*International Journal of Engineering Education* Vol. 36, No. 4, pp. 1302–1311, 2020
Printed in Great Britain
0949-149X/91 $3.00+0.00


# Practical Approach for Teaching Vehicle Design to Engineering Undergraduates*

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We present a new practical methodology for teaching vehicle design to engineering undergraduates. Students, after receiving theoretical lessons, programmed an interactive automatic spreadsheet where inputs were the main geometric and physical parameters of a selected vehicles and as output, the spreadsheet must deliver maximum speed, 0–100 km/h acceleration time, maximum slope, tilt and skid. More than 150 students were taught with this practical methodology, turning the theoretical topic into something really applied, close and attractive for the students. The experience assessment shows that students obtained very good grades in the topic while they were very satisfied with the methodology. They demonstrated an excellent understanding of the vehicle mechanical performance calculation process while their curiosity and interest on the topic was very high because it dealt with their preferred vehicles.

**Keywords:** automotive engineering education; vehicles; road vehicle design; transport engineering

## 1. Introduction

Transport engineering is the application of technology and scientific principles to the planning, functional design, operation and management of facilities for any mode of transportation. It also may involve ethics and social perspective since it affects to all the population [1]. From the mechanical engineering point of view, transport engineering is mainly focused on the mechanical design of vehicles and auxiliary elements. Automobile vehicles design is usually attractive for last courses students since they learn to apply their knowledge into something they find in their daily life.

Nevertheless, we have detected that students are not able straightforward to apply their basic mechanics and physics knowledge to vehicle design and performance calculation. This is mainly due to the large time gap occurring from mechanics courses and vehicle design courses. Hence, we have tried to link both courses through a practical and attractive new teaching methodology.

Previous practical laboratory courses have shown good results in the improvement of the automotive engineering learning process. For example, some universities uses simulation software to make the topic more attractive with excellent results [2, 3]. Even virtual driving lessons have been developed for teaching vehicle dynamics [4]. In another previous work [5], the development of a new hydraulic, hybrid vehicle-based laboratory was presented. Through practical lessons, graduate and undergraduate students enhanced understanding of hydraulics hybrid system while having the possibility also to perform research activities.

The impact of experimental methodologies on the student mechanical automotive engineering education has been assessed in reference [6]. The statistical analysis showed that students following experimental learning methodology performed significantly better than the control group with respect to 'class pass percentage' and 'grade score'. Moreover, when comparing the 'class attendance', significantly higher-class attendance was achieved in the experimental group compared to the control group. Therefore, this study demonstrated the experimental teaching methodology efficacy in enhancing students' learning outcomes. Dedicated labs practical courses has been developed for teaching vehicle electronics, as a complementary element of vehicle design theory courses [7]. Academic cars competitions also enhances the interests of students for vehicle design, again because they directly apply



* Accepted 24 January 2020.

and use theoretical concepts into something practical [8, 9].

It can also be found multidisciplinary practical teaching with road vehicles satellite navigation systems [10]. In this work, the ultimate objective was to develop a driving assistance system based on satellite positioning. This integrative experience makes full use of each group's knowledge so as to be able to offer students a global view of both theory and practice. Students judged the experience positively because they obtained new knowledge that hardly could be achieved without collaborative work in a multidisciplinary team.

We take previous cited methodologies as examples of success, and in this study, we have developed a similar practical approach but applied for the first time to vehicles mechanical performance calculation. We introduce a new practical methodology in which we encourage students to become actively involved in the learning of vehicle performance calculation and design. Students are asked to calculate the mechanical performance of their own vehicles, or a vehicle or their choice, turning the topic into something that could really be applied, making it close and more attractive for them.

The methodology is divided in several teaching steps. Students receive a theoretical description [11, 12] where performance calculation fundamentals are explained in classroom. Then, students are told to gather data of the geometry of the vehicle, physical properties and the performance curve of the corresponding cars engines. It is encouraged to select their own vehicles or the family car to increase the affinity. Afterwards, all these data are used as input of the practical methodology. Students obtain the vehicle performance and its main features as an output result by using an automatic spreadsheet. At the end of the practical session, they compare the results with the expected performance claimed by cars manufacturers.

In this article, both theory and practical methodology description are explained in detail. In addition, we show the experience assessment results gathered from inquiries done by the students during five consecutives courses.

