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Spreadsheet Applications for Control System Modelling and Analysis

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The paper describes a number of dynamic control system modelling applications using a commercial spreadsheet software package. Finite difference formulations of the state variable equations are developed in order to solve some typical control system models. To illustrate the potential of the spreadsheet as a mathematical tool, these systems include a voltage-controlled DC servo-motor using a full PID control strategy, a discretely sampled data control system and a generalized system model with a selectable input range of non-linear control elements. The paper also describes how the spreadsheet can be adapted for stability analyses in the frequency domain by generation of the usual Bode and Nyquist plots for any arbitrary control system transfer functions.

NOTATION

- C controller transfer function
- E error
- G process transfer function
- G' pulse transfer function
- K controller gain
- M modulus of amplitude ratio
- M' overall modulus of amplitude ratio
- SP set point
- T sampling time interval
- T_1 controller integral time constant (sec)
- T_d controller derivative time constant (sec)
- U controller output
- X state variable
- j time increments
- k system gain
- s Laplace variable, i.e. $s = (d/dt)$
- z z-transform, i.e. $z = e^{sT}$
- (s) frequency domain
- (z) z domain
- Δt time increment, sampling time
- ϕ phase angle (degrees)
- ϕ' overall phase angle (degrees)
- τ system time constant (sec)
- ω input signal frequency (rad/sec)
- ω_n natural frequency (rad/sec)
- ζ damping ratio

INTRODUCTION

SPREADSHEET software packages have been readily available for some considerable time, with Lotus 1-2-3 being perhaps the most commonly known spreadsheet used in industrial practice.

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Most users generally associate spreadsheets with commercial business functions, financial accounting and other similar database management techniques. However, since spreadsheets are well structured to handle data in a highly organized manner, then they also lend themselves well to general applications in the physical and engineering sciences. In particular the solution of finite difference equations are eminently well suited for solution by a spreadsheet routine. Fraser and Thorpe [1], illustrate the application of spreadsheets for the solution of two-dimensional Laplace and Poisson type problems by finite difference approximation. Modern texts, for example Krall [2], do not, however, exploit this vast potential for numerical simulation offered in the form of the spreadsheet software package.

For applications in control engineering there are in fact a number of customized software packages available which can accept any input transfer function, continuous or discrete, and produce output responses in the time domain, frequency domain and also root locus, or z-plots. The CODAS package developed by Golten and Verwer [3], is fairly typical and has proven to be very popular with its users. Program CC, by Thompson [4], provides another alternative software package for control system design and simulation. Both of these packages are specific to applications in control engineering, but a more recent development has been the use of the generalized mathematical software packages including MATLAB or MATHCAD, for the analysis of dynamic control systems (see, for example, Ogata [5]). While the benefits and convenience of specific control software packages are apparent, student users at an early stage in their learning curve do not really gain any insight on how the dynamic systems are actually modelled. This is an essential aspect in the teaching of control engineering and students need to have a sound understanding of the mathematical or numerical techniques used in dynamic system modelling. The use of MATLAB, or any popular spreadsheet, requires a basic level of mathematical ability and these software packages can be used as suitable vehicles to develop the finer details of the modelling aspects as applied to control engineering. The main advantages in the use of these packages is the ease with which fairly complex relationships can be defined and accounted for. The mathematical or spreadsheet packages therefore represent viable engineering tools and they offer much quicker alternatives to writing a specific computer program. They also offer full flexibility in examining an entire range of 'what-if' scenarios.

The examples described in this paper were developed using Microsoft's Excel spreadsheet, version 3.0. The files generated were also copied directly to the Quattro-Pro spreadsheet and run successfully without modification. Equivalent macros, however, had to be created within each spreadsheet as these do not appear to be directly transferable.

SECOND-ORDER SYSTEM WITH A PID CONTROLLER

As a first example a generalized second-order system which is to be controlled using a full PID strategy is considered. The system is represented by equation (1):

$$G(s) = k(\omega_n^2)/[s^2 + 2\zeta\omega_n s + \omega_n^2] \qquad (1)$$

The system parameters, k, ω_n and ζ, are related to the load and motor inertia, the drive friction and the inductance and Ohmic resistance of the armature coils. These parameters were obtained from measurements of the open-loop response of the motor and drive system to a step input voltage.

