing, the setting of the research, and the data sources for each of the papers were identified.

4. Results

Each of the papers in the final paper set are listed in Table 4, 5, 6, and 7. For each of the papers in the set,

Table 4. Studies involving the most common individual discipline, electrical and computer engineering

Paper	Description	Discipline	Course
[13]	Two cases of a design intervention using modeling to teach concepts around electric circuits are investigated in order to understand the effects that different representations have on conceptual understanding.	Electrical and Computer Engineering	Linear Circuits Analysis course
[14]	This study presents a graphical user interface that allows students to choose between various mathematical models and how this program impacts student self-reported ability and feedback.	Electrical and Electronics Engineering	A control theory and control laboratory course
[15]	This study presents a modeling and simulation intervention of power electronics systems used in an electrical engineering curriculum.	Electrical Engineering	Industrial Electronics Laboratory course
[16]	This study presents an intervention that allows instructors to approach average modeling of switch-mode power systems with modeling and simulation.	Electrical Engineering	Industrial Electronics Laboratory Course
[17]	This study looks at how neuromorphic systems can be modeled, and how a training course regarding these models can impact students' modeling and simulation skills.	Electrical and Electronics Engineering	Electronic Circuit Hardware and Test Techniques course

Table 5. Studies involving individual disciplines other than electrical engineering

Paper	Description	Discipline	Course
[18]	Using classroom observations, interview data, classroom artifacts, and survey data, this study qualitatively looked at how students approached and modeled problems related to grain growth in a sophomore level materials science course.	Material science/ material science engineering	Microstructural Dynamics course
[19]	This study uses naturalistic inquiry with the theoretical framework of the Lesh translation model to understand how students move between different representations during a model-eliciting activity in an undergraduate engineering setting.	Chemical engineering	Heat transfer course
[20]	This study presented computational modeling modules to be incorporated into various levels of undergraduate engineering coursework in order to look at student learning through the experience	Mechanical engineering	Process and system dynamics, mechatronics, automatic controls
[21]	This study presents an overview of a computational modeling and simulation course used to teach students computational methods to solve differential equations and transport phenomena.	Nanotechnology engineering	Micro and nano systems computer-aided design
[22]	This study presents a modeling intervention for modeling distillation columns.	Chemical engineering	Optimization of Chemical Processes course
[23]	This paper presents a mathematical modeling lab within an online course. Students are given both data files as well as a programmed GUI that allows students to look at multiple different mathematical models to compare to the given data.	Chemical Engineering	Chemical Engineering materials course.

Table 6. Studies involving interdisciplinary or multiple engineering disciplines

Paper	Description	Discipline	Course
[24]	This study investigates how computer modeling is integrated into both undergraduate and graduate geoscience curriculum.	Various majors including civil/ environmental engineering as well as geoscience	N/A
[25]	This study uses a survey of academic and industry professionals to creating a learning progression for modeling and simulation to be used in the engineering classroom.	Various academic and industry engineering disciplines.	N/A
[26]	A typology of different validating activities during a series of modeling exercises is developed through task-based think aloud interviews.	Various engineering majors.	Activity outside of a course
[27]	This study investigates, through a multiple case study analysis, how faculty members instructional beliefs change as a result of implementing model-eliciting activities into their classrooms.	Mechanical, biomedical, industrial, and chemical engineering.	N/A
[28]	This survey study reports on beliefs of engineering and science industry and academic professional on needed skills in industry, desired computational tools, and challenges to implementation into the engineering classroom.	Various industry engineers and academic engineering and science disciplines.	N/A
[29]	The research utilized task-based interviews in order to understand how students transitioned between different modeling activities through modeling-transition-diagrams.	Varying engineering majors: environmental engineering, electrical and computer engineering, mechanical engineering, and general engineering.	Activity outside of a course
[30]	This theory paper proposed model-eliciting activities as a link between mathematics and engineering education research.	Various engineering majors and programs.	N/A

