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Digital Simulation of Dynamic Machinery Systems*

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A technique is presented for modelling prescribed force machinery systems using a digital computer and a simulation language appropriate to the particular computer available. The technique utilizes d'Alembert's principle and loop constraint equations, to determine the nonlinear ordinary differential equations describing the machinery system. The set of differential equations is treated as an initial value problem to determine accelerations, velocities, displacements and reaction forces for a cycle of motion or for whatever time interval is of interest. Utilization of a simulation language for the solution of the equations representing the physical system accomplishes the process of generating accurate, reliable numbers for variables of interest with a minimal involvement in the details of the numerical techniques being utilized. This paper expands the basic development outlined in a previous paper to illustrate the use of the simulation language for modelling dynamic (prescribed force) systems. When utilizing a simulation language in the undergraduate instruction of machine kinematics and dynamics, the transition from kinematic to dynamic models is extremely straightforward. Students have minimal difficulty in understanding the concepts required to implement the more realistic dynamic models. There are just more equations to deal with, but the equations are essentially the same as those used in the kinematic models. The presentation in this paper is such that an instructor can easily extend the concepts involved with simulating kinematic machinery systems, to include dynamic systems. As with the kinematic simulations, the results are generated for any desired time duration, rather than isolated points in time as would be the case if graphical or hand calculation methods were utilized.

INTRODUCTION

A PREVIOUS paper [1] introduced the constructs of using a simulation language appropriate for modelling kinematic (prescribed motion) machinery systems. The basic technique utilized the loop constraint equations in second derivative form for the description of the system motion, and Newton's second law, in the form of d'Alembert's principle, for the force relationships. As is typical for kinematic systems, the motion and force problems were treated separately.

This paper considers the simulation of the dynamic or prescribed force problems using the continuous simulation language ACSL. These problems are characterized by the coupling of the governing motion and force equations, and the system of equations representing the physical system is considered as an initial value problem.

The example given here is an extension of the single-cylinder engine considered previously [1]. In the present investigation, the start-up behavior of the engine is of interest; i.e. how the engine reaches 'steady-state'. This example indicates the ease with which the effects of driving forces may be incorporated into the simulation of a particular device. This is particularly important when developing a realistic model of a prescribed force problem. The reaction forces for the example when operating at 'steady-state' are compared with the reactions from the kinematic model with interesting results.

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THE DYNAMIC SINGLE CYLINDER ENGINE

Figure 1 is the schematic and vector loop representation of the single-cylinder engine [1]. The second order differential equation relating the kinematic variables of the engine were developed in reference [1] and are:

$$-R_2\alpha_2\sin(\theta_2) - R_3\alpha_3\sin(\theta_3) = R_2\omega_2^2\cos(\theta_2) + R_3\omega_3^2\cos(\theta_2) \quad (1)$$

$$R_2\alpha_2\cos(\theta_2) + R_3\alpha_3\cos(\theta_3) - \ddot{S} = R_2\omega_2^2\sin(\theta_2) + R_3\omega_3^2\sin(\theta_3) \quad (2)$$

These are equations (4) and (5) from reference [2], slightly rearranged. The rearrangement groups terms involving unknowns on the left-hand side of the equations and 'known' terms on the right-hand sides. The terms on the right-hand side will be known from initial conditions at the start of the

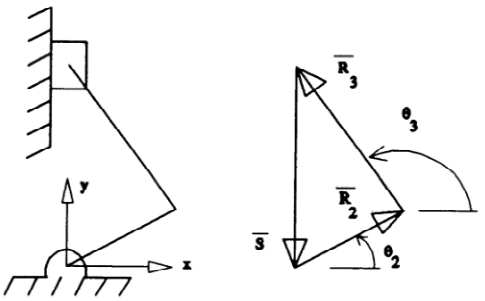

Fig. 1. Schematic and vector equivalent of a single-cylinder engine.

simulation. As the simulation progresses in time these terms will be recalculated from the results of the integration process.

