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Elucidating Negative Feedback during the Introduction to Operational Amplifiers*

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An understanding of stable equilibrium and negative feedback is provided with the introduction of operational amplifiers. Negative feedback is one of the most subtle and elegant intellectual contributions of engineering. By presenting the simplest consistent dynamic model of operational amplifier response, the process of negative feedback can be illuminated. The use of a demonstration greatly increases the number of students who grasp the operation of negative feedback.

INTRODUCTION

CERTAINLY all undergraduates in electrical engineering and many undergraduates in other engineering programs receive a first course in the analysis of electrical circuits. This course is basic to the curriculum, provides an introduction to this important technology and, in the USA, may partially satisfy accreditation requirements. In this first course in circuits, operational amplifiers, or op-amps, are often presented. Op-amps are the building blocks of electronic systems of interest in all disciplines of engineering. A survey of current textbooks on elementary circuit analysis [1–6] shows that each of these authors believes that op-amps should be introduced in the first semester of the study of electrical circuits.

The op-amp becomes a linear circuit element through the use of negative feedback, making an understanding of the workings of negative feedback essential to all but a superficial understanding of the op-amp. Negative feedback—the self-regulating system—has been recognized by philosophers of science as one of the most important intellectual contributions of engineering [7]. It is now appreciated that nearly every responsive system in nature is under the influence of a governing feedback process, including countless regulatory mechanisms of the human body [8]. These feedback processes were neither recognized nor understood until 80–100 years ago, when they were illuminated by the exploration of engineering systems with amplification and feedback.

Current textbooks often use a *static* description of the op-amp [1–6]. Typical of these presentations is [5], which starts with the device relation

$$V_{\text{out}} = A(V^+ - V^-) \qquad (1)$$

where A is the open loop gain and V_{out}, V^+ and V^- are the output and positive and negative inputs, respectively, as identified in Fig. 1. The case is then made that 'if the output is to remain bounded, then as the gain ... approaches infinity, the voltage across the input terminals must simultaneously become infinitesimally small so that as $A \rightarrow \infty$, $V^+ - V^- \rightarrow 0$ (i.e., $V^+ - V^- = 0$ or $V^+ = V^-$)' [5].

Such a description is adequate for explaining design formulae, but does not give a sense of the process of negative feedback. Reviewers of current textbooks in electrical circuits express the belief that the presentation of op-amps should be strengthened [9, 10]; and a number of authors in the education literature present approaches to strengthening this presentation [11–13].

As opposed to a static description, the approach described here relies on a *dynamic* model of the op-amp and a lecture demonstration. The material can be presented in a 1-h lecture.

A DYNAMIC MODEL OF THE OPERATIONAL AMPLIFIER

Motivation of the characteristics of the ideal operational amplifier

In lecture and by incremental steps in homework, the characteristics of the ideal op-amp are motivated by exploring the use of electric signals in transmitting information. The use of voltage rather than current as the information-bearing quantity quickly leads to the desire that the input impedance of each device be high and the output impedance be low. Examples can be given where a variable closed-loop gain is desired, leading to the utility of a negative-feedback amplifier. Finally, it is shown that with the negative-feedback configuration, a high open-loop gain is desired. These things become the definition of the ideal op-amp.

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Equilibrium, movement and negative feedback: the simplest consistent dynamic model

To communicate the workings of negative feedback, a dynamic model must be presented. The simplest consistent dynamic model incorporates an equilibrium condition and a process of returning to equilibrium. This description is embodied in the rules of Table 1. By reference to the balance that exists in ecological and physiological systems, such as population balance between predators and prey and maintaining body temperature, equilibrium can be defined by example. The dynamic is then introduced with the response to being out of equilibrium.