Table 7. Studies involving general engineering or first-year engineering

Paper	Description	Discipline	Course
[31]	This study analyzed classroom artifacts using a qualitative coding strategy with a theoretical framework of model-development sequences to investigate students entering their university engineering studies.	General engineering	Summer bridge program
[32]	A finite element modeling method is presented that uses both computational modeling as well as experimental results and is useful in early undergraduate coursework.	First-year engineering	Multidisciplinary engineering project course
[33]	This study analyzes, through a qualitative coding scheme of instructor feedback, how feedback should be given to students in order to improve student models and confidence with their models during a model-eliciting activity.	First-year engineering	Introductory engineering course on problem solving, design, and computer tools.
[34]	An example of a model-development sequence, an extension of the model-eliciting activity, is presented in relevant engineering education context and is connected to broader learning theories.	First-year engineering	Introductory course to problem solving and computer tools.
[8]	A developed framework is presented in which instructor feedback on modeling problems and activities can be analyzed. A specific case of the frameworks use is demonstrated.	First-year engineering	Introductory problem solving and computer tools course
[35]	This study investigated the effects of a modeling-based mathematics course on the performance of summer bridge program participants in their first undergraduate mathematics course.	General engineering	Summer bridge mathematics course
[36]	This study compares student performance on traditional mathematics assessments with performance on complex mathematical modeling activities.	First-year engineering	Ideas to innovations course
[12]	A framework of how to introduce a model-eliciting activity into a course and the principles necessary to design a model-eliciting activity are overviewed.	First-year engineering	Problem solving and computer tools course
[37]	This study overviews a designed educational intervention using model-based approaches. The intervention's effects on student interpretation of changing rate problems is examined.	General engineering	Summer bridge mathematics course

a brief description of the study is given along with the population it addresses (discipline and setting). The Tables are broken up by field for ease of readability. Table 4 overviews studies within electrical and computer engineering, which was the most common individual discipline in the dataset. Table 5 looks at other papers that focused in on singular disciplines. Table 6 overviews studies that looked at a multidisciplinary audience. Finally, Table 7 overviews studies that looked at general or first-year engineering contexts.

The papers were read and ideas were broken into three categories mapping to each of the research questions; what strategies students take when doing mathematical modeling activities, instructional implementation methods in terms of student learning, and instructor assessment of mathematical modeling and modeling activities. From these three different categories; student approach, instructional implementation, and instructor assessment, major themes were grouped together. These themes from each of the categories are summarized in Table 8.

4.1 Student strategies to mathematical modeling

The first category from the literature discusses the different ways that students approach mathematical modeling exercises in engineering contexts. Three themes emerged from this category: (1) student approach is either mathematical, contextual, or both, (2) student approach to mathematical modeling is diverse and nonlinear, and (3) student approach to modeling involves simplifications.

(1) Student strategies are either mathematical, contextual, or both. Many modeling activities focus on the underlying mathematical structure [31]. Some students preferred approaching the problem from how it was situated in mathematical theory, while others preferred to look at the modeling activity from the contextual theory that surrounded the problem [31]. In rare cases, students were able to connect both situational viewpoints, although this tended to be the most difficult approach given the need for deep understanding of both and the connections required to truly understand both. Without this deep understanding students typically

Table 8. List of themes mapping to each of the study categories derived from the research questions

Category	Theme
Student strategies to mathematical modeling	Student strategies are either mathematical, contextual, or both. Student strategies to mathematical modeling are diverse and nonlinear. Student strategies to modeling involve simplifications.
Instructional implementations in engineering coursework	Mathematical modeling activities should be situated in real-world context. Implementation can be both extension and creation. Background knowledge must be adequate. Modeling activities should be implemented in modules. Modeling activities should include adequately difficult concepts. Modeling activities should be implemented as team exercises.
Assessment of modeling problems	Assessing modeling problems should include solution and process. Feedback type is variable and impactful. Math and mathematical modeling require different assessments.

would begin to conflate physical concepts with abstract ideas [31]. Yet, research reported that students felt that modeling and simulation activities contributed to both mathematical and scientific/disciplinary knowledge gains as well as performed better on class assessments [17, 23].

