Int. J. Appl. Engng Ed. Vol. 7, No. 3, pp. 212–220, 1991
Printed in Great Britain.

0742–0269/91 $3.00+0.00

Kinematic Design of Spatial Mechanisms on the CAD System*

Z. LIANG†

Indiana University-Purdue University at Fort Wayne, 2101 *Coliseum Blvd E., Fort Wayne, IN* 46805; *U.S.A.*

The paper discusses graphical methods of designing spatial mechanisms on the CAD (Computer Aided Drafting) system through a function generation problem and a path generation problem extracted from a well adopted textbook on spatial mechanisms [5]. The results are compared with those by analytical methods. It is shown that spatial mechanisms can be designed by graphical methods on the CAD system effectively and accurately.

INTRODUCTION

SINCE graphical methods of mechanisms synthesis provide the designer with a fairly quick, straightforward method of design, it is normally considered important to have experience in graphical techniques for use in the initial phases of kinematic synthesis [1]. For planar mechanisms, synthesis by graphical methods have been studied extensively [2, 3, 4]. For spatial mechanisms, on the other hand, graphical methods were seldom addressed. The procedures for planar mechanisms synthesis can be extended to three dimensions without great difficulty. For example, the problem of defining a circle from three points is extended to the one of defining a sphere from four points. But presenting and manipulating three dimensional geometric elements can be very tedious and inaccurate. These problems, however, would not exist when working on a CAD (Computer Aided Drafting) system.

In some sense, graphical design on a CAD system is a hybrid of graphical approach and analytical computation. Correct dimensions and relations of geometric elements rely on the digital computation of the computer system such that measurements and views of a mechanism are available at high speed and with high accuracy.

With computer visualization, a designer can visualize the kinematic model of the mechanism being designed. The designed mechanism can be 'played back' to see if it satisfies certain design requirements. Some design considerations other than the kinematic aspects, such as working space, link proportions and so on, can be observed on the computer as well. The designer can modify the design interactively.

Two examples will be offered in the paper. The first one is a function generation problem. The second one is a path generation problem. The both are extracted from a textbook on spatial mechanisms [5], where analytical solutions of these examples are available for comparison. Presented in the examples will be procedures only, from which computer commands can be written based on the CAD system used. This paper is not intended to supply ready-to-run computer programs, but to discuss a way of design.

We prefer, in mechanism design, wire-frame models to surface models though the latter could make procedures simpler. A wire-frame model includes fewer points and curves than the corresponding surface model. Hence, using wire-frame models makes construction and manipulation faster, results more accurate, and drawings more readable.

Unfortunately, as is well known, not every three-dimensional, or even two-dimensional mechanism can be designed graphically in a straightforward manner. Iterations are very common for analytical methods, but are often too complex for graphical methods.

We hope that this paper will lead to more interest in using the CAD system for three-dimensional manipulation and visualization in different subjects even though only spatial mechanisms will be discussed here in detail.

A FUNCTION GENERATOR

Background

A function generation problem is to design a mechanism whose output link displacement vs. input link displacement will follow the relation of a given function within a specified angle range.

Shown in Fig. 1 ([5], Fig. 7.2) is a planar mechanism. With locations of the joints a_0, b_0, and a_1 given, the location of the joint b_1 is to be determined such that output angles ϕ_{1j} will correspond to input angles θ_{1j} ($j = 2, 3$). The design procedure

* Paper accepted 17 October 1990.
† Visiting Assistant Professor.