On the left-hand sides of equations (1) and (2) there are three unknowns, the angular accelerations of the crankshaft and coupler, α_2 and α_3, and the acceleration of the piston, $\ddot{S}$. For the dynamic model of the single-cylinder engine the angular acceleration of the crankshaft, α_2, is not known. Thus, it is impossible to solve for the kinematic variables at this point, as was done for the prescribed motion problem described previously [1]. It will be necessary to consider the forces acting on the system simultaneously with the motion equations.

The applied and reaction forces acting on the dynamic engine are indicated in Fig. 2. The gas force acting on the piston which drives the engine will be denoted as F_{ext} and is due to an air standard Otto cycle. The viscous plus viscous squared drag torque acting on the crankshaft will be denoted as T_0.

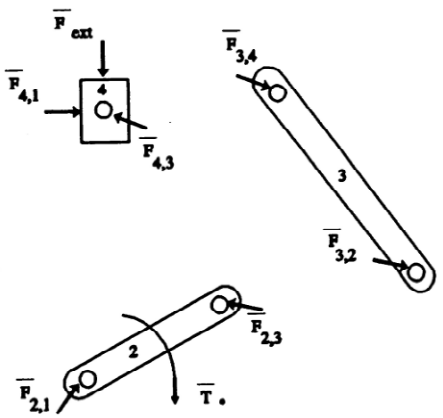

Fig. 2. Applied and reaction forces acting on the dynamic engine.

From reference [1], the scalar equations representing the force interactions between the individual parts of the engine are:

$$-F_{3,4y} - m_4\ddot{S} = F_{ext} \quad (3)$$

$$F_{3,4x} - F_{2,3x} - m_3 a_{3x} = 0 \quad (4)$$

$$F_{3,4y} - F_{2,3y} - m_3 a_{3y} = 0 \quad (5)$$

$$F_{2,3x} + F_{2,1x} - m_2 a_{2x} = 0 \quad (6)$$

$$F_{2,3y} + F_{2,1y} - m_2 a_{2y} = 0 \quad (7)$$

$$R_2\cos(\theta_2)F_{2,3y} - R_2\sin(\theta_2)F_{2,3x} - I_2\alpha_2 - R'_2\cos(\theta_2)m_2\alpha_{2y} + R'_2\sin(\theta_2)m_2\alpha_{2x} = T_0 \quad (8)$$

$$R_3\cos(\theta_3)F_{3,4y} - R_3\sin(\theta_3)F_{3,4x} - I_3\alpha_3 - R'_3\cos(\theta_3)m_3\alpha_{3y} + R'_3\sin(\theta_3)m_3\alpha_{3x} = 0 \quad (9)$$

To equations (1) and (2) representing the kinematic relationships of the system, are added the above force interaction equations for a total of nine independent equations. There are now three unknown motion variables (α_2, α_3 and $\ddot{S}$), six unknown reaction force components ($F_{3,4x}$, $F_{3,4y}$, $F_{2,3x}$, $F_{2,3y}$, $F_{2,1x}$, and $F_{2,1y}$,) and four unknown components of mass center acceleration (a_{2x}, a_{2y}, a_{3x} and a_{3y}). Each of these components of mass center acceleration can be developed in terms of the unknown angular accelerations and 'known' angular velocities and angles.

Two strategies are available at this stage. Expressions for the mass center accelerations could be developed and substituted into equations (4)–(9). This would result in a 9×9 set of simultaneous equations to solve at least once for every integration step. Alternatively, the nine equations above could be supplemented with four equations for the components of the mass center accelerations. This results in a 13×13 set of simultaneous equations. Adopting the philosophy that it is easier to get a simulation running correctly with a lot of relatively simple equations, than it is to reduce the number of equations through extensive algebraic manipulations, the model of the engine will be implemented as a 13×13 problem. The equations for the components of mass center accelerations are:

$$a_{2x} - R'_2\alpha_2\sin(\theta_2) = -R'_2\omega_2^2\cos(\theta_2) \quad (10)$$