Additionally, students struggled to align language around the physical context of the problem with the corresponding mathematical language [37]. This constant movement of thought during the modeling process, pairing up the physical real-life context of the model with the mathematical symbols and equations, was also reflected in a study done by Moore et al. [19] looking at the representational fluency of Chemical Engineering students on a modeling task. Students who were able to move more easily through multiple representations of the phenomena being modeled, were able to develop a deeper conceptual understanding of the material being taught [19]. Likewise, Ortega-Alverez et al. [13] found that working with multiple representations of information when modeling built conceptual understanding within an electrical circuits context. Modeling transition diagrams have been shown as a useful tool for instructors to map out how their students are moving through these multiple representations [29]. Through the investigation of the modeling process with modeling transition diagrams, it has been shown that we may be able to better map where students are moving throughout the modeling process and how they are thinking through the modeling problem [29].

When discussing student "Liz", Blikstein and Wilensky [18, p. 102] showed that students were able to understand complex mathematical concepts from interpretation of their physical understanding and meaning. This interplay was seen even more when Czocher [26] showed that students change their modeling assumptions and variables when they did not agree with the output of their model, adjusting their own real-world interpretation of the model to make it more usable to their mathematical analysis. Thus students look at these problems through both the lens of mathematics and their own understanding of the real-world, and both may need adjusting during a mathematical modeling activity. However, Czocher [26] found no evidence that students were switching between pure real world thinking and mathematical thinking, but rather, that students were constantly using both during modeling activities.

(2) Student strategies to mathematical modeling are diverse and nonlinear. This means that there may not be one expected or preferred method to solving mathematical modeling problems. Although many have attempted to outline a linear modeling process, Czocher [29] asserted that "instead of being universal, individuals' modeling routes are idiosyncratic" (p. 78). This could in part, be due to the fact that students often do not adequately plan or strategize for the modeling process prior to building the model [13]. Further, the individualistic approach taken by each student may be attributed to "individual mathematical thinking styles" that "impact the choices students make during mathematical modeling" [29, p. 81]. These individual paths had to intersect with six different steps in the modeling process in order for the student to have a successful solution; "understanding the problem", "simplifying/structuring", "mathematizing", "working mathematically", "interpreting", and "validating" [29, p. 88]. This was seen in the vastly different modeling approaches taken by students in Blikstein and Wilensky's [18] study examining different student pathways in an atomic and molecular computational modeling assignment. Modeling pathways to solution are as diverse as the students solving them.

Not only is each path individual, but also cyclical in nature and not progressing forward at all points [29]. For example, validation is often a useful step at the end of the modeling process. However, students may engage this process "early and often" throughout their solution of modeling problems [29, p. 95]. The occurrence of validation of the model often does not mark the end of the modeling cycle, and was used as a continuous monitoring activity by the student [26]. This means that students are constantly moving through various stages of the modeling process throughout the entire modeling cycle, creating a nonlinear process.

The cyclical nature of continuous improvement during the modeling process can be understood as the self-assessment principle, where modeling problems need to be set up so that students can evaluate their progress [12]. As students cyclically improve their models, they pass through multiple representations of the modeling information, nonlinearly going between the representations and potentially increasing conceptual understanding [19]. Hamilton et al. [30], when reporting on the findings from the modeling literature indicated that the final solution of the model is better due to its nonlinearity too because when students "iteratively develop, express, and test models in a solving scenario, they produce new approaches and cognitive structures that are often far more sophisticated than what might be taught in a classroom" [30, p. 11].