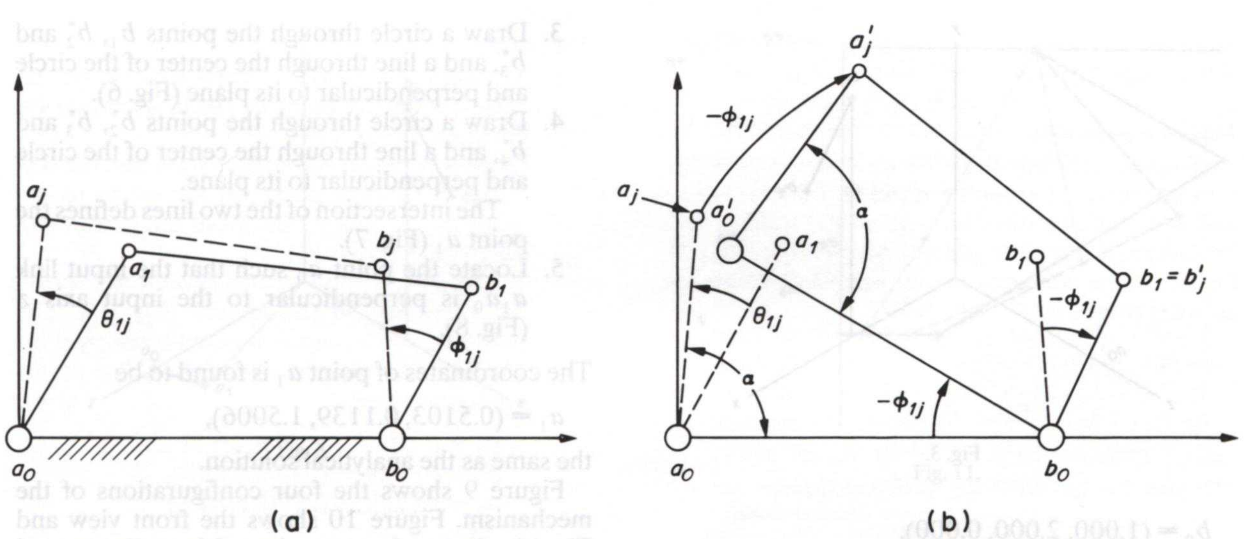

Fig. 1.

is based on a concept called kinematic inversion, as shown in Fig. 1(b). Suppose b_1 were located and the mechanism at θ_{1j} and ϕ_{1j} were rotated around point b_0 by an angle $-\phi_{1j}$ such that b_j' and b_1 coincide, point b_1 would have a same distance to point a_1 and point a_j'. Utilizing this property in design, once the locations of a_j' are found by rotating a_0a_1 by θ_{1j} around a_0 and by $-\phi_{1j}$ around b_0 ($j = 2$, 3), point b_1 is the center of the circle of a_1, a'_2 and a'_3.

It can be seen that if the given conditions of this problem change, it may not be solved graphically in a straightforward manner. If, for example, we let a_0, b_0, a_{1y} and b_{1x} be given, and a_{1x} and b_{1y} the unknown, then some graphical iteration will be needed even though the two-dimensional problem looks simple.

Problem statement

A spatial *RSSR* (Revolute-Sphere-Revolute-Sphere) mechanism (Fig. 2) is to be designed for generation of the exponential function e^x. Without the details ([5], Example 7–8), the displacement angles are computed as

$$\theta_{12} = -30.0 \text{ deg} \qquad \phi_{12} = -19.078 \text{ deg}$$
$$\theta_{13} = -60.0 \text{ deg} \qquad \phi_{13} = -47.540 \text{ deg}$$
$$\theta_{14} = -90.0 \text{ deg} \qquad \phi_{14} = -90.000 \text{ deg}$$

The input axis will be on the z axis and the output axis will be on the line $x = 1$ and $z = 0$. It is specified that

$$b_1 = (2.000, 2.000, 0.562).$$

Find the location of a_1.

Point b_0 is located by a perpendicular through b_1 to the output axis (Fig. 3):

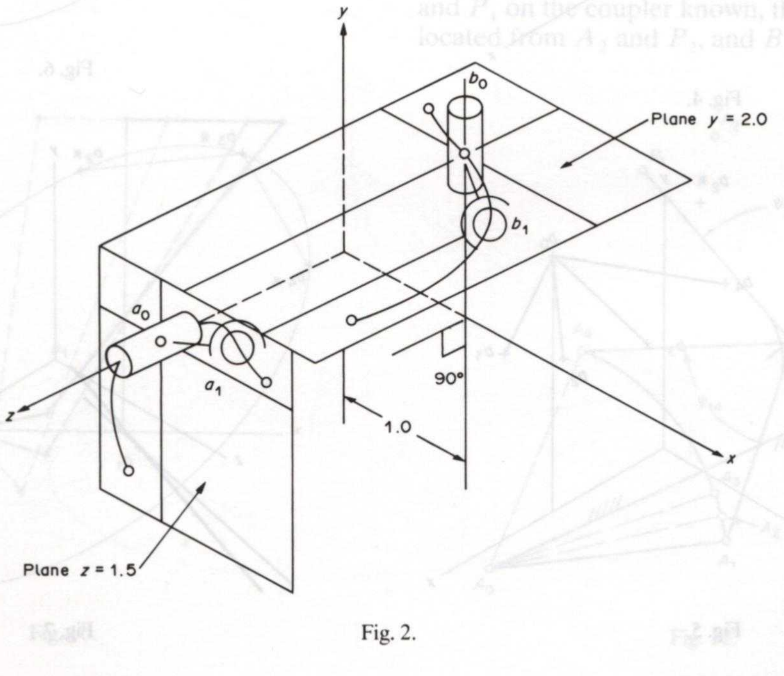

Fig. 2.

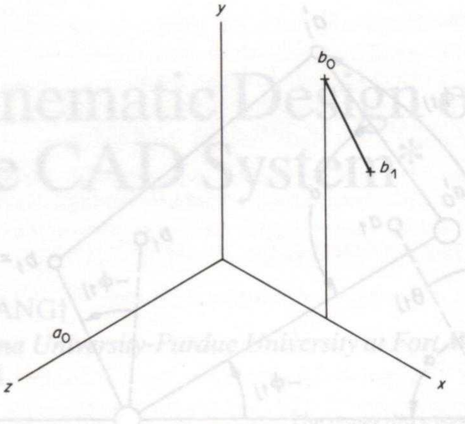

Fig. 3.

$$b_0 = (1.000, 2.000, 0.000).$$

Point a_0 on the input axis will be located in a similar way after point a_1 is located.