Table 1. Rules defining the dynamic operation of the operational amplifier—an infinite gain, finite slew rate approximation

$V^+ = V^-$:	At Equilibrium	V_{out}	Fixed
$V^+ > V^-$:	Out of Equilibrium	V_{out}	Increasing
$V^+ < V^-$:	Out of Equilibrium	V_{out}	Decreasing

The rules of Table 1 define the infinite open-loop gain, finite slew rate approximation to the behavior of an op-amp. The open-loop gain is infinite because, in the absence of negative feedback and saturation limits, the output will commence slewing and continue slewing indefinitely. The model is a reasonable approximation to an operational amplifier for large step inputs. It was chosen over a linear dynamic model because it is considered the simplest consistent model. It can be shown to operate in everyday examples of negative feedback, such as the regulation of home heating, as well as in ecological and physiological examples.

With the concepts of equilibrium and response in place, the students next see that when the response to being out of equilibrium is such as to bring the system back into equilibrium, a stable equilibrium—a self-restoring balance—is achieved. The response toward equilibrium is shown to be brought about through negative feedback in ecological examples as well as op-amps. The authors use ecological and physiological analogies to define equilibrium and illustrate stable and unstable responses, both to draw on intuitions about these areas and to show connections between these concepts and the larger world. Other approaches may, of course, be equally viable.

Turning to the op-amp, equilibrium and out-of-equilibrium conditions are defined in terms of V^+ and V^-, as shown in Table 1. With $V^+ \simeq V^-$ established as the consequence of a dynamic process, the closed-loop gain relationships can be derived as a logical consequence.

THE DEMONSTRATION

The material described above was presented once without a demonstration. To test the degree to which the students could articulate the process of negative feedback, an unprompted essay question was posed on an exam: '*How* does the circuit of Fig. 1 operate to maintain $V^+ \sim V^-$?' Twenty percent of the students were able to show an understanding of the negative-feedback process. Negative feedback is subtle: natural philosophers of the stature of Aristotle and Leonardo da Vinci failed to appreciate its role in the behavior of natural systems. A demonstration was developed as a means to improve the communication of these ideas.

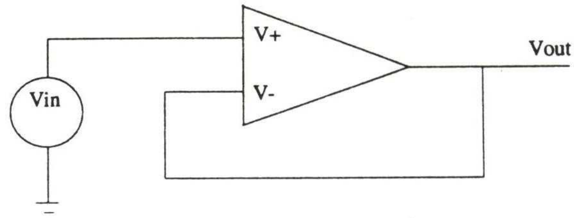

Fig. 1. An operational amplifier configured as a voltage follower.

Demonstration: making the abstract concrete

The use of demonstration is important in transforming an abstract idea into a concrete experience. Demonstration plays an important role in the *direct instruction model*, which is the educational terminology for the instruction format used in engineering education [14–17]. (Rosenshine, [15], provides an interesting survey of empirical studies of the effectiveness of teaching method.) Equilibrium cannot be seen. But a system in equilibrium can, and one may thereby come to understand the abstract notion. Gunter *et al.* [16] discuss the components of an effective demonstration. Ideas should be presented in small segments, with each of the main points illustrated by example; and the teaching, especially explanation, should be redundant, to ensure that all of the students grasp the basics. The demonstration has the advantage of forcing the students to focus, which they must do in order to explain and interpret the process of demonstration. When a student has a misunderstanding, the reality of the demonstration will conflict with expectations, and a process of 'misconception reformation' is possible. In cases where the student has a prior, but misconceived, understanding of the material to be learned, it has been shown that the process of 'unlearning' the misconceived understanding is essential, and that demonstration plays a central role in this learning process [18, 19].

The stable equilibrium point and the dynamic process that guides the system to equilibrium must be visualized in a way accessible to students without a developed intuition of the behavior of electrical systems—it should be noted that students of many departments take a first course in circuits—and causality must be clearly visualized by the workings of the demonstration.