(3) Student strategies to modeling involve simplifications. Whether correct or incorrect, students make many simplifications during the mathematical modeling process [18, 29]. One example would be students trying to simplify the problem too much or take fewer steps creating solutions that are unstable or will not converge [22]. Through the modeling process, if the output of the model does not match how the student expects it to be, students may simply add mathematical structures (such as a negative sign if the output is negative of the expected value) in order to match the output with the expected result [13]. Thus rather than wading back through the complex mathematical model, they may simply opt for the path of least resistance, regardless of correctness of the change.

However, it should be noted that simplifications are not all bad or born out of misunderstandings of the situation. Czocher [29] wrote that simplification was seen in students when "the individual frequently considered the real-world context of the task" [29, p. 94]. Much like professional engineering work, simplifications can be born out of assumptions used to make the model an easier approximation of the realistic task at hand. Heuristics, often referred to as "rules of thumb", are useful for thinking about a problem in context but can lead to inaccuracies in the model if the limitations are not understood fully [18].

4.2 Instructional implementation in engineering coursework

The second category from the literature resulted in different ways that mathematical modeling was implemented into the engineering curricula in order to better promote student learning. The themes from this category speak to how to structure pedagogy effectively. The six themes that emerged in this category are: (1) mathematical modeling activities should be situated in real-world context, (2) implementation can be both extension and creation, (3) background knowledge of the student must be adequate, (4) modeling activities should be implemented in modules, (5) modeling activities should include sufficiently difficult concepts, and (6) modeling activities should be implemented as team exercises.

(1) Mathematical modeling activities should be situated in real-world context. One such common type of modeling activity is the model-eliciting activity (MEA) that has been shown to be useful in engineering context, by connecting simple modeling tasks to real-world engineering problems [30, 33, 35]. Situating these modeling problems in realistic contexts can be referred to as the reality principle [12]. This realistic context benefits even more if the situation has personal meaning to the students performing the activity [34]. Modeling problems benefit from going as far as creating fictional stakeholders in the solution of the given problem [34]. Modeling activities pair well with a problem based learning approach through both reasoning and personal reflection, allowing students to learn through self-construction of their own knowledge [17]. Thus, pairing this with real-world context gives students an even better context to construct their knowledge from previous experiences.

MEAs can be further improved and implemented into the classroom by branching them into model exploration activities and model application/adaptation activities, which have the students extend their created models to more contexts, as well as deliver potential deeper understanding of the mathematical concepts underlying the model [31, 34]. Regardless whether an MEA or not, a mathematical modeling activity, in theory, "renders a real-world problem as a mathematically well-posed problem conducive to mathematical analysis" [29, p. 78]. By doing modeling activities in real-world contexts, students can learn science during modeling as opposed to learning the science prior to modeling, which may be better for student learning in both regards [18].

(2) Implementation can be both extension and creation. Prior to this, all results have been around

model creation. When modeling activities are discussed, many jump to thinking of model creation, although the extension of existing models is an important engineering skill as well [28]. For example, Czocher [29] showed that success in the process of modeling, and the final model submitted, may not be closely linked. This would indicate that obtaining an acceptable solution and understanding of the physical process are two separate entities. Thus, extension of the model requires an understanding of the process and may be just as critical depending on the goal of the overall problem. Such situations, as pointed out by Magana and Coutinho [28], may reflect genuine engineering practice in that engineers "use computation by applying or modifying existing numerical tools" [28, p. 64]. This concept of model extension being important also shows up as the generalizability principle, where a necessary aspect of the modeling solution is that it is useful to other scenarios [12]. This is also the basis for model application activities which require students to extend their own created models to other significant problems [31].

Hence, extension of the model may not only be as important as model creation, but more important for the student's skillset as it relates to professional engineering practice. When looking at the full modeling cycle, Magana [25] wrote that the full modeling learning progression not only includes building the model, but also using, evaluating, and revising the model. Additionally, faculty believe that building may not be all of it, but that in many cases building the model is of little concern early in undergraduate engineering education as compared to choosing correct modeling methods, assumptions, and boundary conditions [25]. Additionally, just the use of models may create motivation in students to want to build models [14].