Procedure

1. Rotate b_1 about the output axis by angles θ_{12}, θ_{13}, and θ_{14} for the points b_2, b_3, and b_4 respectively (Fig. 4).
2. Rotate b_2, b_3, and b_4 about the input axis by angles $-\phi_{12}$, $-\phi_{13}$, and $-\phi_{14}$ for the points b_2^*, b_3^*, and b_4^* respectively (Fig. 5).

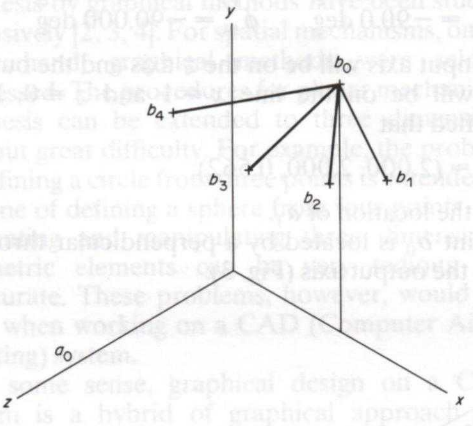

Fig. 4.

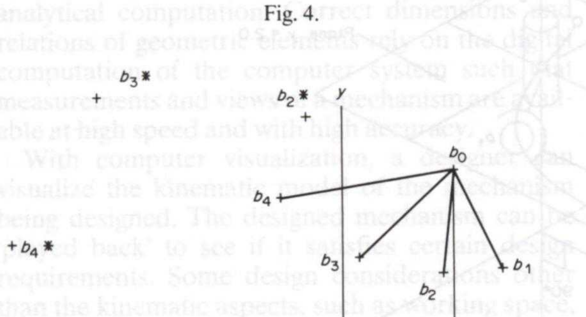

Fig. 5.

3. Draw a circle through the points b_1, b_2^* and b_3^*, and a line through the center of the circle and perpendicular to its plane (Fig. 6).
4. Draw a circle through the points b_2^*, b_3^* and b_4^*, and a line through the center of the circle and perpendicular to its plane.

   The intersection of the two lines defines the point a_1 (Fig. 7).
5. Locate the point a_0 such that the input link a_1a_0 is perpendicular to the input axis z (Fig. 8).

The coordinates of point a_1 is found to be

$$a_1 = (0.5103, 0.1139, 1.5006),$$

the same as the analytical solution.

Figure 9 shows the four configurations of the mechanism. Figure 10 shows the front view and Fig. 11 shows the top view. Many linear and angular dimensions, such as the angles between the coupler and the input link or the output link, can be measured in these views.

It is noted that the signs of ϕ_{12}, ϕ_{13}, and ϕ_{14} are incorrect for its solutions in reference [5]. The errors are not seen so easily with the analytical method as with the graphical method.

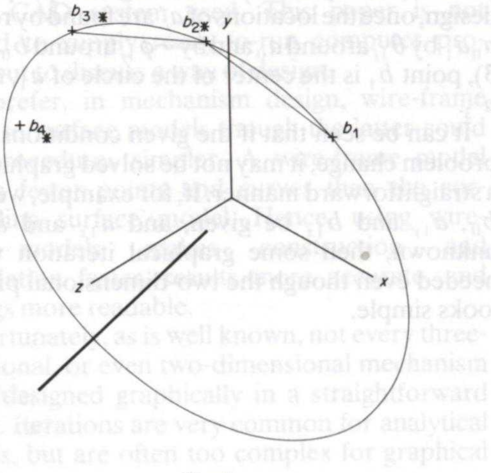

Fig. 6.

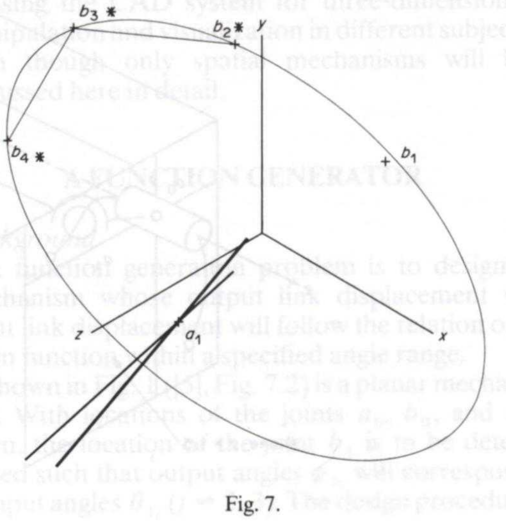

Fig. 7.

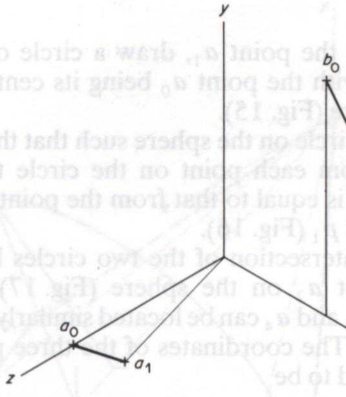

Fig. 8.

