

# Development of a Computer-Aided Experiment in Classic Control Theory\*

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*This paper describes a computer-aided control experiment for an introductory course in classic control theory. The experiment has been running in the Department of Electrical and Electronic Engineering at the University of Melbourne, Australia, since 1988. Over the years, the experiment has made a valuable contribution to the teaching of control in the department. A general description of the experiment is given, and its relevance to the teaching and understanding of the basics of linear control systems theory is then established.*

## INTRODUCTION

THE teaching of control theory at the undergraduate level begins with the study of the dynamical behaviour of second and higher order single-input-single-output (SISO) linear time-invariant systems. This involves both time and frequency domain analysis. In the time domain, the analysis is concerned with the study of transient and steady state responses to standard test signals and the determination of performance measures such as damping coefficient and frequency of oscillation. In the frequency domain, the analysis is carried out by exciting systems with a sinusoidal test signal and plotting the output response on graph paper. Well-established methods such as Bode (magnitude and angle) curves, Nyquist plot, and Nichols chart are then used for performance evaluation, in terms of gain and phase margins. In the case where the performance of a system is found to be unsatisfactory, a compensation scheme is designed for it, so that the overall system performance satisfies certain performance criteria. This usually involves the determination of compensation networks such as lag, lead, and lag-lead. Other standard compensation schemes which may also be used are proportional (P), proportional plus integral (PI), proportional plus derivative (PD), and proportional plus integral plus derivative (PID) controllers.

Before the advent of digital computers, control laboratory experiments were carried out using analog circuits and measuring devices. All the calculations and drawing of performance graphs were performed manually. This was very laborious and time consuming. In addition, the amount of calculations involved in those experiments, did not leave the students with enough time to concentrate on the analysis and design aspects of the experiment.

The rapid advancement in the computer industry over the past two decades has led to the development of computer software programs able to perform such complex tasks as analysis, design, and simulation of dynamical systems of orders that could not have been conceivably considered before. Software development in control engineering has been so advanced that commercial software packages of all kinds have found their way into university undergraduate and research laboratories, industrial institutions, and research establishments. At present it is rather unusual for teaching or research-orientated institutions not to have developed, acquired and used such packages. Therefore the rapid advancement in computer technology has made it possible to design more advanced experiments in control. In this paper a computer-aided control experiment for a first course in control engineering will be presented. The experiment has been designed to demonstrate the basic principles of classical control theory in a unique and effective way.

## GENERAL PURPOSE ANALOG CONTROLLER SIMULATOR

Experience has shown that although the use of commercial analog computers in the control laboratory introduces flexibility in simulating different kinds of systems of different dynamics, their programming is not a straightforward matter and is in most cases time consuming, especially when undergraduate students in the first years of their studies who have just been introduced to control theory are required to use them. Therefore it was felt necessary that a simplified, but as versatile, version should be developed for use in the control laboratory of the Department of Electrical and Electronic Engineering at the University of Melbourne. This has led to design and construction of the gen-

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eral purpose analog controller or simulator (GPACS) shown in Fig. 1. As shown in the figure, the GPACS is capable of simulating transfer functions of dynamical systems represented by up to third order numerator and denominator polynomials. The top row of the GPACS represents the nominator polynomial, while the bottom row represents the denominator polynomial. The potentiometers in each row represent the corresponding polynomial coefficients. This implies that by a proper adjustment of these potentiometer settings, systems of varying dynamics as well as controllers of various descriptions may be simulated. Thus, it is possible, for example, for one GPACS to act as a plant and for another GPACS to act as a controller. The versatility of the GPACS thus provides simple, effective, and inexpensive teaching aid for use in the control laboratory for the study of open- and close-loop performances of systems of various dynamics.

### EXPERIMENTAL SET UP

In order to effectively utilize the laboratory time, by minimizing the time spent on measurements and calculations and maximizing the time spent on analysis and design, the set up shown in Fig. 2 is used. This comprises (1) GPACS; (2) IBM-compatible personal computer; (3) A/D and D/A interfacing boards; (4) software package for data acquisition and display; (5) software package for control system analysis and design; and (6) test signal generator. This set up provides the necessary means for experimenting with real-time control systems of various descriptions and allows the responses of these systems to standard test signals to be monitored in real time, stored for future use, and printed for analysis.