The demonstration developed at UWM is illustrated in Fig. 2. The demonstration is mechanical, with the voltages V^+, V^- and V_{out} implemented in

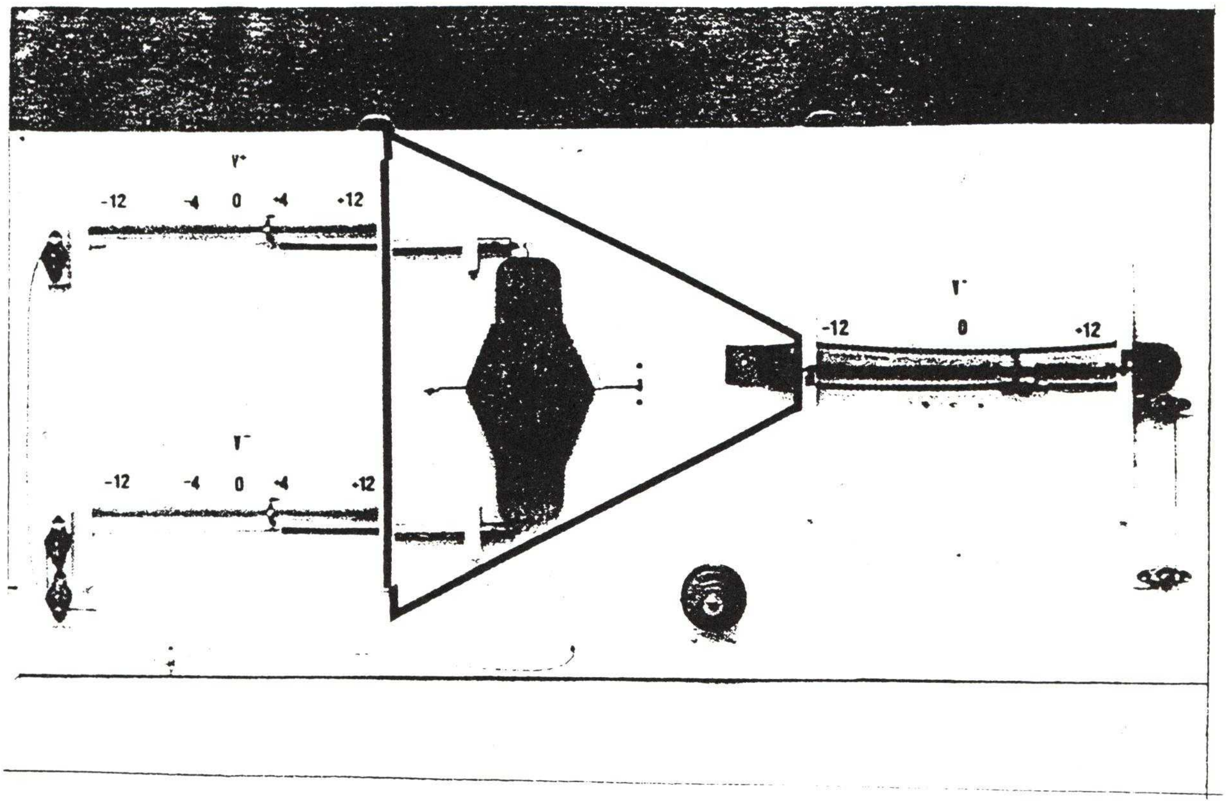

Fig. 2. A lecture hall demonstration of the operational amplifier.

cylindrical sliders moving on horizontal rails. The positions of the slides correspond to the voltages. The sliders an be seen in Fig. 2: the two on the left represent the two input voltages, V^+ and V^-; the slider on the right represents the output voltage, V_{out}. The system is shown operating in Fig. 2, in equilibrium with $V^+ = V^- = V_{\text{out}} = +3.5$ V. For scale, the length of the entire demonstration is 2 m, and the length of each slider travel from −12 volts to +12 volts is 50 cm.

A detail of the V^+ slider is seen in Fig. 3. A spring is seen to the right of the slider. The cable seen to the left of the slider implements a mechanical feedback connection and is used to demonstrate *positive* feedback.

The output voltage slider is driven by a reversible 120 VAC motor, seen in detail in Fig. 4. The motor and gearbox move the slider with a speed of 5 cm/s, implementing an output slew rate of 2.4 V/s. The total slew time from −12 to +12 volts is 10 s.