## 2. Academic Context

The practical methodology proposed is part of Transport Engineering subject which is given in the second course of the MSc in Industrial Engineering at Universidad de Alcalá. The contents of this subject are:

1. Introduction to Transport Engineering and Transport Modes.
2. Road transport: Introduction, performance, consumption, braking and lateral dynamic.
3. Rail transport: Introduction, performance, track, wheel-rail contact and dynamics.
4. Industrial maintenance: Introduction, rollers, conveyors and industrial trucks.
5. Cranes, elevators and lifters.
6. Automatic transport.
7. Management and transport control: Traffic and Safety.

Specifically, the proposed methodology is applied within Topic 2 related to road vehicles. Along this topic, the students follow a general overview of road vehicles design process and it is explained towards the final aim of being able to calculate and select the adequate engine, geometric and physical parameters of the vehicles. The proposed methodology can be applied to different types of roads vehicles, not only cars, but also trucks, motorcycles or even bicycles since the analysis is quite similar in conceptual-design stages. However, at first, it is recommended to apply it to common cars. In fact, the final project of the subject consists in applying the opposite methodology, i.e. designing a road vehicle to fulfill some mechanical requirements defined by the students.

Transport engineering assessment is divided as 20% of the final grade the practical lessons deliverable, 40% of the final grade the grade in a final project and 40% comes from the final exam.

## 3. Theory of Vehicle Mechanical Performance Calculation

Basic knowledge of mechanics is enough to calculate accurately the expected performance of a vehicle provided that geometrical and physical characteristics parameters are realistic and known.

The calculation initiates from the free body diagrams of the vehicle, solved for the movement direction, in our model defined as X direction. Z-axis represents the vertical axis corresponding with vehicle height. There are two types of forces acting on the vehicle: forces that reduce the motion and forces that enable the motion. In addition, ground reaction forces also appear in the free body diagram, as shown in Fig. 1. Ground reaction forces determine maximum adherence capacity of the vehicle to traction.

Forces can be itemized as:

- Motion resistances: Aerodynamic resistance ($R_{ax}$), depending on vehicle aerodynamics; Rolling resistance ($R_r$), depending on tire features; and gradient resistance ($R_g$), depending only on mass vehicle and road slope. Details of the calculation are given in annex B.

$$R_T = R_{ax} + R_r + R_g \ [N] \tag{1}$$

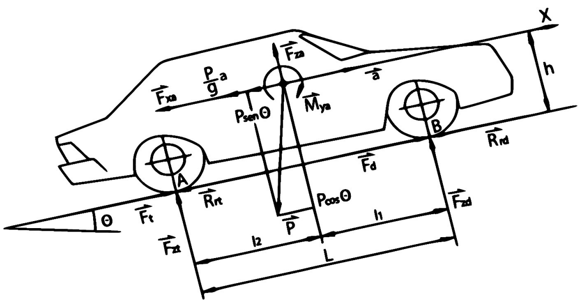


**Fig. 1.** Vehicle Free body diagram model.

- Tractive force ($F_t^m$) on the rim which is related to engine and its performance, gearbox and transmission, drive type (front-wheel drive, rear-wheel drive or $4 \times 4$) and mass distribution of the vehicle). Details of the calculation of traction force given by the motor are given in annex C. Maximum tractive force is limited by motor but also is limited by the maximum adherence force achievable between wheel and road. Details of the calculation of adherence limits are given in annex D.

Then, by applying Newton's third law, the instantaneous vehicle acceleration can be determined as:

$$F_t^m - R_T = m \cdot a_x \tag{2}$$

However, to obtain a certain tractive wheel effort is required to have enough adhesion with the ground to avoid tire slipping. The maximum tractive force that a vehicle can have has two upper limits: that one that provide the engine and the transmission and that one that allows wheel-asphalt adherence. Resistance, tractive forces and adherence limits can be plotted together in a single diagram as a function of the speed as shown in Fig. 2.

Dashed lines represent different adherence limits according to adherence coefficient, point lines represent total motion resistance (so the sum of aerodynamics, rolling and grade resistance) and continuous lines represent tractive force depending on the gear ratio and engine output torque. Notice that rightwards Y-axis is scaled in percentages, showing the motion resistance for different slopes, X-axis measured in $\frac{km}{h}$ and leftwards Y-axis measured in N. Basic parameters that define vehicle mechanical performance are: maximum speed on plain road, $v_{max}$; maximum slope at low speed, $j_{max}$; acceleration time from 0 to 100 $\frac{km}{h}$, $t_{0-100}$ and tilt and skid speed, $v_T$ and $v_s$.

Vehicles maximum achievable speed on a plain road is the maximum speed wherein tractive forces are higher or equal to resistance forces. Thus, in order to get the maximum speed, it is required to locate the intersection point between the tractive force curve in the highest gear ratio and the motion resistance curve for $j = 0$ (0% slope).