### TRANSIENT RESPONSE ANALYSIS EXPERIMENT

This is a basic control experiment which covers the fundamentals of linear control theory. It demonstrates quite clearly the concepts of poles and zeros of dynamical systems and the relationship between their location in the complex plane and the transient response of the systems they represent. It also demonstrates the role of the dominant poles, the effect of zero-pole cancellation, and advantages of compensation schemes. All these are linked together so that an enhanced understanding of the basics of liner control theory is developed at the early stages of the control engineering course.

Although the set up shown in Fig. 2 may be used to perform a wide range of control experimentation, the experiment proposed in this paper comprises the following four parts. Part A deals with a standard second order system with no zero. Part B introduces the concept of zero and discusses its effect on the dynamical behaviour of the second order system. Part C deals with the concepts of dominant poles and pole-zero cancellation. Part C is concerned with the advantages of compensation schemes.

Throughout the experiment, the following applies. The test signal is a step change in the reference input,  $R$ . The output response,  $Y$ , is acquired by using an IBM personal computer equipped with an A/D and D/A interfacing card and a data acquisition software package. A hard copy of the response is obtained, by a simple screen dump on an Epson dot matrix printer. A computer-aided control system design (CACSD) software package is used to verify the experimental results, through computer simulations. The same package is also

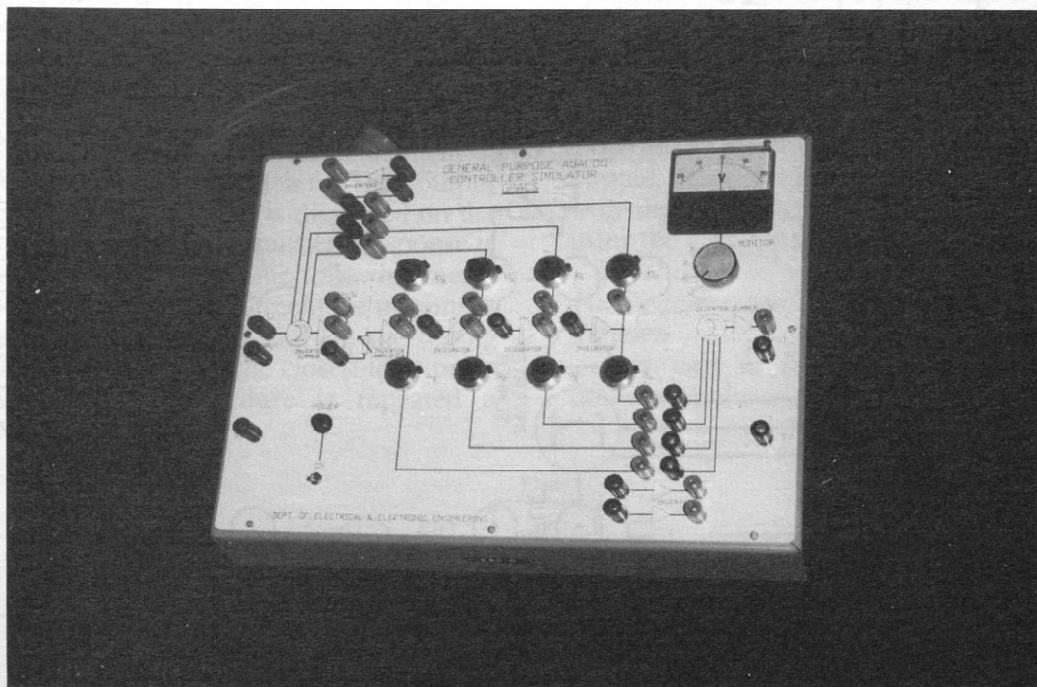


Fig. 1. (a) A view of the GPACS.