(3) Background knowledge must be adequate. Although it may seem trivial, students often struggle with materials that are less familiar to them. For example, Arleback and Doerr [37] mentioned motion and light being conceptually easier during modeling activities than less familiar concepts such as light intensity and voltage drop. This may cause one of the biggest struggles of modeling activity implementation which is getting students to find their own errors and to appropriately validate their model [29]. When the student is less familiar with the source material, this validation process may become more difficult.

This background knowledge that is needed is not static. The more complex a model gets, the more background knowledge is needed. Abramovitz [15] mentions that the more complex or realistic a system gets, the more prerequisite knowledge is needed for the modeling activity. One instructor in the study conducted by Magana [25] said that students constructing mathematical models "would only work if they have the technical knowledge required" [25, p. 7]. The instructor went on to say that more complex mathematical modeling should only be expected of junior/senior level students.

Additionally, when mathematical modeling moves into computational modeling, the knowledge required becomes even more complex with the additional of programming. Furat [14] used a pre-programmed graphical user interface (GUI) for their modeling activities instead of command-line programming because the GUI allowed for their students to focus on the objectives of the modeling activity. Campbell, Overeem, and Berlin [24] wrote that introductory courses may need GUI driven computational models so that modeling activities can account for different levels of programming ability. Thus what the activity looks like should be weighed against both the disciplinary knowledge as well as the programming knowledge of the students.

(4) Modeling activities should be implemented in modules. Many of the papers in the data set used modeling as a sort of module to come alongside an existing class or laboratory. For example, Abramovitz [15, 16] used the modeling activities within an existing laboratory course. Pairing modeling exercises with existing laboratories in module allows for students to compare their theoretical results with real-world results [16]. These modules should allow time up front to the activity for students to digest material so that all members of a solution team can contribute to the solution [34]. When comparing the use of a finite-element modeling activity in a module form as opposed to a typical experimental lab exercise, Ural [32] reported a higher level of self-reported intellectual stimulation and development of knowledge by the students through the mathematical modeling activity. In addition, allowing the students to choose what project they worked on was seen in the data set [18].

These modeling activities may be most effectively implemented in the form of "spiral curriculum", where mathematical modeling activities and projects are implemented into labs and coursework that span across a student's degree program [28]. In addition, scaffolding the projects so that students are exposed to multiple modeling program types and projects allow for a more well-rounded student at the end of the degree program [28]. Spiral curriculum and scaffolding allow for the projects to fit into existing coursework, of which students and instructors benefit, as projects become more relevant, easier to incorporate into the classroom, and avoids the creation of a new class altogether [20, 28]. This is important in that modeling currently is applicable and relevant to courses within university contexts that it is currently not in [24]. It should be noted that of all hurdles to inclusion of these activities in class, the most prevalent reason for their exclusion is limited class time and a bloated engineering curriculum; specifically because modeling activities can take multiple hours of class time if done within class [20, 28]. This movement from traditional lecture format has also been shown to change faculty members towards more student-centered beliefs in the classroom [27]. A more student-centered approach with modeling activities creates a desire within instructors to have students "take more responsibility for and to advance their own learning" [27, p. 293].

(5) Modeling activities should include adequately difficult concepts. As seen previously, students must make simplifications during the modeling process. However, instructors must include both enough difficulty and simplicity for the modeling problem to be solved. Computational modeling works well in subjects that are complex, such as multiphysics problems where numerical solution is required; this gives students the ability to solve problems without having to make assumptions that limit the models likeness to the real world [21].When including difficult material, modeling activities may provide instructors the ability to be able to identify the common misconceptions that students have in the given subject [27]. An additional benefit of adding modeling activities in later years of undergraduate coursework is the necessity of using the computer during the process. Adding computational aspects to the mathematical modeling activity, which is common in upper division engineering courses such as transport phenomena, can open students eyes to the use of computational modeling through numerical methods in future applications [21].