An example of intersection can be shown in Fig. 2. Maximum speed corresponds with the point of intersection between the motion resistance and the traction force in firth gear, resulting at 155 $\frac{km}{h}$.

This intersection point also defines the required power that the engine must deliver in order to reach this speed. As power is resistant forces at maximum speed multiplied by car speed.

To calculate this maximum slope, we assume constant and low speed (< 10 km/h), so the aerodynamic resistance can be neglected from the equation (1). In addition, Constant speed means that both maximum traction force and total motion resistance cancel each other. Assuming $P = m \cdot g$ (vehicle weight, in N), slope angle $\theta_g$ as and small-angle approximation, then:

$$F_{Tmax} = R_T = P \cdot \theta_g + P \cdot f_r(v) \tag{3}$$

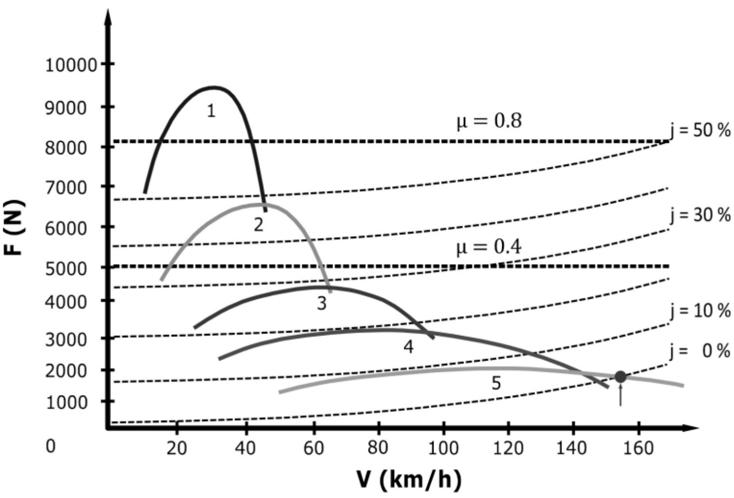


**Fig. 2.** Tractive, resistances and adherence limits vs vehicle speed.

This maximum traction force will be the minimum value between maximum tractive force on the rim provided by the engine torque and maximum tractive capacity due to adherence. Then, the maximum slope in radians can be calculated as:

$$\theta_{max} = \frac{F_{Tmax} - P \cdot f_r(v)}{P} \quad (4)$$

Generally, we use the tangent of the angle ($j$) when referring to slopes. Tangent value can also be found in percentage values. So, maximum slope in percentage can be written as $j_{max} = 100 \cdot \tan\ (\theta_{max})\%$.

Another important parameter that defines vehicle performance is its capacity to achieve 100 km/h in a short time. Vehicle acceleration force capacity can be determined by the instantaneous difference between maximum tractive force and resistance at a certain speed. This is named available force for acceleration. Hence, the available force for acceleration ($F_{aa}$, in N) of the vehicle is related to its acceleration and mass. Considering the acceleration in a road with no slope, the required time to accelerate from $v_1$ to $v_2$ with the same gear ratio is:

$$t_{1,2} = m \cdot \int_{v_1}^{v_2} \frac{dv}{F_t^m(v) - R_T(v)} \quad (5)$$

Gear ratios changes must be taken into account during the acceleration time calculation. During this change time (typically 1 to 2 s in manual change) the vehicle undergoes a deceleration due to motion resistances.

Last important parameters are tilt and skid speeds. They are two useful parameters that define safety limits when driving in curves. Assuming that there is no camber on the curve, tilt speed $v_T$, and skid speed $v_s$ can be calculated as:

$$v_T = \sqrt{g \cdot R \cdot \frac{B}{2h}} \text{ and } v_S = \sqrt{g \cdot R \cdot \mu} \quad (6)$$

where R is the curve radius (m), B the vehicle width (m), $\mu$ is the maximum friction coefficient between wheel and road and h the vehicle center of gravity height of the (m).

## 4. Practical Teaching Methodology

The main objective of the practical methodology consists in applying the theoretical concepts learnt in classroom to a specific example. In order to increase the interest of the students, they are told to calculate the performance of their selected vehicles. It is encouraged to select their own, or family, cars because this increase the level of proximity of the students with the subject and also, from a practical point of view, because they use to have access to car manual where most of the required data can be found.

The lessons are organized in three lessons of two hours of duration each of them, in total 6 hours for this topic.