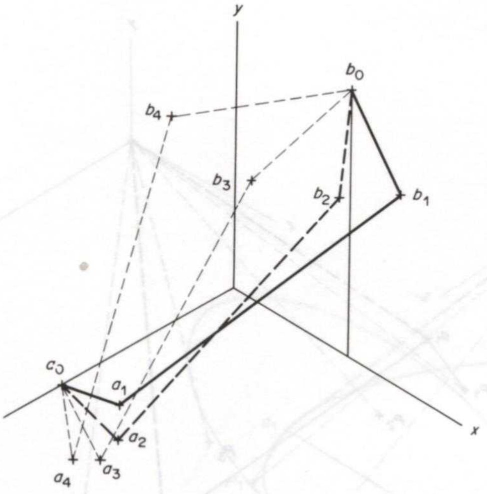

Fig. 9.

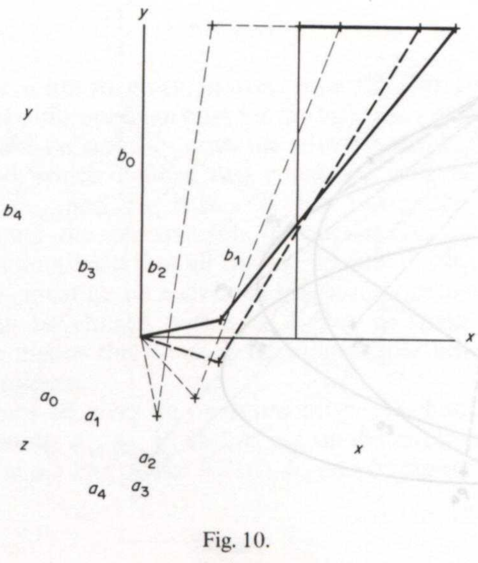

Fig. 10.

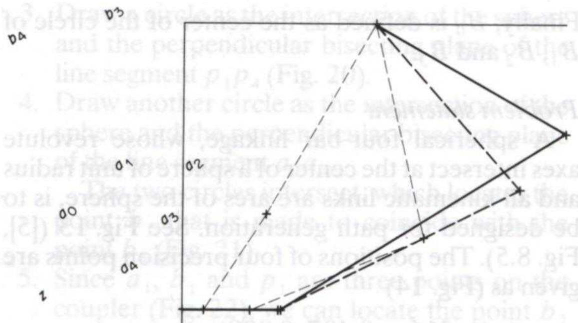

Fig. 11.

A PATH GENERATOR

Background

A path generation problem is to design a mechanism such that a point on its coupler will go through a certain number of specified points in sequence.

Shown in Fig. 12 is a planar mechanism ([1], Fig. 2.46). The two joints A_0, A_1, and the four points P_1, P_2, P_3, P_4 are specified. The joints B_0 and B_1 are to be located. We can first locate the points A_2, A_3, and A_4 since $A_2P_2 = A_1P_1$ and so on. It is then thought that if locations of the four points B_1, B_2, B_3, and B_4 would all be different, then location of the point B_0 may be over-determined. The number of different positions of the four points can be reduced to three if we make two of the four points, say B_1 and B_4, coincident. And B_1 and B_4 will be coincident if $P_1B_1 = P_4B_1$ and $A_1B_1 = A_4B_1$. Hence, point B_1 can be located as the intersection of the perpendicular bisector of A_1A_4 and that of P_1P_4. The three points A_1, B_1 and P_1 on the coupler known, the point B_2 can be located from A_2 and P_2, and B_3 from A_3 and P_3.

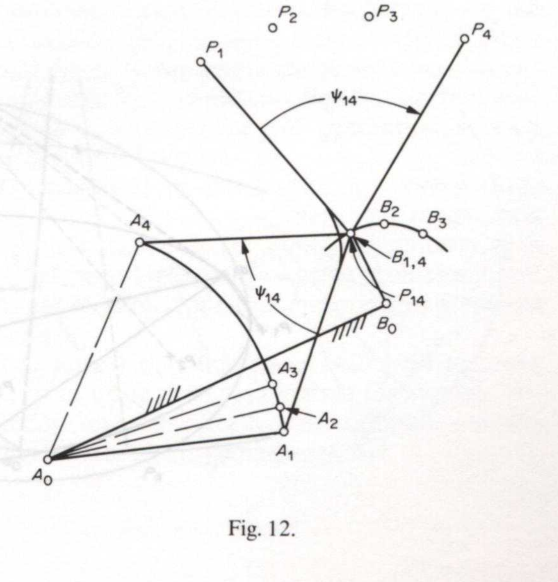

Fig. 12.

Finally, B_0 is defined as the center of the circle of B_1, B_2 and B_3.