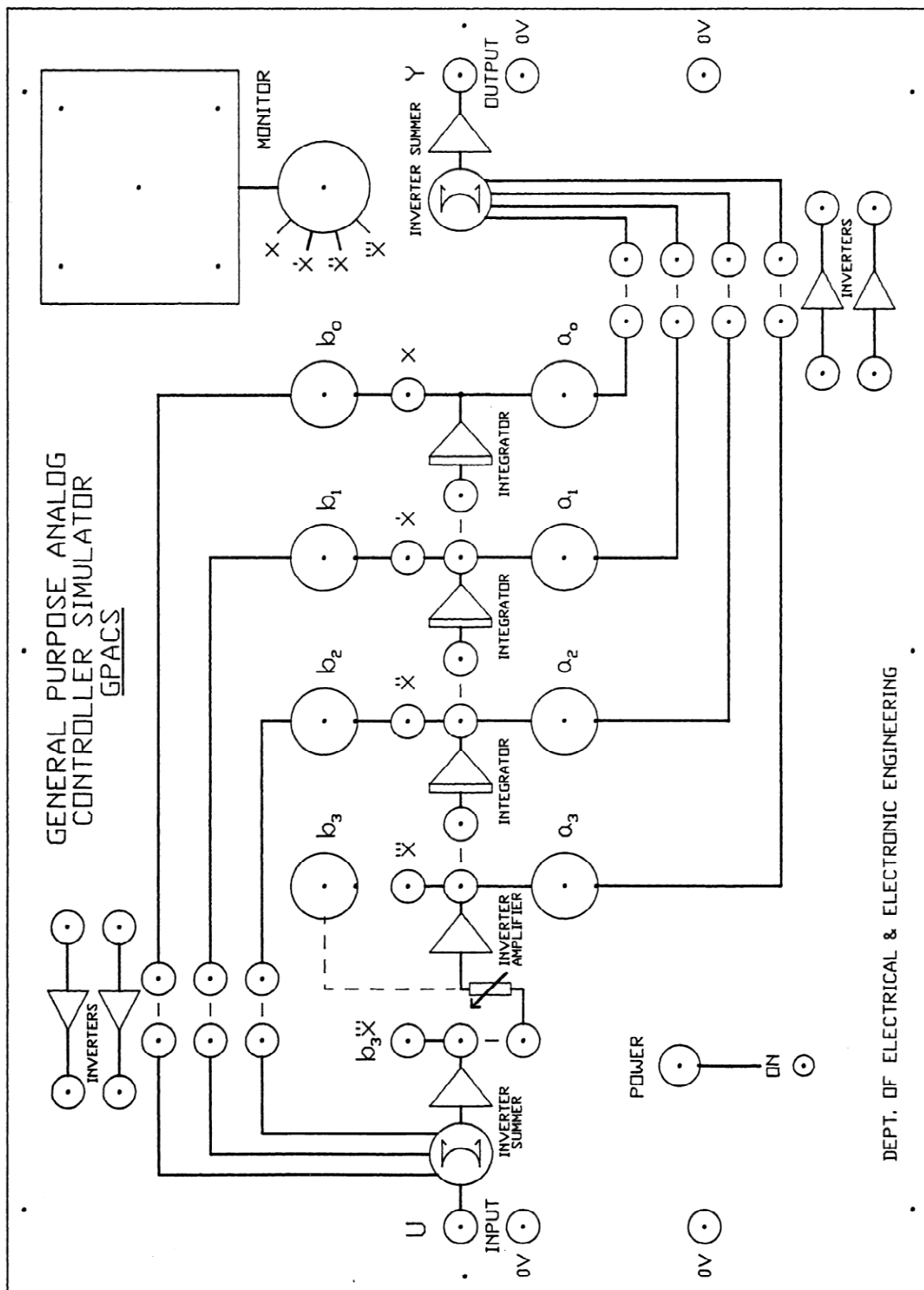


Fig. 1. (b) Front panel layout of the GPACS.