First lesson is the theoretical concepts explanation in a common classroom. Slides and examples in the blackboard are shown. This lesson lasts 2 hours. At the end of this first lesson, we explain to the student how the practical lesson are going to be and which type of geometrical and physical data of their cars they have to gather. At this moment, students have access to a non-programmed template of the spreadsheet.

Second lesson is the first practical lesson. This is a 2 hours session given in a computers room. Each student have one computer so they can develop the practice alone. This session is divided in three parts:

- First part may take approximately 30–40 minutes and it is about to gather all the necessary data of the vehicles from car manual of instructions or from internet. The geometrical and physical data of the vehicles can be easily found, but the characteristics of the engines (torque-speed curves and gear ratios) are sometimes harder to obtain. In case that the student cannot find the torque-speed curve of the exact engine of the car, it is suggested to the student to find torque-speed curve of another engine from different manufacturer but with the same type of fuel and similar cubic capacity and nominal power. At the end, the objective of the practice is to teach the student the process, it is not to obtain very precise results. Another critical issue related with data are gear ratios. Sometimes gear ratios are given as speed of the car for at 1000 rpm of engine, which implies that students have to do a conversion. In case of doubt or difficulties it is suggested to use default values given in annex A. Once students have found all the required data to continue, they start performing parametric calculations by programming an automatic spreadsheet in a software as for example Microsoft Excel as shown in Fig. 3. Most of students obtain all the data without too much problem.
- Second part may take between 50–60 minutes which are devoted to calculate resistant forces as a function of speed and adherence limits. It is suggested to calculate resistant forces for three different slopes (0%, 10% and 30%). As long as the student gets the results, they should plot them so the teacher can rapidly detect any anomaly in the results. It is very useful to highlight that resistant forces curves must be quadratic functions and that the curves between slopes are shifted curves and of course, they will never

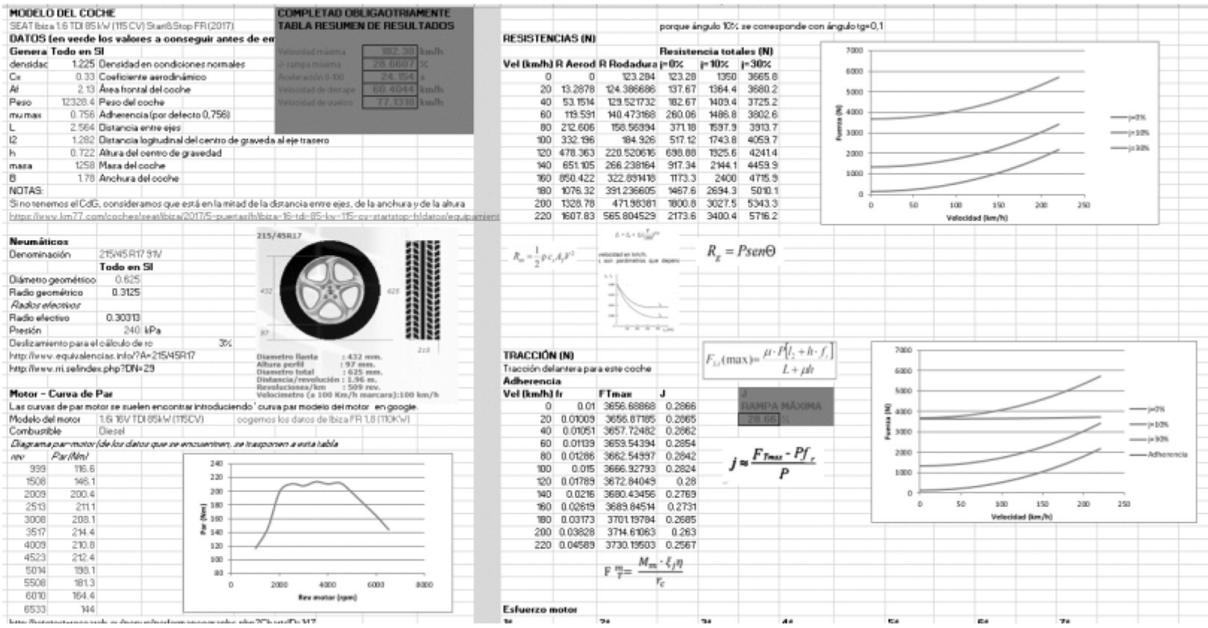

**Fig. 3.** Automatic spreadsheet made in Excel screenshot.

intersects. It is also important to make the students ask themselves if the obtained results makes physical sense. For conventional cars adherence limits use to be between 3500–5000 N and resistant curves of 30% slope must, for most of the cases, start slightly below adherence limits. With adherence limits, maximum slope can be obtained. Typical range for maximum slopes are 35–40%. This part is the most complicated for students within this second lesson.