Problem statement

A spherical four-bar linkage, whose revolute axes intersect at the center of a sphere of unit radius and all kinematic links are arcs of the sphere, is to be designed for path generation. See Fig. 13 ([5], Fig. 8.5). The positions of four precision points are given as (Fig. 14)

$$p_1 = (-.316, -0.497, 0.808),$$
$$p_2 = (0.117, -0.280, 0.953),$$
$$p_3 = (0.551, -0.208, 0.808),$$
$$p_4 = (0.695, -0.497, 0.519).$$

The crank is specified for its first position:

$$a_0 = (0.100, -0.975, 0.200),$$
$$a_1 = (-.600, -0.693, 0.400).$$

Find the locations of b_0 and b_1.

Procedure

1. Through the point a_1, draw a circle on the sphere with the point a_0 being its center on the sphere (Fig. 15).
2. Draw a circle on the sphere such that the distance from each point on the circle to the point p_2 is equal to that from the point a_1 to the point p_1 (Fig. 16).

The intersection of the two circles locate the point a_2 on the sphere (Fig. 17). The points a_3 and a_4 can be located similarly (Figs 18, 19). The coordinates of the three points are found to be

$$a_2 = (-.277, -0.586, 0.762),$$
$$a_3 = (0.124, -0.528, 0.840),$$
$$a_4 = (0.252, -0.520, 0.817),$$

the same as those by an analytical method [5], p. 198).

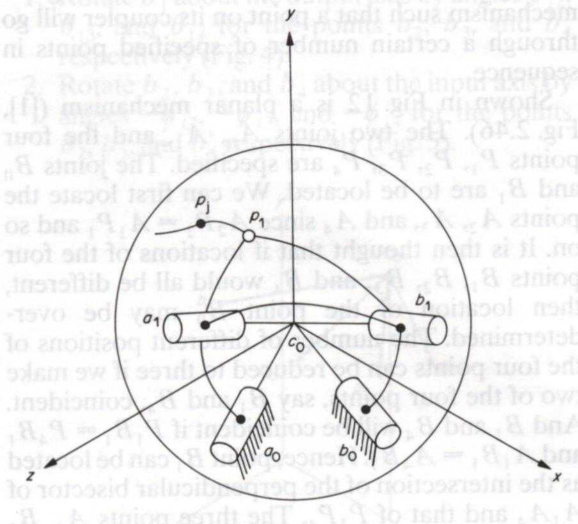

Fig. 13.

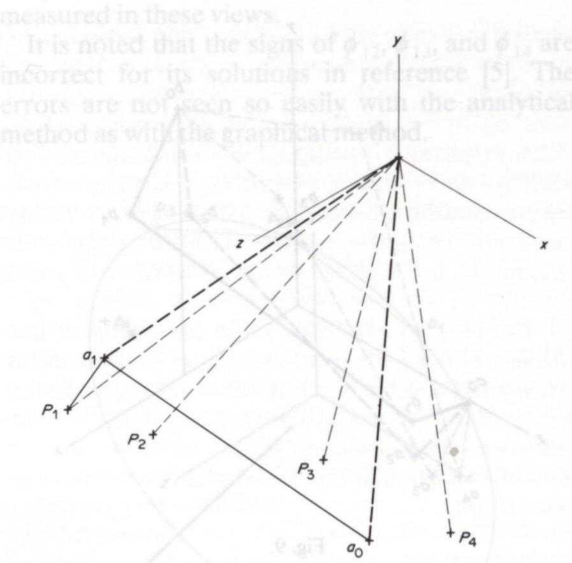

Fig. 14.

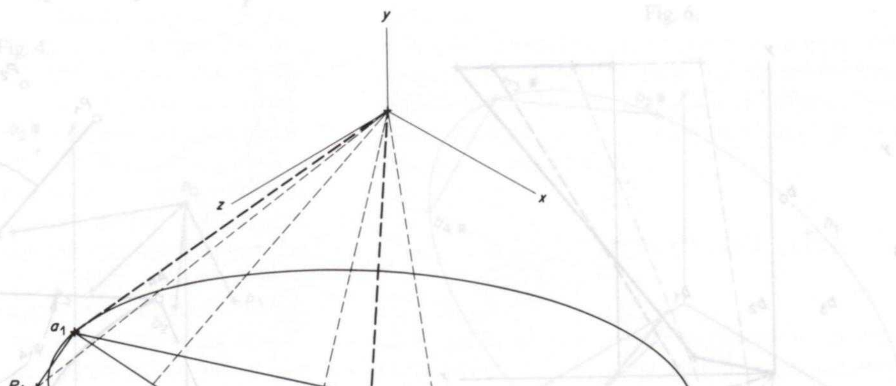

Fig. 15.

3. Draw a circle as the intersection of the sphere and the perpendicular bisecting plane of the line segment p_1p_4 (Fig. 20).
4. Draw another circle as the intersection of the sphere and the perpendicular bisecting plane of the line segment a_1a_4.