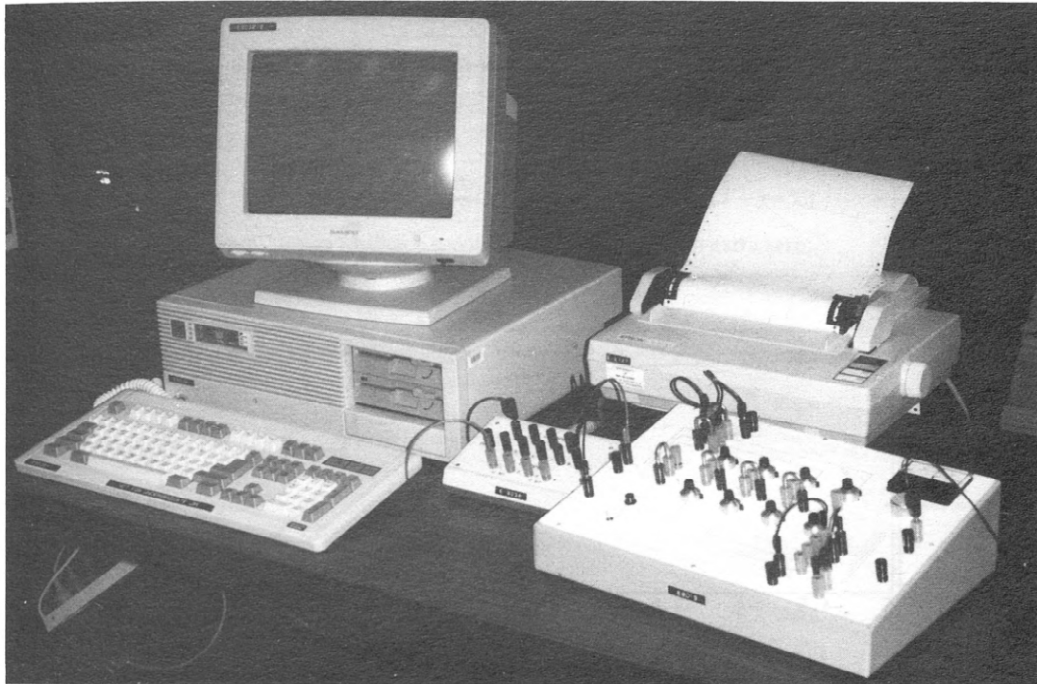


Fig. 2. A view of the control experiment set up.

used for the computer-aided design of compensation schemes.

#### *Part A: transient analysis of second order systems*

In this part, measures that characterize a standard second order dynamical system, such as d.c. gain, damping coefficient, and natural frequency of oscillation, are studied. For this purpose, the unity feedback control system shown in Fig. 3 is considered, where the GPACS is used to simulate a second order plant.

*Root locus: changing the damping coefficient.* This exercise involves keeping the gain,  $K$ , constant while changing the damping coefficient,  $\eta$ . This is achieved by changing the location of the open-loop pole,  $P_o$ , as  $P_o = 2\eta\omega_n$  where  $\omega_n = \sqrt{K}$  is the natural frequency of oscillation. A change in  $\eta$  can easily be affected by a simple adjustment of the corresponding potentiometer. The output response for each change in  $\eta$  is displayed on the monitor. A hard copy of the transient response is obtained. From the response the following measurements are taken: (1) the percentage maximum overshoot; (2) the peak-time; and (3) the settling time. Using these measures, the closed-loop poles are calculated. This procedure is repeated a

number of times until a specified maximum value of  $\eta$  is reached. Finally, the location of the closed-loop poles vs  $\eta$  are plotted to produce the root locus, as shown in Fig. 4. The resulting root locus is then compared with the one produced by computer simulation, using the CACSD software.

*Root locus: changing the gain.* In this exercise, the open-loop pole,  $P_o$ , is kept constant while the gain,  $K$ , is changed from a value of zero, where the closed-loop poles are the same as the open-loop ones, to a maximum value, where the closed-loop poles have small negative real part but large imaginary part, i.e. small  $\eta$  and large  $\omega_n$ . Each time an increase in  $K$  is effected, the output response is displayed on the monitor and a hard copy is obtained. Here again the closed-loop poles are calculated and the root locus, i.e. the location of the closed-loop poles vs  $K$ , as shown in Fig. 5, is plotted. The resulting root locus is then compared with the one produced by computer simulation, using the CACSD software.

#### *Part B: adding zero to the open-loop transfer function*

This part is concerned with the notion of dominant poles and the effect of adding zero to the

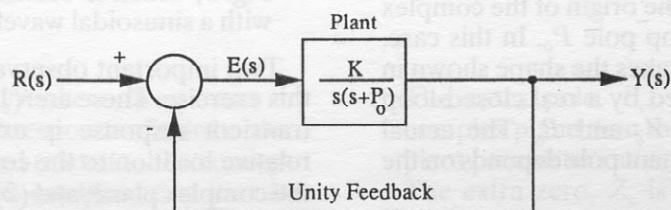


Fig. 3. Unity feedback standard second order system.