- Third part of this session takes between 10–15 minutes and it is devoted to calculate skid and tilt speed which are simple calculations. The main issue found in this part is that students sometimes mismatch the units of the parameters.

Third lesson is the second practical lesson. This is a 2 hours session given in a computers room. Each student have one computer so they can develop the practice alone. This session is divided in two main parts:

- First part takes almost one hour and it is devoted to create traction curves, one curve per each gear ratio, and to add them in the resistant and adherence diagram. Once they are added, maximum speed can be obtained by searching for the fastest speed point of the intersection between resistant forces at 0% slope and traction force. Again it is important here to analyze if the value of the speed makes sense and if the value is close, or not, with the value of maximum speed given in car catalogues. One common and significant issue that students find is that traction forces in the first gear ratio use to overpass widely adherence limit. This would imply that wheel of most of the cars would drift if the driver push to the bottom the throttle. In normal modern cars, slide of the wheel is electronically controlled so when they detect this behaviour, in the first gear ratio, the central electronic unit reduces the amount of fuel, so the real effective torque is much smaller. This is why it is suggested to apply a reduction coefficient of 0.6–0.8 in the torque for the first gear traction curves calculations.
- Second part of this third lesson requires 30–40 minutes. This part is devoted to the calculation of the 0-100 km/h acceleration time. This part can be done by integrating equation 5, making a piecewise integral, one per each gear ratio and adding the time devoted to changes of gears. This integral can be done by rectangular approximation of the integral calculation is a simple and fast way.

At the end of the practice, it is suggested to play a little with the automatic datasheet by changing some parameters as for example aerodynamic coefficient of friction coefficient and see how these affects to vehicle performance. When all operations are finished, students must summarize performance results in a small table, compare the results with manufacturer catalogue values, and analyze the differences they may find. Then, they deliver the spreadsheet for its grading.

## 5. Experience Assessment

This practical methodology has been implemented during the last five courses, starting on the year 2015, with excellent results. During these years courses, more than 150 students have delivered adequately the practical lessons. The experience has been assessed from three points of view: grades obtained in the practical lessons deliverables, success rate in the final exam questions related to the vehicle design and by a satisfaction survey done to the students after the experience.

The grades of the practical lessons deliverables are in general very good. The mean value of the grades is 9.45 over 10 with a $3\sigma$ deviation of $+-0.5$. Although it is a practical lessons very guided, these grades indicates that students have assimilated correctly theoretical concepts and practical procedures during the sessions.

In the final exam, students must answer several theoretical test questions and test exercises related to the vehicle design calculation. 86% of the answers were right, being 96% of the exercises correctly done.

This methodology was implemented from the beginning of the new Master in industrial Engineering in Universidad de Alcalá, so we cannot compare the results with a methodology without the practical lessons. In any case, grading results indicates that students acquire correctly the contents taught.

Besides this assessment, we have performed a satisfaction survey among the students asking for their opinion about this methodology. In general, students consider that they have learnt a lot by following this method, much more than they had learnt with simple theoretical or exercise lessons. In addition, they enjoyed and felt satisfied about the teaching method which always reinforce the understanding of the concepts.

Four questions were formulated to the students:

Q1: Indicate your previous knowledge about vehicle performance and features calculation.
Q2: Indicate the level of learning you feel you have acquire in vehicle performance calculation
Q3: Indicate the level of confidence you have to apply vehicle performance calculation to other types of road vehicles like bicycles, motorbikes, trucks, etc. . .
Q4: Indicate the level of satisfaction you have with this methodology.

Statistical results are listed in next Fig. 4.

According to the results we can extract several conclusions.

Q1 results indicates that student did not have significant previous knowledge about the theoretical contents. Linking this with the grading results, then we can determine that methodology serves for a proper learning.
Q2 results show that students feel sure before facing the final exam and this is endorsed by the excellent success rate in exam exercises.