   The two circles intersect, which locates the point b_1 that is made to coincide with the point b_4 (Fig. 21).
5. Since a_1, b_1 and p_1 are three points on the coupler (Fig. 22), we can locate the point b_2 from the points a_2 and p_2 (Fig. 23), and the point b_3 from the points a_3 and p_3 (Fig. 24).
6. Through the points $b_1(b_4)$, b_2 and b_3, draw a circle. Its center on the sphere locates point b_0 (Fig. 25), whose coordinates are determined to be

$$b_0 = (0.298, -0.938, 0.176).$$

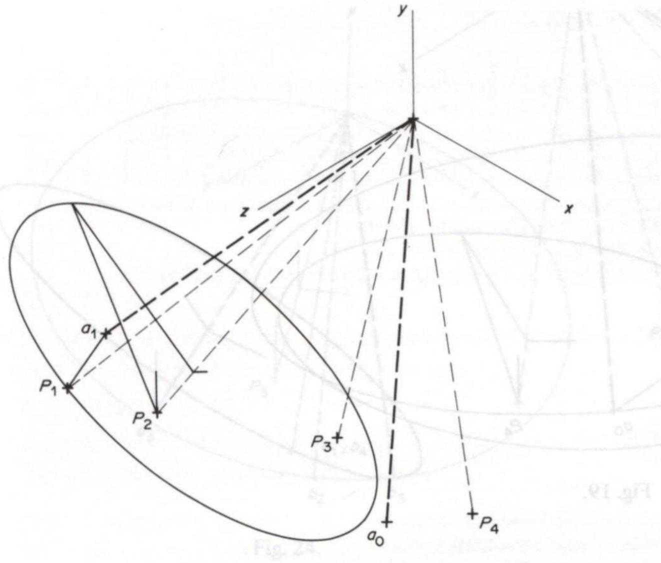

Fig. 16.

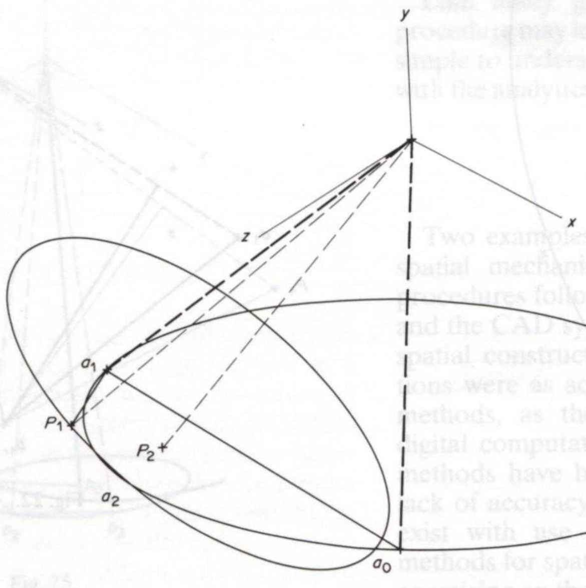

Fig. 17.

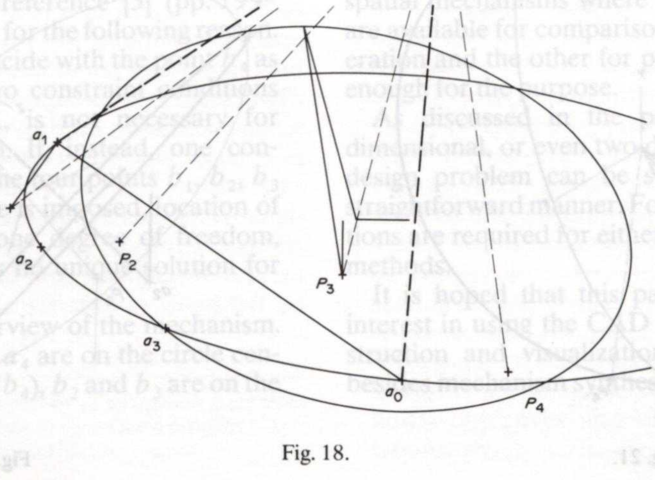

Fig. 18.

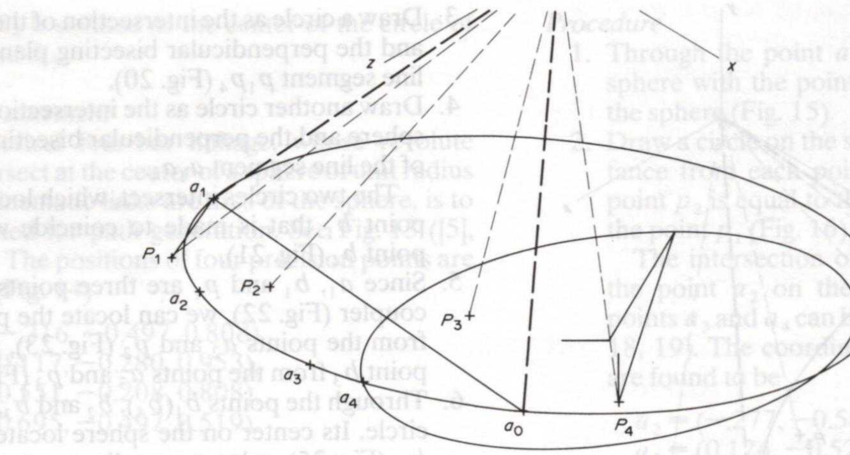

Fig. 19.