Control of the motor is electro-mechanical. Control is carried out by a pivot mechanism and two electrical contacts, shown in the center of Fig. 2 and schematically in Fig. 5. The vertically oriented wood plate is the pivot and rides on a ball bearing. The pivot point, to the right of the pivot bearing, sits between two electrical contacts, one or the other of which will close when the pivot is rotated. The contacts, through relays, select motor direction, as described in Table 2.

Table 2. Rules defining the dynamics of the operational amplifier demonstration

$V^+ = V^-$:	Pivot Centered	Motor Off
$V^+ > V^-$:	Pivot Rotated Clockwise	Motor Running CW
$V^+ < V^-$:	Pivot Rotated Counter Clockwise	Motor Running CCW

At the top and bottom of the pivot are the main springs, which connect to the V^+ and V^- sliders. If the V^+ slider is to the right of the V^+ slider, simulating the voltage condition $V^+ > V^-$, the pivot rotates clockwise, closing the lower electrical contact and running the motor clockwise to move the V_{out} slider to the right. Thus, in simulated voltages, an increasing V_{out} is the result of $V^+ > V^-$. When $V^+ < V^-$, the pivot rotates counter-clockwise, closing the upper contact and moving the V_{out} slider to the left. And when $V^+ = V^-$, the two spring tensions are in balance, the pivot is centered so that neither contact is closed and the motor does not run: V_{out} remains at rest.

The equilibrium process operates in the springs and pivot. When the spring tensions balance, the pivot is centered and the motor does not run. When one or the other of the springs exerts a greater tension, the pivot rotates, closing a switch which runs the motor to either augment or diminish the position of the V_{out} slider. Thus, the open loop

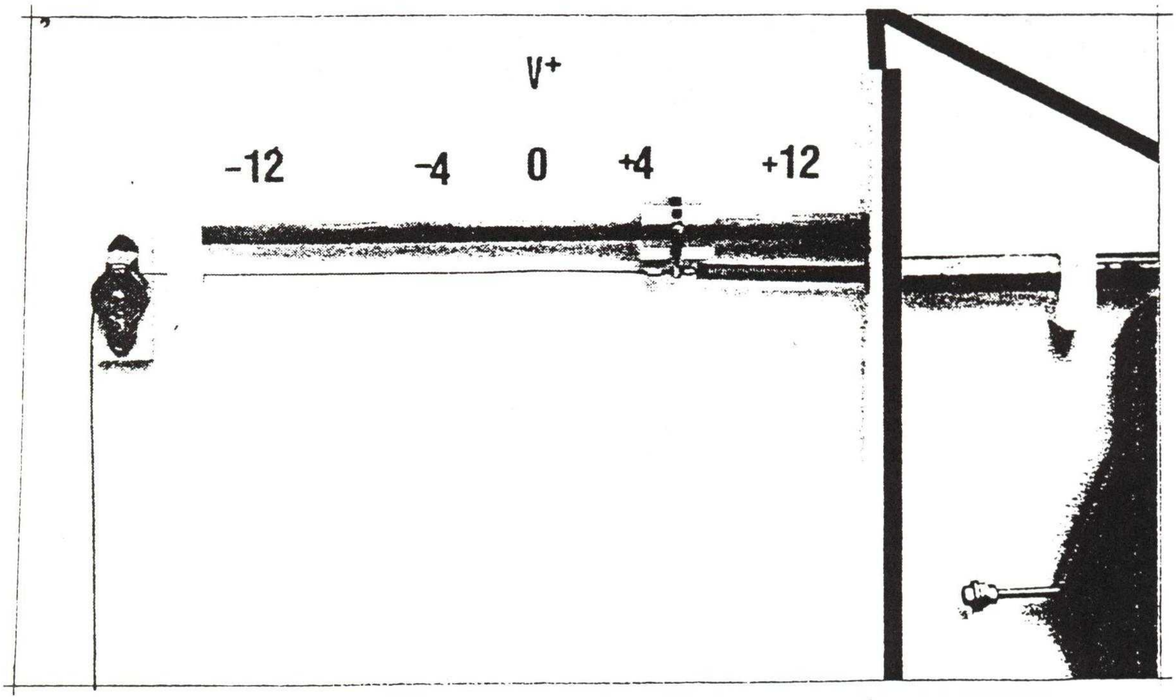

Fig. 3. Detail of the V^+ slider.