Although difficulty of the concept is necessary for the modeling activity, a modeling activity that is too difficult for the students may require simplifications by the instructor in order to avoid a model that is impossible for the student to evaluate against real-world conditions [12]. This need for simplification is especially true in early engineering classes where "student often have very little experience with engineering and some of the tools of engineering" [34, p. 65]. Caution must be given when adding simplifications to the problem. If a modeling problem is conceptually too difficult, students may have trouble questioning assumptions or procedures of the model, however it must be complex enough to feel like a real-world engineering problem to the students [12, 34]. A balance of these two is necessary to create the cyclical learning that leads to increased student learning through the self-assessment principle [12].

(6) Modeling activities should be implemented as team exercises. Many times, modeling activities involve difficult subjects. Many of the studies used teams rather than individual assignments, with a few examples being [19, 34, 36, 37]. Teaming can be a useful approach to implemented modeling activities, and benefits are seen in the multiple perspectives that are brought to the table through having multiple students work on the problem together. Each team member may think about the problem in a different light increasing representational fluency and solution progress [19]. This multiplicity of perspective keeps students from being stuck on a part of the problem for too long and Moore et al. [19] showed that "the multiple interacting perspectives of teams during model development led to increased use of representations and representational fluency" [19, p. 167]. That is to say, that students were able to cognitively move more quickly throughout the problem space when multiple perspectives were involved in the problem-solving process. Additionally, when students operate as a team they bring multiple different skill sets which allows them to solve very complex and realistic problems [12].

4.3 Assessment of modeling problems

The third category found from the literature were topics dealing with the assessment of modeling problems. This was the least prevalent of the three categories. Three themes emerged from the literature relating to this category: (1) assessing modeling problems should include both the solution and the process, (2) feedback type is variable and impactful, and (3) traditional math and mathematical modeling require different assessments.

(1) Assessing modeling problems should include solution and process. Czocher [29] maked this argument by showing that the path students take in solving the modeling problem is highly diverse through the use of modeling transition diagrams, and that there is no one correct process to solve a modeling problem. The need for this as the primary goal is also reflected in the second highest desired quality of students on modeling and simulation activities from both academics and industry professionals being students able to "Choose an appropriate modeling approach or method for a given problem or situation" [28, p. 67]. This would be an indication of better understanding of the process as opposed to an absolute solution to the modeling problem. In addition, Moore et al. [27] showed that

when used in the classroom, modeling activities changed instructors "use of formative assessment more than summative assessment" [27, p. 295]. This shows that modeling activities require different assessment methods and the different methods instructors deem required emphasize formative assessment.

(2) Feedback type is variable and impactful. Jung et al [33] showed that positive feedback, as opposed to negative feedback, increased performance on modeling assignments and modeling activities. In addition, redundancy in the feedback has been shown to decrease student improvement on future assessments of the model [33]. Instructor feedback was shown to be variable based on perceived quality of the student solutions [8]. In addition, feedback varied greatly with the specific criteria or portion of the problem that was being assessed [8]. For example, one hypothesis among the studies was that instructors made more "open suggestions or ask more questions" when a high level of interpretation was required when giving the students feedback [8, p. 399].

(3) Math and mathematical modeling require different assessments. Kartal et al. [36] showed that performance on mathematical modeling assignments does not line up with traditional mathematical performance, potentially pointing to a need for new assessments for mathematical modeling activities. This aligned with Hamilton et al. [30] who discussed that the broader research literature has shown that "students who traditionally underperformed in more traditional mathematics curriculum settings were very successful performers in team modeling sessions" [30, p. 11]. Although, the two types of problems may require different types of assessments, Doerr et al. [35] found that engineering students who participate in mathematical modeling activities during a bridge program did significantly better in their first collegiate math experience. This implies that the skillsets are related even though they are testing different skills. However, because they are distinct, one consequence is that students who may have succeeded in engineering or more broadly STEM, may not be pushed towards STEM due to their performance on standardized mathematics tests when those may actually be a poor indicator [36].