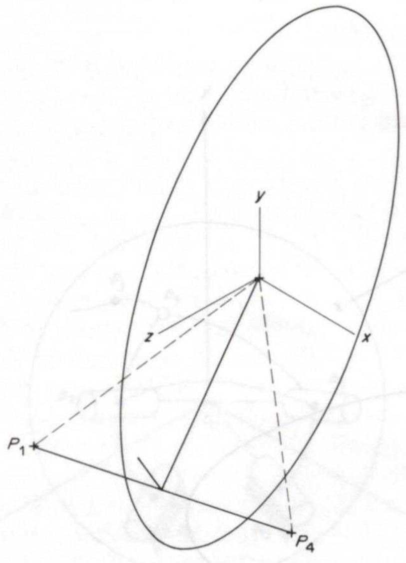

Fig. 20.

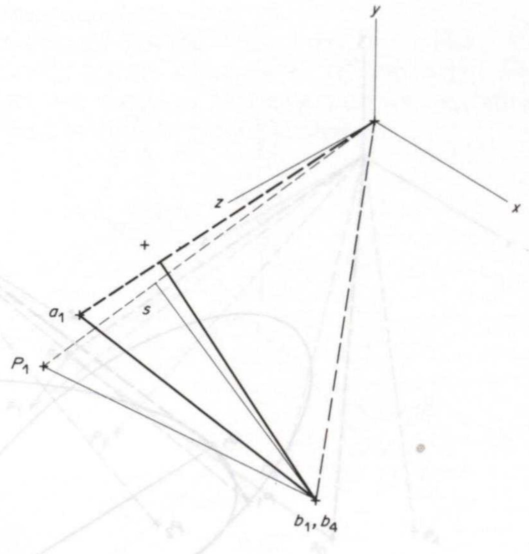

Fig. 22.

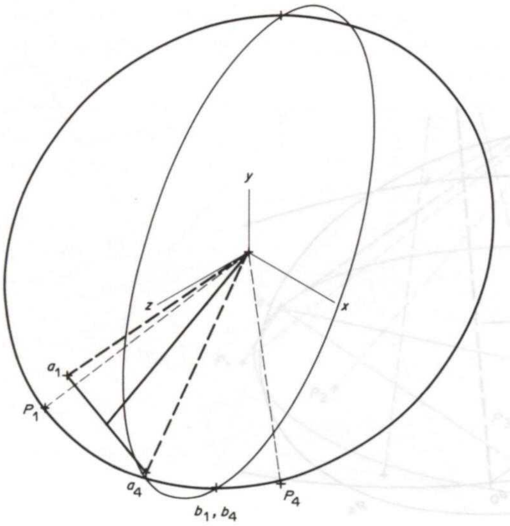

Fig. 21.

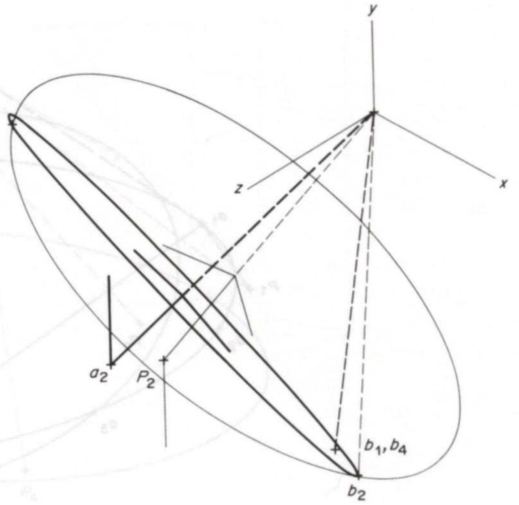

Fig. 23.

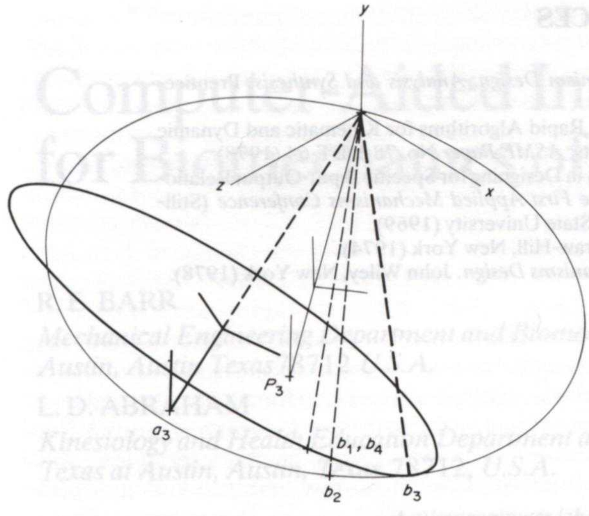

Fig. 24.