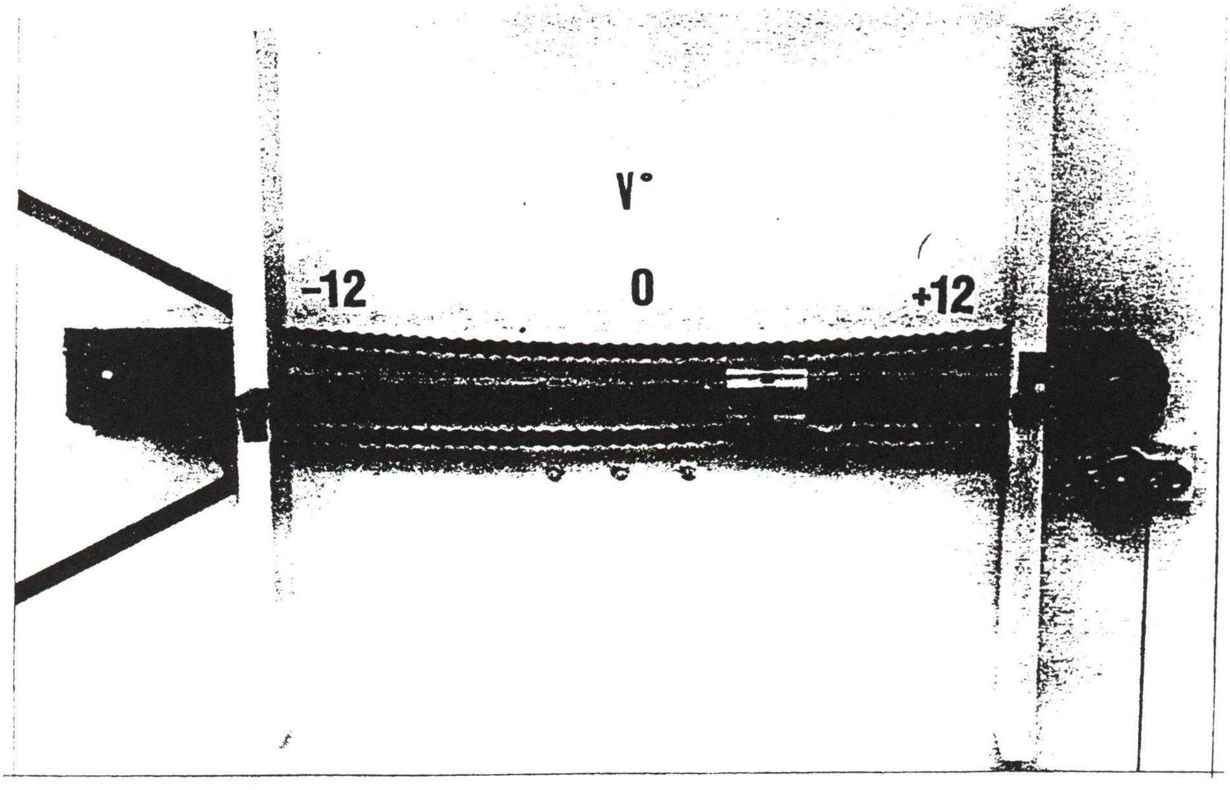

Fig. 4. Detail of the motor, chain drive and V_{out} slider.

operation of an op-amp is implemented in an electro-mechanical analog, with the equilibrium process realized in an observable mechanical process. Successful use of the demonstration rests largely on careful explanation of the spring tension, contact mechanism, much as outlined by Gunter *et al.* [16]. To increase the visibility of the pivot movement, LEDs, shown above and below the electrical contacts in Fig. 5, indicate when the pivot point is rotated and one or the other of the