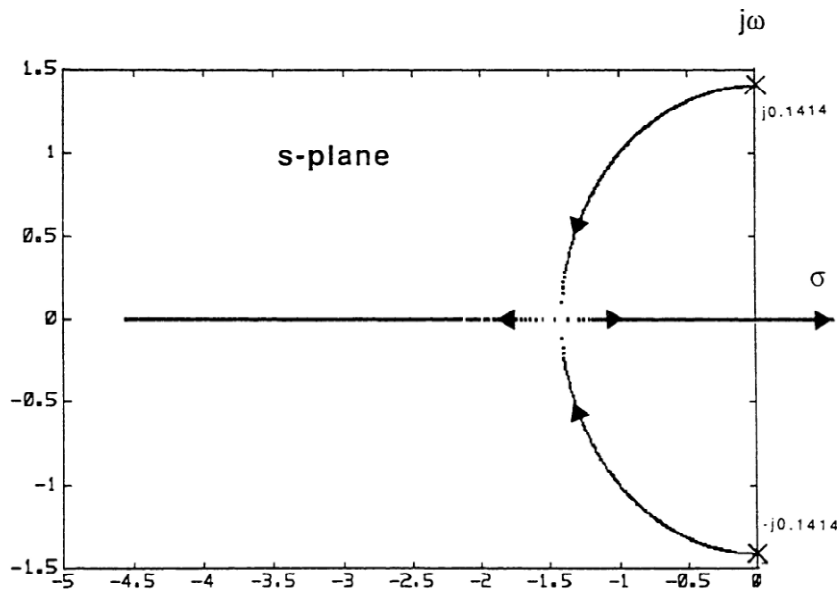


Fig. 4. Root locus of the system of Fig. 3.  $K = 2$  and  $P_o = 0 \rightarrow 5$ .

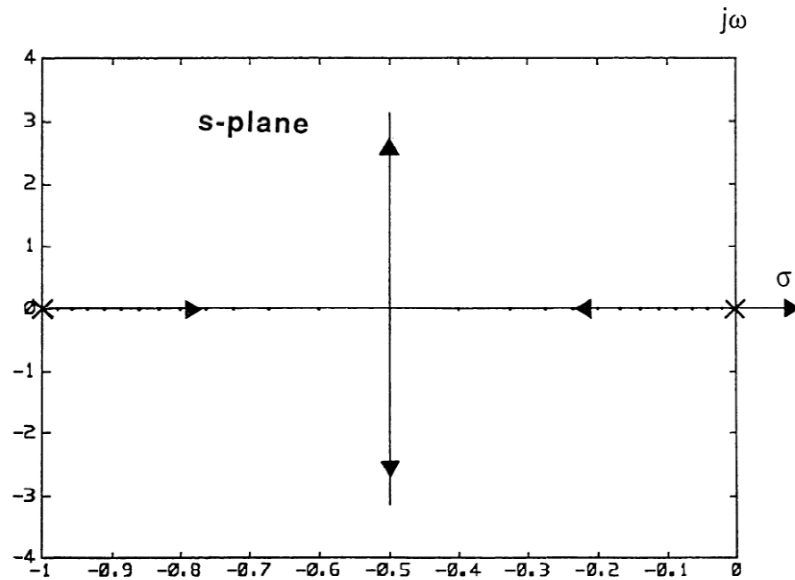


Fig. 5. Root locus of the system of Fig. 3.  $P_o = 1$  and  $K = 0 \rightarrow 10$ .

open-loop transfer function. For this purpose, the control system shown in Fig. 6 is used.

With the system parameters  $K$  and  $P_o$  kept constant, the location of the extra zero,  $Z_e$ , along the negative part of the real axis is changed. Two conditions are tested.

1. The zero lies between the origin of the complex plane and the open-loop pole  $P_o$ . In this case, the transient response takes the shape shown in Fig. 7, as it is dominated by a real closed-loop pole located between  $Z_e$  and  $P_o$ . The actual magnitude of this dominant pole depends on the value of the gain  $K$ .
2. The zero lies between the open-loop pole  $P_o$  and  $-\infty$ . In this case, the dominant closed-loop

poles are a pair of complex conjugate poles. The location of these poles in the complex plane, and hence the frequency of oscillation and the damping coefficient, depends on the placement of  $Z_e$  and also on the value of the gain  $K$ . The transient response for this case is as shown in Fig. 8, which is basically an exponential curve with a sinusoidal waveform superimposed on it.

Two important observations can be made from this exercise. These are: (1) the effect of zero on the transient response is explained in terms of its relative location to the complex conjugate poles in the complex plane; and (2) the transient response is always dominated by those poles which are closest to the imaginary axis.