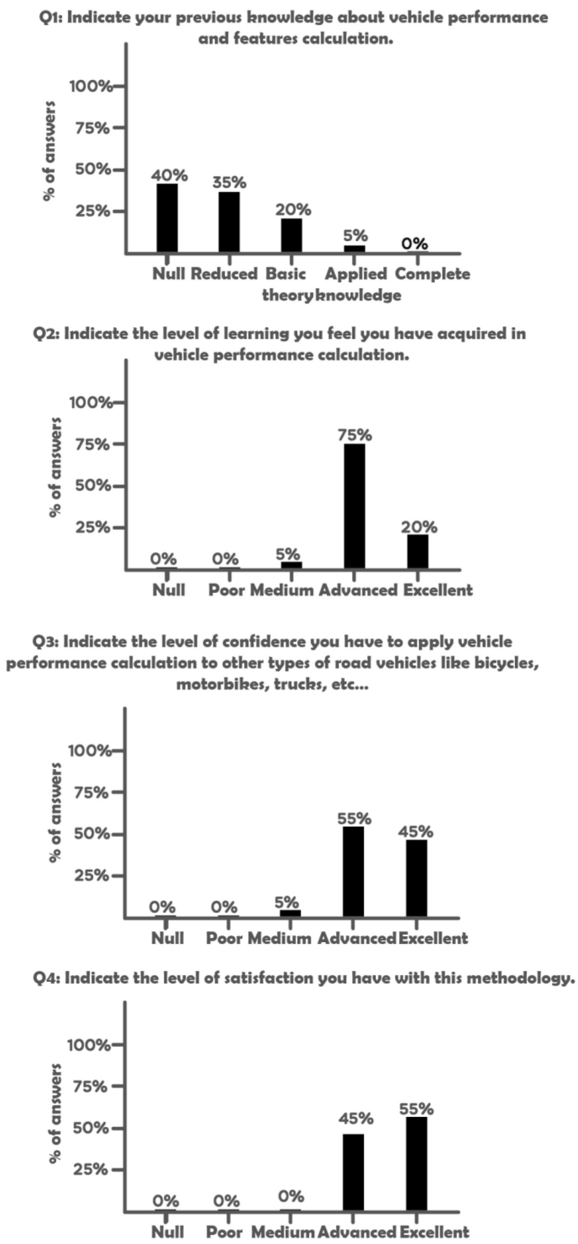


**Fig. 4.** Students satisfaction survey results. Total number of students: 155.

Q3 results demonstrate that students feel confident about their new competences. It is so that more than a third of the students choose to develop vehicle design applied to other types of vehicle in the final project of the subject. Besides 2% of the students have chosen vehicle design as the topic for their final master thesis.
Q4 indicates in general that students do like the methodology and that they feel satisfied about the method. For most of the students is very satisfying to check that the performance values they obtain coincides, or are very close, to those given by the car manufacturer. Besides, they consider that programming an automatic excel file can be

very helpful for their future works because they can rapidly check and optimize any time of calculation of process.

Therefore, the experiences assessment shows that the methodology is really effective and attractive to the students. This encourage us to continue with its implementation in future courses.

We propose as potential improvement of the methodology to further develop the automatic spreadsheet to obtain other values of interest as for example motor-braking, consumption calculation or optimal weight distribution. If consumption calculation is included, we could demonstrate numerically some of the recommendation commonly given for an ecological driving. Moreover, we will try to complement this practical experience with the real driving practices in a closed circuit that is near the university. In this real driving practices students could try to reach the limits of their vehicles in a safe way and check calculation with real experience.

## 6. Conclusions

In this article, we present a new practical methodology in which we encourage students to be involved in the learning of vehicle performance calculation and design. Students are asked to calculate the mechanical performance of their own vehicles, turning the topic into something really applied, close and attractive for them. Students gather geometric and main mechanical parameters of their vehicles and calculate the maximum speed, 0–100 km/h acceleration time and maximum slope that their vehicles can reach.

The experience assessment results show that students have achieved an excellent understanding of the vehicle mechanical performance calculation process while their curiosity and interest on the topic is also increased because it is applied to their selected vehicles. After five years of application and more than 150 students, results show that students have acquired a good learning, with a broader overview, so that they can apply the knowledge learned in a reversed way. That is, designing a vehicle based on certain proposed characteristics, using critical thinking and common sense) or even applying the same knowledge into other road vehicles, such as trucks, motorbikes or even bicycles. Some of the students gets so attracted by this topic that they chose it for their final master thesis.

## 7. Appendices

*7.1 Nomenclature and typical geometric and physical values for passengers cars*

The complete set of parameters that students must gather are listed in table 1, including their nomenclature and abbreviations. If the student cannot find or determine some of these values, default general values can be used. These default values are also listed in Table 1.