$$a_{2y} - R'_2\alpha_2\cos(\theta_2) = -R'_2\omega_2^2\sin(\theta_2) \quad (11)$$

$$a_{3x} + R_2\alpha_2\sin(\theta_2) + R'_3\alpha^3\sin(\theta_3) = -R_2\omega_2^2\cos(\theta_2) - R'_3\omega_3^2\cos(\theta_3) \quad (12)$$

$$a_{3y} - R_2\alpha_2\cos(\theta_2) - R'_3\alpha_3\cos(\theta_3) = -R_2\omega_2^2\sin(\theta_2) - R'_3\omega_3^2\sin(\theta_3) \quad (13)$$

The parameter R'_2 is the distance from the center of the main bearings to the mass center of the crankshaft. In the present example, this will be zero to be consistent with the example from reference [1]. Equations (10) and (11) will be included in the model for ease of evaluating situations where the crankshaft is not perfectly balanced. The mass center of the connecting rod is assumed located at the midpoint of the rod, distance R'_3.

Equations (1)–(13) represent the mathematical model of the engine and were implemented to simulate the dynamic engine using the digital simulation language ACSL. No additional constructs from the ASCL language beyond those introduced in reference [1] are required to imple-

ment the dynamic model of the single-cylinder engine. A procedural block is required which accepts position and velocity information, and the physical parameters of the system (lengths, inertias and masses). The output of the procedural block is the reaction force and acceleration information. The procedural block performs the function evaluation process discussed in reference [1]. This is accomplished by solving the 13×13 set of simultaneous linear equations using an external library routine called from the ACSL program. Remember that it was not necessary to solve any simultaneous equations for the kinematic system. The mathematical model incorporates the same air standard Otto cycle to drive the engine and the same viscous plus viscous squared load torque acting on the crankshaft that were used in the kinematic model of reference [1].

At this point it is perhaps appropriate to indicate the major philosophical difference between the present work and that of Timm [2]. The approach developed by Timm results in fewer equations to solve at each integration step, at the expense of considerable algebraic manipulation on the part of the individual implementing the simulation. The Timm approach might start by reducing the problem to a 9×9 by utilizing equations (10)–(13) to eliminate the unknown components of mass center acceleration from equations (4)–(9). This would be followed by the selection of one of the motion variables and four of the reaction force variables as primary variables and further reduction of the nine simultaneous equations to a model based on five simultaneous equations. If any of the variables which are eliminated in the algebraic process are of interest they must be developed with explicit algebraic equations after the solution of the 5×5 set of simultaneous linear equations. Models based on fewer simultaneous equations tend to be more computationally efficient. However, locating algebraic errors in the model becomes tedious as the number of equations is reduced, because of the algebraic complexity of the equations. In addition, the model based on a larger number of equations need not be computationally cumbersome if sparse matrix techniques are utilized. This was pointed out by Orlandea *et al.* [3]. Models based on a large number of relatively simple equations tend to be quite sparse as indicated by the above example. Sparse techniques have not been applied to the example simulation since satisfactory accuracy and execution times were obtained without them. The basic philosophical difference between the present work and that of Timm is that here the burden of unravelling the relationships between variables is relegated to the simultaneous equation solver, used with the simulation package. The technique developed by Timm places a considerable proportion of this effort on the individual who is implementing the simulation.