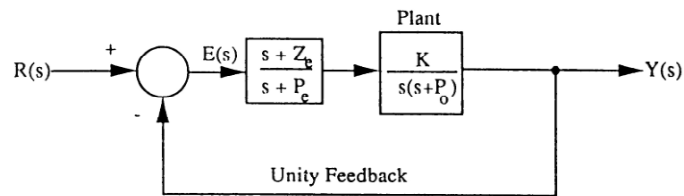
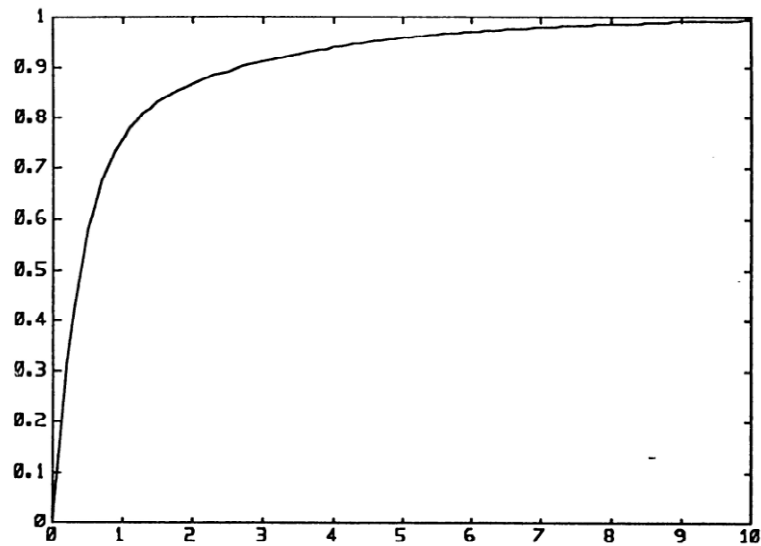
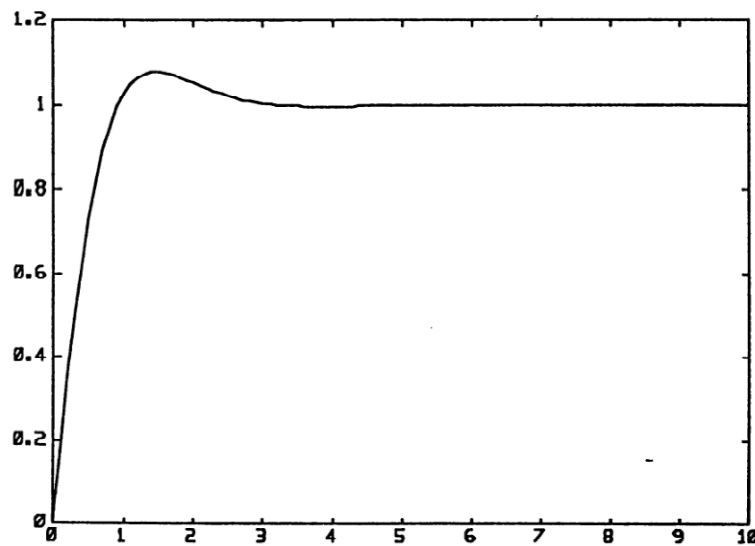


Fig. 6. Unity feedback second order system with an extra zero.

Fig. 7. A step response of the system of Fig. 6.  $K = 2$ ,  $P_o = 1$ , and  $Z_c = 0.5$ .Fig. 8. A step response of the system of Fig. 6.  $K = 2$ ,  $P_o = 1$ , and  $Z_c = 2$ .

### Part C: transient response of higher order systems

In this part the transient response of third order linear systems is studied and related to the concept of dominant poles that was established in Part B. The system shown in Fig. 9 is used for this purpose.

Two cases are investigated. These are:

1. The extra pole,  $P_c$ , is placed deeper in the complex plane than the open-loop pole  $P_o$ . The plant parameters as well as  $P_c$  are kept constant. The extra zero,  $Z_c$  is located (a) between the origin of the s-plane and  $P_o$ ; in this case the transient response will be dominated by a real

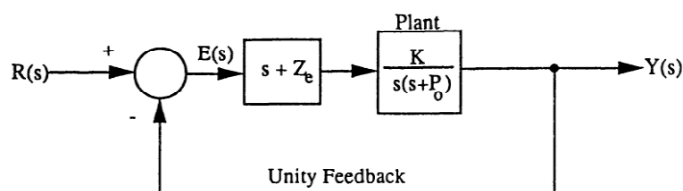


Fig. 9. Unity feedback second order control system with extra zero and pole.

closed-loop pole as shown in Fig. 10; and (b) between  $P_c$  and  $P_o$ ; in this case the transient response will be dominated by a pair of complex conjugate closed-loop poles, as shown in Fig. 11. Here again the location of these poles depends on the value of  $K$ .