*7.2 Calculation of resistant forces*

The calculation of the resistant forces can be done as follows:

- Aerodynamic resistance, $R_{ax}$: Aerodynamic resistance is the force a body undergoes as the vehicle moves through the air, and in particular to the component of that force in the direction of the relative speed of the body in respect to the medium. Aerodynamics Resistance is always opposite to that of speed, so it is usually said that, in a manner analogous to friction, is the force that opposes the advancing of a body through the air. It can be expressed as

$$R_{ax} = \frac{1}{2} \cdot \rho \cdot Af \cdot Cx \cdot v^2 \,[N] \quad (7)$$

Where $\rho$ is the density $(\frac{kg}{m^{-3}})$ of the medium (air in the present work), $A_f$ the frontal area $(m^2)$ of the vehicle, $C_x$ the aerodynamic coefficient (dimensionless), characteristic of the vehicle to study and v $(\frac{m}{s})$ the speed vehicle.

- Rolling resistance, $R_r$: Force composed mainly by the resistance from tire deformation (90%), tire penetration and surface compression (4%), tire slippage and air circulation around the wheel (6%). The calculation expression of this force is:

$$R_r = m \cdot g \cdot cos(\theta_g) \cdot f_r(v) \,[N] \quad (8)$$

Where m (kg) is the vehicle mass, g $(\frac{m}{s^2})$ the gravitational constant, $\theta_g$ (radians) the slope $j$ of the road and $f_r$ the rolling friction coefficient whose value depends on the speed of the vehicle and tire pressure, as illustrated in Fig. 5 for common car tires.

In that sense, $f_r$ can be expressed as follows:

$$f_r = f_o + f_s \cdot \left(\frac{v}{100}\right)^{2.5} \quad (9)$$

Where v $(\frac{km}{h})$ is the vehicle speed and $f_o$ and $f_s$ are parameters depending on the tire pressure (kPa).

- Gradient resistance, $R_g$: the force generated by the gravitational force acting on the vehicle.

$$R_{ax} = m \cdot g \cdot sin(\theta_g) \,[N] \quad (10)$$

Where m (kg) is the vehicle mass, g $(\frac{m}{s^2})$ the gravitational constant and $\theta_g$ (radians) the slope of the actual road, if tilted. Therefore, sum of these

**Table 1.** List of geometric and physical parameters to be gathered for vehicle performance calculation. Default values can be used if the actual value is not found

| **Parameter denomination** | **Sym.** | **Default value** | **Comments** |
|---|---|---|---|
| Air density | $\rho$ | 1.22 kg/m$^3$ | NTP condition |
| Frontal area | $Af$ | – | Mandatory |
| Aerodyn. coef. | $Cx$ | 0.35 | Family car |
| Mass | $m$ | – | Mandatory |
| Distance between axles | $L$ | | Mandatory |
| Distance from CoG to rear axle | $l_2$ | $\frac{L}{2}$ | Mandatory |
| Total Height | $H$ | – | Mandatory |
| CoG height | $h$ | 0.4 H | Mandatory |
| Width | $B$ | – | Mandatory |
| Wheel-road sliding friction coefficient | $\mu$ | 0.756 | Exact value depending on road condition. |
| Tire denomination | – | 195/65R15 | Easy to find |
| Tire pressure | $p_i$ | 240 kPa | Easy to find |
| Tire deformation | $s_t$ | 3 % | Use default. |
| Gear box | | | |
| 1st gear | $\xi_1'$ | 1:4 | Easy to find but sometimes found as wheel development instead of gear ratio. |
| 2nd gear | $\xi_2'$ | 1:2.25 | |
| 3rd gear | $\xi_3'$ | 1:1.5 | |
| 4th gear | $\xi_4'$ | 1:1.15 | |
| 5th gear | $\xi_5'$ | 1:0.9 | |
| 6th gear | $\xi_6'$ | 1:0.7 | |
| Differential ratio | $\xi_c'$ | 1:4 | Sometimes included in wheel development. |
| Engine torque curve | $M_m(\omega)$ | – | Easy to find for common vehicles on internet. |
| Transmission performance | $\eta$ | 0.9 | Use default. |

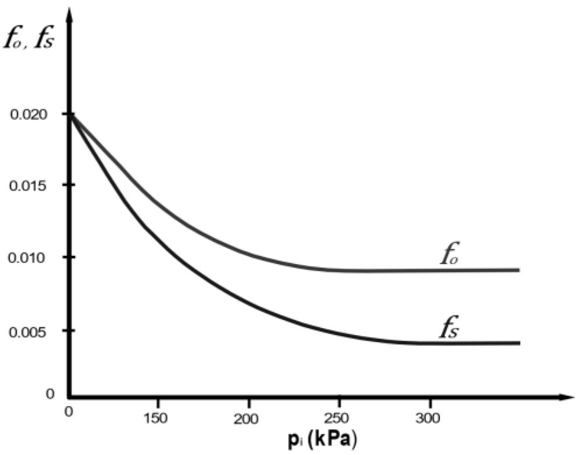


**Fig. 5.** Coefficients $f_o$ and $f_s$ required to calculate the rolling coefficient fr as a tire pressure function.

three resistances as a function of the speed of the vehicle has a parabolic behavior as shown in Fig. 2.