5. Discussion and Implications for Teaching and Learning

Multiple themes within each of the categories were found within the results of the review and an overview of implications of the results are discussed here. The most common population addressed in the final data set was first-year or introductory engineering coursework. Instructor implementation was the highest discussed theme throughout the papers. The high prevalence of instructor implementation is likely the result of traditional engineering researchers, who do research in their own classroom through the scholarship of teaching and learning, seeming to typically focus on instructional methods and student satisfaction rather than assessment of the problem or student approach. Many of the papers that were excluded from the study were papers around the scholarship of teaching and learning presenting specific modeling activities in specific contexts without data on student learning. This is in line with other education literature reviews which have found that rather than student learning, student perceptions and motivation is often reported at a higher rate [38]. This discussion is broken down by research questions, followed by ideas from each of the themes and how they connect to form a holistic picture of modeling in the engineering classroom.

5.1 What strategies do students take when solving mathematical modeling problems?

Students may use mathematical, contextual, or a combination of strategies to mathematical modeling problems (theme one). Modeling problems create a situation where the physical world meets mathematical knowledge, and that intersection can be difficult for students to traverse. Because of this, students typically use strategies that involve simplifications of the problem statement and forthcoming solution (theme three). This is beneficial, but can lead to pitfalls if not done carefully. Above all, students use a vast and diverse set of strategies to modeling problems, in part because it is not a linear process (theme two). Rather, a cyclical process that allows a continuous improvement mindset that improves student learning of the conceptual material by allowing them to work through their own conceptual misunderstandings.

These themes are consistent with literature external to this review. For example, research has shown that even professional engineers can find it difficult to reconcile the physical reality with the limitations of a modeled solution, and thus can struggle with what simplifying assumptions they need to make as they go through the modeling process [2]. In addition, when looking at how students moved through a modeling problem, Galbraith and Stillman [39] characterized a modeling process that was anything but linear, but rather a messy process where steps of the process are being moved between and iterated upon. However, this nonlinear process can lead to additional inquiry in the student which can sustain interest in the student as well as the student learning within a broader problem solving process [10]. Lesh and Harel [40] asserted that students who were able to make sense between the mathematical and physical representations of information through this iterative modeling process ultimately were are able to comprehend much more elaborate mathematical constructs than they would have through traditional lecture/textbook formats. However, previous research has reported that mathematical modeling is very difficult for students as compared to other types of modeling because of how mathematics is traditionally taught in the classroom [41] or that mathematical modeling is often not thought of by students as a powerful tool when engaged in design [42].

5.2 How can mathematical modeling activities be structured to promote learning in the undergraduate engineering classroom?

As opposed to traditional lecture type activities, mathematical modeling activities allow for instructors to introduce real-world concepts and meaning to the work to the students are doing (theme one). By doing so, students can connect to issues they may face in the future but also have a personal connection to the material they are learning. The modeling activities can be implemented in modules that span topical boundaries in engineering with difficult concepts such as multiphysics problems or electricity and magnetism (theme four, theme five). However, simplifications may be necessary by the instructor, or provide additional background knowledge to students, if content is too difficult (theme three).

In addition, mathematical modeling activities can be used to teach students how to extend and adapt existing models (theme two). This extension and adaptation allows them to overcome common misconceptions regarding difficult subject matter by giving the students the opportunity to refine their own models or critically evaluate the models of others. Other workplace skills, such as teaming, are not only easily applied to modeling activities, but may increase learning outcomes (theme three).