RESULTS AND OBSERVATIONS ABOUT THE ENGINE EXAMPLE

The single-cylinder engine example was run to determine various parameters and items of interest about the system. One of these items of interest is the determination of the 'steady-state' speed of the engine with the given driving thermodynamic cycle and load torque. Steady-state must be carefully interpreted, since the dynamic model does not exhibit a constant crankshaft r.p.m. as was assumed for the kinematic version of reference [1]. Here, 'steady-state' will be taken to mean when the time for a complete cycle settles out to a constant value. The model was started from rest with the crankshaft at 95° to the positive x axis of Fig. 1. The air standard Otto cycle was 'turned on' and allowed to accelerate the engine up to speed. The first second of the process is shown in Fig. 3. Notice that as the engine comes up to speed, the time for a cycle decreases. The cycle-time stabilizes to 0.0248 s in about 1.5 s after starting from rest. This cycle-time would correspond to a constant angular velocity of 252.336 radians per second. However, the crankshaft angular velocity during a cycle is definitely not constant as was assumed for the kinematic model. The crankshaft angular velocity varies from about 202 rads s^{-1} during the compression phase of the cycle to a high of about 344 rads s^{-1} at the end of the expansion phase of the driving thermodynamic cycle.

The reaction forces during a single 'steady-state' cycle are indicated in Fig. 4. The timescale was selected so that the crankshaft is passing through a horizontal 0°, rotating counterclockwise, at the start of the plot. The top dead-center of the cycle is at $T \simeq 1.935$ s and bottom dead-center is at $T \simeq 1.965$ s. The plots stop at $T \simeq 1.972$ s as the crankshaft again rotates through the horizontal zero. This scale corresponds to the abscissa of

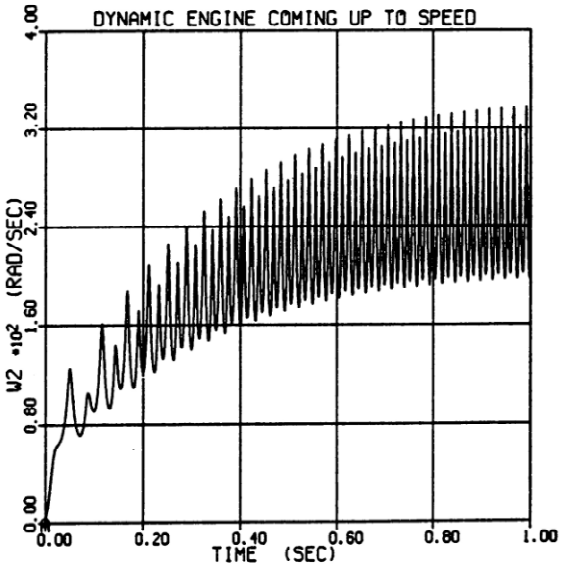

Fig. 3. The dynamic engine coming up to speed.

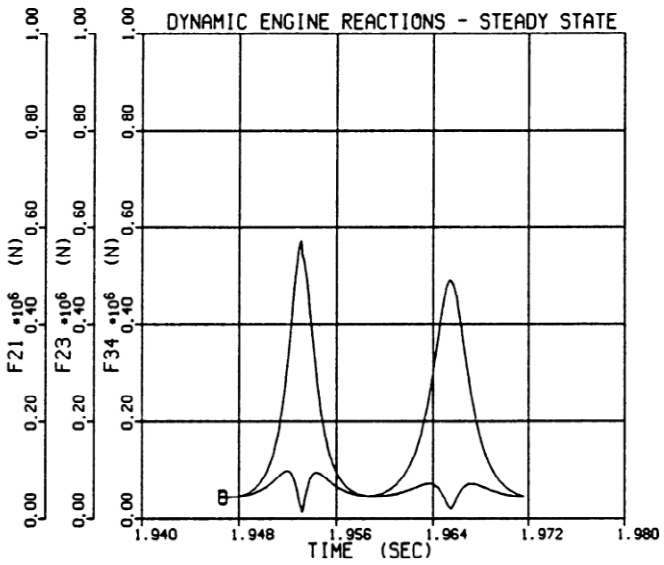

Fig. 4. Steady-state dynamic engine reactions.

Fig. 5 from [1], i.e., both plots start with the crankshaft horizontal and rotating counterclockwise. The system parameters used for the dynamic model are the same as those used for the kinematic model and are presented in Table 1 of [1]. In Fig. 4 the top curve is a plot $F_{2,1}$ and $F_{2,3}$. Since the crankshaft was modelled as perfectly balanced, the magnitude of the reaction at the main bearings will be exactly equal to the reaction at the crankshaft connecting rod connection. The lower curve is a plot of the reaction of the wrist pin connection between the piston and connecting rod.