### *7.3 Calculation of traction forces and adherence limits*

Traction forces must surpass resistances in order to accelerate the vehicle or, at least, they must be equal to maintain the speed. The whole mechanical transmission system of the car must be considered to calculate the total traction force applied between wheel and asphalt. A combustion engine powered vehicle generally includes the next main elements:

- Engine/engine: Delivers power, i.e. input torque $M_e$ (N·m) and angular speed $\omega_e$ ($\frac{rad}{s}$).
- Clutch: Enables / interrupts power transmission.
- Gearbox: Multiply/divide torque speed. $M_s$ and $\omega_s$ represent the torque and speed at the gearbox output.

$$\xi_f = \frac{M_s}{M_e} = \frac{\omega_e}{\omega_s} = Gear\ ratio \tag{11}$$

- Joint: Flexible couplings to join both bevel differential and gearbox with the driveshaft.
- Driveshaft: Rotating shaft that transmits the torque from engine to bevel differential.
- Bevel differential: Mechanism that allows speed differences between wheels at the same axle, to accommodate different paths marked by the trajectories of the wheels. 90$^0$ angular forwarding may involve reduction ratio.

Notice that the overall relationship ($\xi_j$) between engine and wheels is $\xi_j = \xi_c \cdot \xi'_j$, where $\xi_c$ is the gear ratio provided by the bevel gear and $\xi'_j$ each gear ratio at the gearbox assuming 'q' ratios ($\xi'_1 \ldots, \xi'_q$), both dimensionless. Gear ratio with the lowest reduction occurs faster and is used so as to calculate the maximum speed. Gear ratio with greater reduction produces higher torque and is used to calculate the maximum slope.

The effort transmitted by the engine ($F_t^m$) to the drive wheels can be written from the torque at the wheel as:

$$F_t^m = \frac{M_m \cdot \xi_j \cdot \eta}{r_c} \; [N] \tag{12}$$

Where $M_m$ is the engine output torque (Nm), $\xi_j$ the gear ratio (dimensionless), $\eta$ represents the overall performance of the transmission (dimensionless) and $r_c$ (m) the effective tire radius. Therefore, all these variables must be known to calculate tractive forces diagram. Torque-speed relationship may vary depending on the combustion engine type (electric, diesel or gasoline) and its power capacity.

It is important to link both torque and speed from the motor with torque and speed at the wheels. This is done through gear ratios and it depends on the gear combination that is active at any moment. So, in the tractive forces diagram, different tractive forces appear as combustion engine vehicles use to have several gear ratios in the gearbox. Overall transmission performance depends on the partial performance of each transmission chain member. This performance also depends on time since element performance decays with wear and use.

In order to obtain vehicle's effective ratio (assuming all tires have the same effective ratio), it can be derived from geometric radius $r_g$ (m). Geometric radius is calculated by using tire information. This information is usually found either in the manufacturer's catalogue or on the popper tire, resulting in an expression that follows next expression 215/65R16 98 H. This expression can be translated into geometric values easily in online website convertors.

Finally, the effective ratio is equal to the product of the above calculated geometric ratio and the tire deformation:

$$r_c = r_g \cdot \left(1 - \frac{s_t}{100}\right) [m] \tag{13}$$

Where $r_g$ (m) is the geometric ratio and $s_t$ (dimensionless %) the tire deformation.

Adherence maximum tractive force can be obtained by calculating torque around the tractive wheel. Hence, it depends on the type of drive as:

- 4 × 4 drive

$$F_{Tmax} = \mu \cdot P \cdot \cos(\theta_g) \tag{14}$$

- Front-wheel drive

$$F_{Tmax} = \mu \cdot P \cdot \frac{l_2 + h \cdot f_r(v)}{L + \mu \cdot h} \tag{15}$$

- Rear-wheel drive

$$F_{Tmax} = \mu \cdot P \cdot \frac{l_1 - h \cdot f_r(v)}{L - \mu \cdot h} \tag{16}$$

Where $\mu$ is the sliding coefficient of friction (dimensionless), $h$ the height of the gravity center of the vehicle (m), $l_1$ the distance from the gravity center to the front wheels axle (m) and $l_2$ the distance from the gravity center to the rear wheel axle (m), as illustrated in Fig. 1.

*Acknowledgments* – Authors want to recognize the work of Alba Martínez Pérez in figures preparation.

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