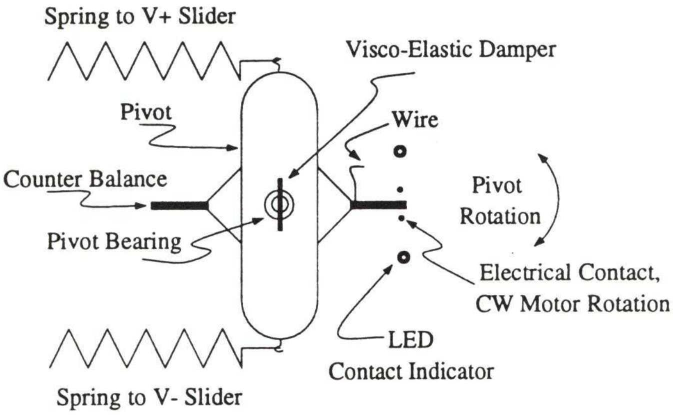

Fig. 5. Schematic of the pivot and switches controlling the motor.

electrical contacts is closed. Stability is not addressed in this course, but left for later courses that develop the appropriate mathematical tools. Stability of the demonstration is achieved with the use of hysteresis and visco-elastic damping of the pivot.

The open loop op-amp analog is made to operate in a closed loop by connecting the V_{out} and V^{-} sliders with cable, a mechanical analog to the wire connection of a voltage follower. The cable implements mechanical feedback. The system is shown cabled for unity negative feedback in Fig. 2. The cabling may be configured in several ways to demonstrate:

- open loop operation;
- negative feedback, unity gain;
- negative feedback, 3× gain;
- positive feedback.

The cable hanging loose on the left of Fig. 2 is used to demonstrate positive feedback. A detail of the cable drum is seen in Fig. 6. Before implementing each configuration, it is our technique to pose questions to the class exploring what the con-