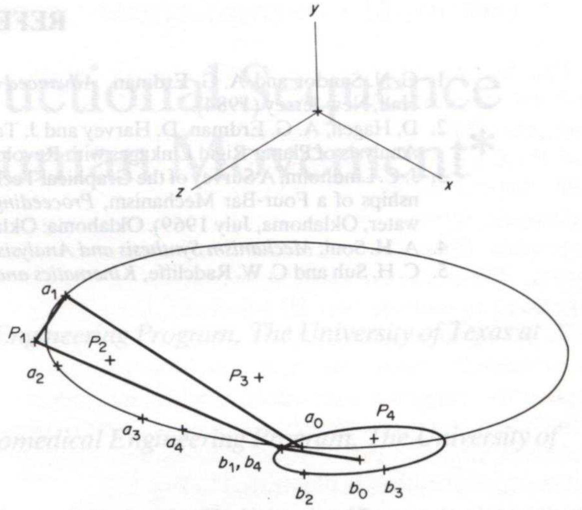

Fig. 26.

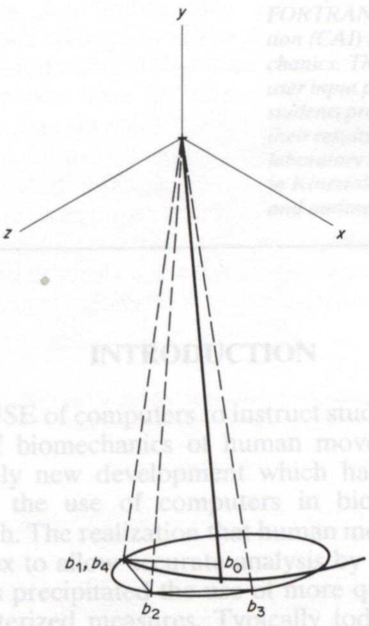

Fig. 25.

The solution given in reference [5] (pp. 199–200) is different from ours for the following reason. To make the point b_1 coincide with the point b_4 as we did, which implies two constraint conditions $b_{1x} = b_{4x}$ and $b_{1y} = b_{4y}$, is not necessary for designing the mechanism. If, instead, one constraint condition that all the four points b_1, b_2, b_3 and b_4 must be on a circle is imposed, location of b_1 can be chosen with one degree of freedom, which means that there is no unique solution for this problem.

Figure 26 gives an overview of the mechanism. The points a_1, a_2, a_3 and a_4 are on the circle centered at a_0. The points $b_1(b_4)$, b_2 and b_3 are on the circle centered at b_0. When the crank rotates, point p_1 will come to p_2, p_3 and p_4 in sequence.

Like many graphical procedures, the above procedure may look hard to read. But it is relatively simple to understand and to implement compared with the analytical procedure.

CONCLUSION

Two examples have been shown for designing spatial mechanisms on the CAD system. The procedures followed those for planar mechanisms and the CAD system provides a powerful tool for spatial construction and visualization. The solutions were as accurate as those by the analytical methods, as the measurements were given by digital computations of the computer. Graphical methods have been generally criticized for their lack of accuracy. This problem should no longer exist with use of the CAD system. Graphical methods for spatial mechanism design looked very promising on the CAD system.

This paper illustrated effective implementation of graphical design of spatial mechanisms on the CAD system. Two examples from a textbook [5] on spatial mechanisms where the analytical solutions are available for comparison, one for function generation and the other for path generation, seemed enough for the purpose.

As discussed in the paper, not every three dimensional, or even two dimensional mechanism design problem can be solved graphically in a straightforward manner. For some problems, iterations are required for either analytical or graphical methods.

It is hoped that this paper will lead to more interest in using the CAD system for spatial construction and visualization in different subjects besides mechanism synthesis.

REFERENCES

1. G. N. Sandor and A. G. Erdman, *Advanced Mechanism Design: Analysis and Synthesis*. Prentice-Hall, New Jersey (1984).
2. D. Hagen, A. G. Erdman, D. Harvey and J. Tacheny, Rapid Algorithms for Kinematic and Dynamic Analysis of Planar Rigid Linkages with Revolute Joints, *ASME Paper* No. 78-DET-64 (1978).
3. J. C. Lindholm, A Survey of the Graphical Techniques in Designing for Specific Input-Output Relationships of a Four-Bar Mechanism, *Proceedings of the First Applied Mechanisms Conference* (Stillwater, Oklahoma, July 1969). Oklahoma: Oklahoma State University (1969).
4. A. H. Soni, *Mechanism Synthesis and Analysis*. McGraw-Hill, New York (1974).
5. C. H. Suh and C. W. Radcliffe, *Kinematics and Mechanisms Design*. John Wiley, New York (1978).