Comparing the reactions from Fig. 4 above with the kinematic reactions from Fig. 5 of reference [1] it is noticed that only the general shape of the curves is the same. The particular angular velocity selected for the kinematic model yields the same period for the system as the dynamic engine at 'steady state'. Calculations of peak reaction forces would be seriously in error if the kinematic model were used to predict the magnitudes of the reaction forces. There are two reasons for this. The kinematic model assumed a constant crankshaft angular velocity or $\alpha_2 = 0$. The dynamic model allowed α_2 to vary in response to the external and reaction forces and torques acting. Variations in α_2 result in accelerations of the connecting rod and piston in accordance with equations (1) and (2). The second effect of considering the variation in crankshaft angular velocity is the resultant changes in the angular velocities ω_2 and ω_3. Terms involving these quantities squared occur through equations (10)–(13); explicitly in equations (1), (2) and (10)–(13); and implicitly in the center of mass acceleration terms of equations (3)–(9). Thus, considerable difference will occur during a cycle between these terms for a constant crankshaft angular velocity and for the time-varying crankshaft angular velocity associated with the dynamic model. This example indicates the need for an investigator to ascertain the apppropriateness of a kinematic assumption. For the engine example, the use of a kinematic model for selection of bearings or member cross-sections could result in a less than satisfactory design.

CONCLUDING REMARKS

A technique has been presented for modelling both kinematic and dynamic machinery systems which is simple and straightforward to apply. The basic strategy is to establish the differential equations appropriate to the physical device and integrate these equations through a time interval of interest; one cycle for kinematic systems.

The integration process requires accurate initial conditions. For kinematic devices using a velocity-based simulation, only one set of accurate positions is necessary. The velocity information is generated during the function evaluation process. Acceleration-based kinematic simulations require an accurate set of positions and velocity information for initial conditions. All reaction force and acceleration information is generated during the function evaluation process. The example presented for kinematic simulation indicated the accuracy of the process, since the end of a cycle did indeed look like the start of the same cycle to several significant digits.

Some of the extensive programs for modelling mechanical systems which were mentioned in the Introduction of reference [1] are very suitable for modelling kinematic systems but lack the capability of modelling the dynamic system. It has been demonstrated that the simulation language approach presented here can model dynamic systems as easily as kinematic systems. Terms which were known in the kinematic simulation become unknowns in the dynamic simulation.

These are readily found by the solution of the system of equations which results.

Another advantage of the methodology presented here is that any item of interest in the physical system is generally available as a parameter somewhere in the problem formulation. This parameter can be accessed without resorting to additional auxiliary equations for its evaluation. Part of the ease of implementing a simulation with the methodology presented is the emphasis on a large number of relatively simple equations rather than few, algebraically complex, equations.

The authors have found, from their educational environments, that students are most willing to accept the digital simulation approach to modelling machinery systems, once they realize the shortcomings of more conventional approaches. It is most gratifying to find these same students utilizing the simulation language in other courses (heat transfer, circuits, vibrations, automatic control, etc.), even when linear, closed-form techniques may be available there.

It is hoped that the detail and example provided in these two papers will encourage other academics to incorporate digital simulation techniques into their kinematics and dynamics of machinery courses. Copies of the ACSL programs are available from the authors on request.

SUMMARY

This paper details the use of a continuous system simulation language for modelling dynamic machinery systems. The authors use the same equations that were used to describe kinematic machinery systems, and apply them to the simulation of dynamic systems. The resultant force-acceleration model is treated as an initial value, differential equation problem. The transition from kinematic to dynamic models is straightforward and utilizes no fundamentals beyond those encountered in a first dynamics course. In an undergraduate education environment this approach reinforces the student's basic mechanics background while experiencing a solution methodology for nonlinear differential equations which is applicable to many technical areas beyond dynamics of machinery.

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