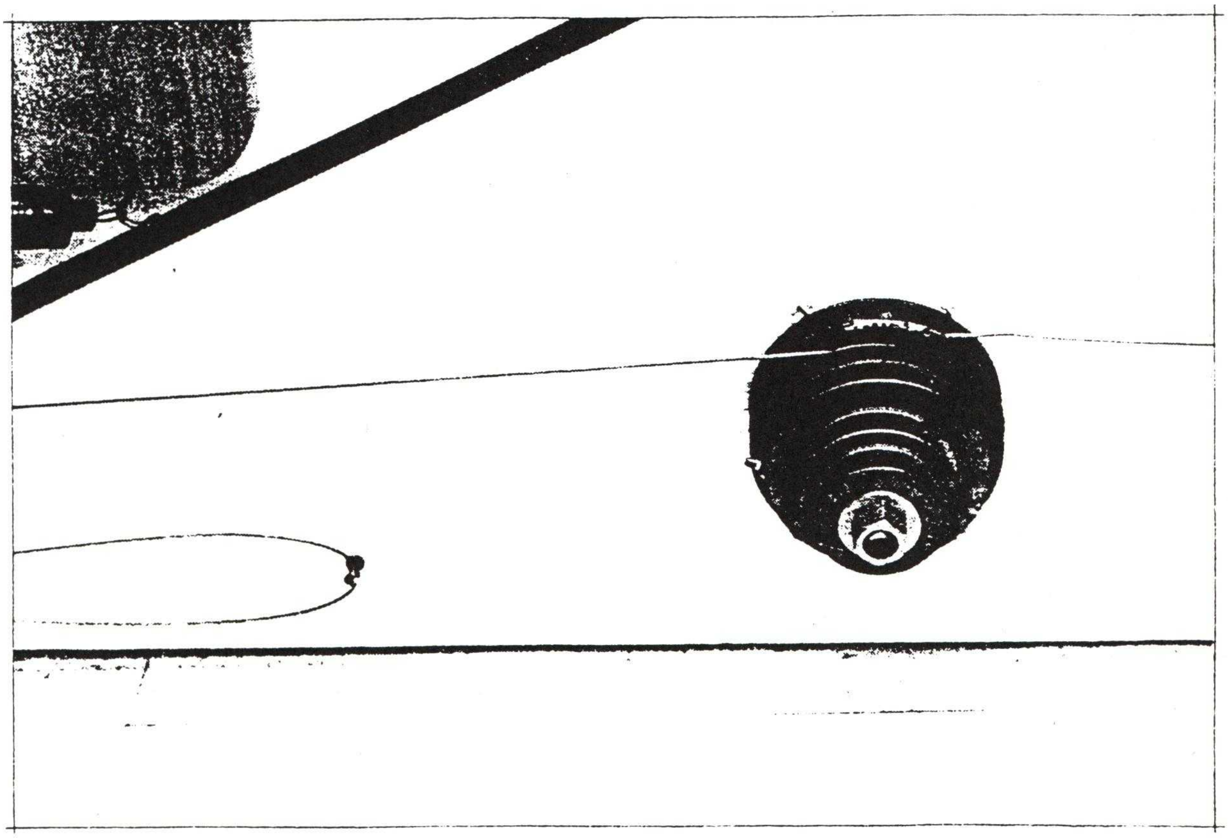

Fig. 6. Cable drums arranged for unit gain.

figuration is expected to do. In a group of 80 this often results in silence, but the pause allows the student to give some thought to how the process works and to take an active rather than a passive role. Perhaps the most effective learning occurs when the student develops expectations, but the reality is shown to be different. This is the process of misconception reformation described above. The demonstration was not initially arranged to support positive feedback, but classroom use of the demonstration suggested positive feedback as a point about which misconceptions may exist, and on which the student's understanding should be challenged and confirmed.

CONCLUSIONS

By introducing the simplest consistent model of feedback system dynamics and illustrating the process with a demonstration, the portion of students who were able to explain the operation of negative feedback was greatly increased. To make the equilibrium process tangible, a mechanical analog of the electronic operational amplifier is used for the demonstration. The mechanical analog offers related advantages in that the operating quantities are directly visible and operate at visible rates. As seen in Fig. 7, the portion of students able to articulate the process of negative feedback went from 4% to 20% when equilibrium and slewing were emphasized but no demonstration was made, and to 72% with the aid of demonstration. Equilibrium in spring tension is tangible; and by demonstration, the negative-feedback process is made visible. After seeing the demonstration, many students left the course with an appreciation of this fundamental process.

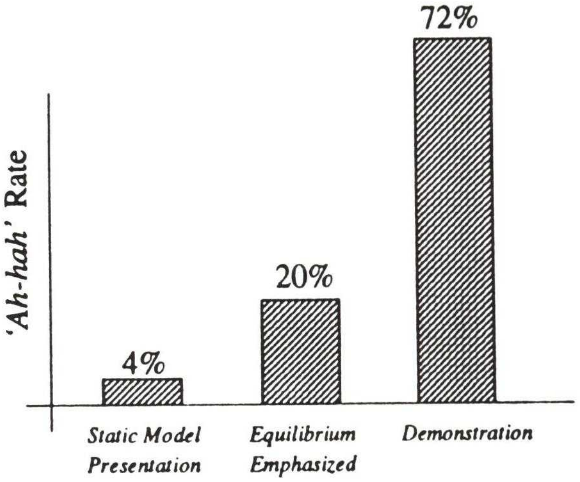

Fig. 7. Percentage of students giving a satisfactory explanation of the operation of negative feedback, the 'ah-hah' rate.

The cost of building the demonstration was substantial—roughly $1000 in materials and hourly labor, and 70 h of faculty input. Seventy hours is comparable to the faculty input required to teach a semester course, but we will be introducing students to negative feedback for many years to come, and the investment will be thoroughly recouped.

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