2. The extra zero  $Z_c$  is placed deeper in the complex plane than the open-loop system pole  $P_o$ . The plant parameters as well as  $Z_c$  are kept constant. The extra pole,  $P_c$ , is located (a) between  $P_o$  and  $Z_c$ ; and (b) between  $Z_c$  and  $-\infty$ . In both of these cases the transient response will be dominated by a pair of complex conjugate closed-loop poles as shown in Figs 12 and 13 respectively.

The results of this part demonstrate clearly the concept of dominant poles and their relation to the transient response of the system they represent. They also illustrate the role that extra open-loop zero and pole play in the closed-loop dynamical behaviour of the system.

#### Part D: compensation schemes

In this part the advantages of various compensation schemes are illustrated. The standard second order system of Fig. 3 is considered. The system parameters are chosen so that highly underdamped oscillations are exhibited. The task is to design a

compensation scheme such as lag, lead, lag-lead, P, PI, PD, or PID for the systems so that these oscillations are damped out in a reasonable space of time. The compensation design is carried out by using the observations made in Parts A–C above and with the help of the CACSD software. Once the design process is finished, the resulting compensator is simulated on the GPACS and the closed-loop compensated system is tested. The response is then evaluated and compared with the desired one.

## CONCLUDING REMARKS

In this paper an experiment is introduced for a first course in control engineering. The experiment uses a general purpose analog control simulator, a personal computer, interfacing equipment, and computer software for the data acquisition, display, analysis, and compensator design. The experiment covers the basic idea of classic linear control theory. It clearly explains the fundamentals of dynamical systems behaviour, by relating it to the systems poles and zeros.

To allow more time for analysis and design, the time-consuming tasks of data recording, manipulation, and graph plotting have been shifted to computer hardware and software. This allows the

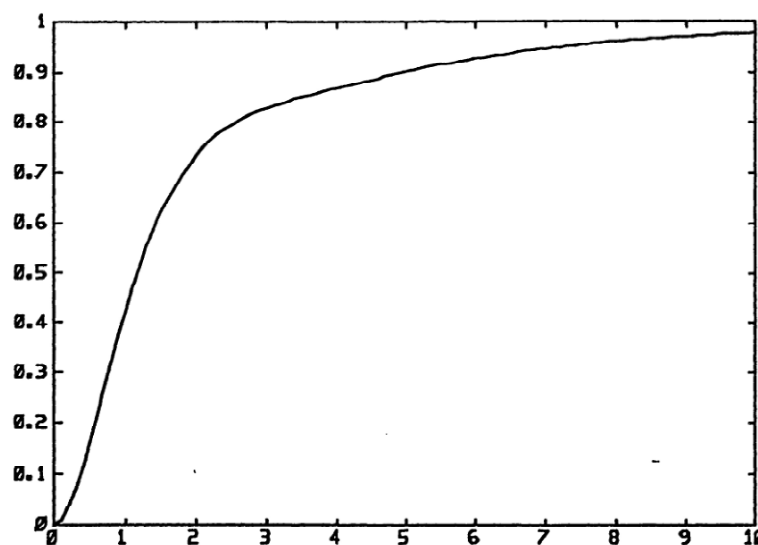


Fig. 10. A step response of the system of Fig. 9.  $K = 2$ ,  $P_o = 1$ ,  $P_c = 2$ , and  $Z_c = 0.5$ .