In previous research, it has been shown that introducing computational modeling activities in the form of modules allows for learning of both the process of computational modeling but also the discipline specific information as well [43–45]. Research has shown that mathematical modeling in instructional units in a science course increased student understanding more than during instructional units where mathematical modeling was absent [46]. However there is a cost, a bloated engineering curriculum means likely a few topics that would need cut from the curriculum with the incorporation of mathematical modeling broadly into the engineering classroom as "curriculum will need to provide ample time for students to discuss, conjecture and validate, while spending less time on drill and practice, memorization, and lecturing" [10]. That is to say, mathematical modeling activities do take more time than a traditional lecture format might.

The need for simplifications from the instructor make sense in that even in the professional world engineers must make simplifying assumptions to the underlying modeled phenomenon, and these assumptions come from the background knowledge and experience the engineer has [2]. In addition to disciplinary background knowledge, if the mathematical modeling problem is using computation, insufficient background knowledge in computing can interfere with the learning gains of the student [47]. Because of this, a spiral curriculum where these skills are revisited and integrated together into discipline specific context is likely the best approach [28]. However, no studies in this review of the literature investigated the systematic implementation of modeling activities across an entire engineering curriculum.

5.3 What techniques do instructors have for assessing the work from these open-ended modeling problems?

Assessments of modeling problems should include both the solution and process to the problem because of the diverse nature of the resulting solutions (theme 1). This sets traditional math problem assessment apart from mathematical modeling assessments (theme 3). Through these mathematical modeling activities, feedback should be given to the students and the format of that feedback can look variable (theme 2). What that feedback looks like to students can be very impactful in their ability to improve their own work. Because modeling activities are cyclical, positive and non-redundant feedback on modeling problems can spur students to overcome their misunderstandings, whereas negative or redundant feedback can discourage students from continuing to try and improve their solution.

The research on assessment seemed to be the scarcest of the three questions. This may be because many of the studies focused on self-reported student perceptions rather than student learning. This remains a large opportunity for further research. One of the biggest struggles is that with open-ended problems such as modeling problems, the grading seemingly can become more subjective and labor intensive if class size is large [4]. Continued research is needed to understand how best to assess solutions to open-ended modeling problems.

6. Limitations

Additional literature outside of peer-reviewed journal articles was not considered for the review. Documents such as textbooks, grey literature, and conference papers were consequently excluded from the final set. The authors acknowledge the importance of this literature, and the fact that its exclusion may leave some ideas and voices that would otherwise contribute to this work left unheard. In addition, there were decisions made in the searching process that could have had a limiting factor if researchers used did not have descriptive titles or abstracts. This study limited the research relating to engineering undergraduates, however, mathematical modeling activities do not exist exclusively to this population. Understanding how modeling exercises are used in other disciplines as well as other age groups may give new insights into the entirety of their utility. Another limitation was the authors needing to interpret the level of which certain computer software packages allowed students to truly mathematically model the process, or if simulation was the main driver of the activity. For example, some papers using PSPICE and SIMULINK were included, whereas papers using building information modeling were excluded.

In addition, much of the research focused on class-specific interventions or a general discussion of the implementation in single classrooms. Although this is needed information, this review illuminated little research on systematic implementation of modeling activities or what the results are when these modeling activities are added to curriculum department-wide. A deeper knowledge of this may very well improve our understanding of the totality of the impact that these methods may have, but in addition, help us to understand how the development of these skills contributes to the engineering workforce.

7. Conclusion

The described review of the literature aimed to answer three research questions; (1) What strategies do students use when solving mathematical modeling problems? (2) How can mathematical modeling activities be structured to promote learning in the undergraduate engineering classroom? and (3) What techniques do instructors have for assessing the work from these open-ended modeling problems?

The findings indicate that there is more research to be done, specifically in assessment of mathematical modeling problems. Further, these findings were more prevalent in general engineering or early engineering as opposed to upper division engineering classrooms. While implementation of the modeling problems and student strategies seemed to form a clear picture through the research of what these activities should look like, assessment strategies were scarce in the results. The research illuminated feedback strategies. However, little research showed and discussed assessment strategies and impact on instructors and students of new and novel assessment strategies.

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