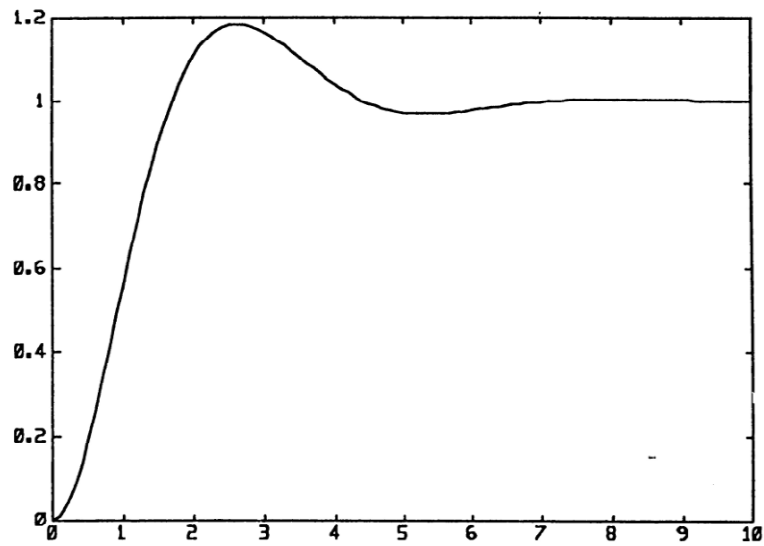


Fig. 11. A step response of the system of Fig. 9.

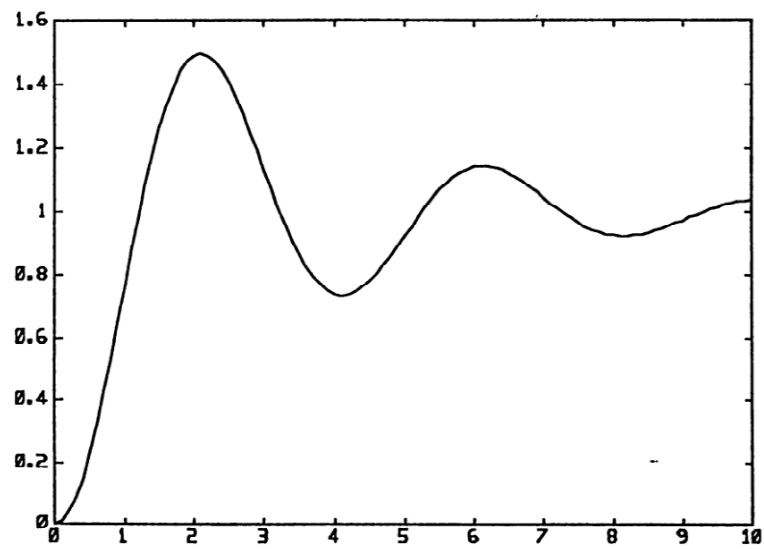


Fig. 12. A step response of the system of Fig. 9.  $K=2$ ,  $P_o=1$ ,  $Z_c=3$ , and  $P_c=2$ .

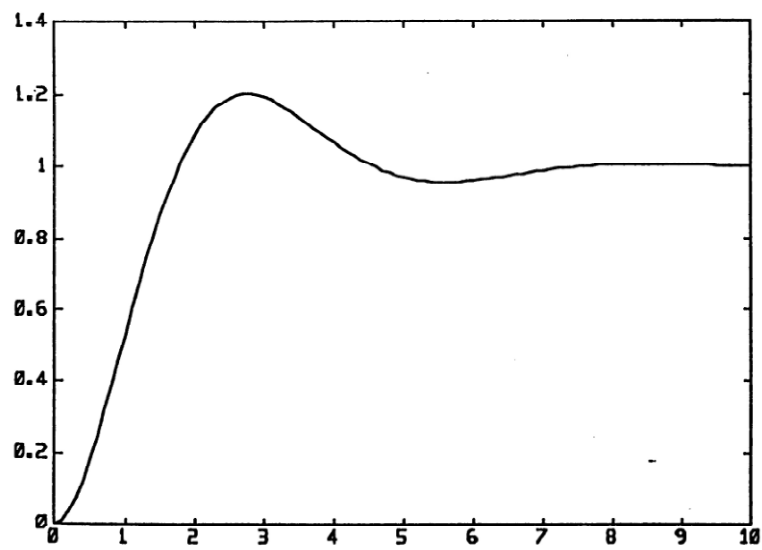


Fig. 13. A step response of the system of Fig. 9.  $K=2$ ,  $P_o=1$ ,  $Z_c=3$ , and  $P_c=4$ .



students to concentrate on analysing the various results and relating them to the theory. The experience of the past 4 years has shown that a significant improvement in the understanding of the principles of control theory has been achieved as a direct result of this experiment.

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