The transfer function for the PID controller is:

$$C(s) = K[1 + (1/T_i s) + T_d s] \qquad (2)$$

For a system with unity feedback, the closed loop transfer function becomes:

$$\frac{PV(s)}{SP(s)} = \frac{k\omega_n{}^2K\{(1/T_i) + s + T_d s^2\}}{[s^3 + \omega_n(2\zeta + k\omega_n KT_d)s^2 + \omega_n{}^2(1+kK)s + k\omega_n{}^2K/T_i]} \qquad (3)$$

Defining the terms $A = k\omega_n^2 K$, $B = \omega_n(2\zeta + k\omega_n KT_d)$ and $C = \omega_n^2(1 + kK)$, equation (3) may be written in the more compact form:

$$PV(s)/SP(s) = A\{T_d s^2 + s + (1/T_i)\}/[s^3 + Bs^2 + Cs + A/T_i] \qquad (4)$$

Equation (4) shows that the closed-loop system is third order and does not therefore lend itself amenable to solution by analytical techniques. The state variable method, however (see Fraser and Milne [6]), can be employed to write the governing relationship as a system of three first-order state equations which are:

$$\dot{X}1 = X2 \qquad (5)$$

$$\dot{X}2 = X3 \qquad (6)$$

$$\dot{X}3 = d^3(PV)/dt^3 = (A/T_i)SP + A\dot{SP} + AT_d\ddot{SP} - BX3 - CX2 - (A/T_i)X1 \qquad (7)$$

where $X1 = PV$ = the load speed in rad/sec, $X2 = \dot{PV}$ and $X3 = \ddot{PV}$.

Using a simple Euler finite difference approximation, equations (5)–(7) can be written in the discrete forms:

$$X1_{j+1} = X1_j + \Delta t[X2_j] \qquad (8)$$

$$X2_{j+1} = X2_j + \Delta t[X3_j] \qquad (9)$$

$$X3_{j+1} = X3_j + \Delta t[(A/T_i)SP + A\dot{SP} + AT_d\ddot{SP} - BX3_j - CX2_j - (A/T_i)X1_j] \qquad (10)$$

where j denotes the time intervals.

The first and second time derivatives of SP can also be approximated by suitable finite difference equations, i.e.

$$\dot{SP} = [SP_{j+1} - SP_{j-1}]/(2\Delta t) \qquad (11)$$

and

$$\ddot{SP} = [SP_{j-1} - 2SP_j + SP_{j+1}]/(\Delta t)^2 \qquad (12)$$

In using such basic numerical approximations, the finite time step, Δt, must necessarily be kept very small in order to ensure reasonable accuracy in the computed results.

For a step input, $SP_j = 0.3$ rad/sec at time equal to zero, equations (11) and (12) return finite values, but these both become equal to zero at any time after the step input. A ramp input can be simulated by making $SP = Qt$, where Q is some suitable constant. A sinusoidal input may also be simulated but the analysis of sinusoidal inputs is more appropriately dealt with in the frequency domain.

Figure 1 shows the modelled system layout on the spreadsheet, where the system parameters are allocated to convenient cells for user friendliness. The initial values of zero are assigned to all parameters at time levels $(0 - \Delta t)$ and $(0 - 2\Delta t)$. The solution is implemented by writing equations (8)–(12) in the appropriate cells at time level $t = 0$. These equations are then copied as a block and pasted to as many cells along the spreadsheet as is necessary to cover the transient time period. In writing the finite difference equations, specific cells containing the system parameters are referred to in the equation in the form, for example, C9. All other parameters held in adjacent cells are relative to the cell where the equation is written and these are referenced without the '$' signs. The relation-

Spreadsheet Simulation of a "PID" control strategy for a DC Servo-motor

The Servo-motor is modelled as :- G(s) = k(wn)^2/(s^2 + 2z(wn)s + (wn)^2)

The Controller uses a PID control strategy, ie. C(s) = K(1 + (1/Ti.s) + (Td)s)

Input System Parameters	
system gain, k	0.1059
Natural frequency, wn	2.0075
Damping factor, z	1.3406
Enter time increment	0.1

Input Controller Parameters	
Gain, K	100
Integral time, Ti	0.6
Deriv time, Td	0.05

Derived Parameters	
A =	42.6783
B =	7.516424
C =	46.70835

Finite Difference Solution :-	time (s)	-0.2	-0.1	0	0.1	0.2
	SP	0	0	0.3	0.3	0.3
	d(SP)	0	0	1.5	0	0
	d2(SP)	0	0	-30	0	0
	X1	0	0	0	0	0.021339
	X2	0	0	0	0.213391	0.266389
	X3	0	0	2.133915	0.529974	0.738847

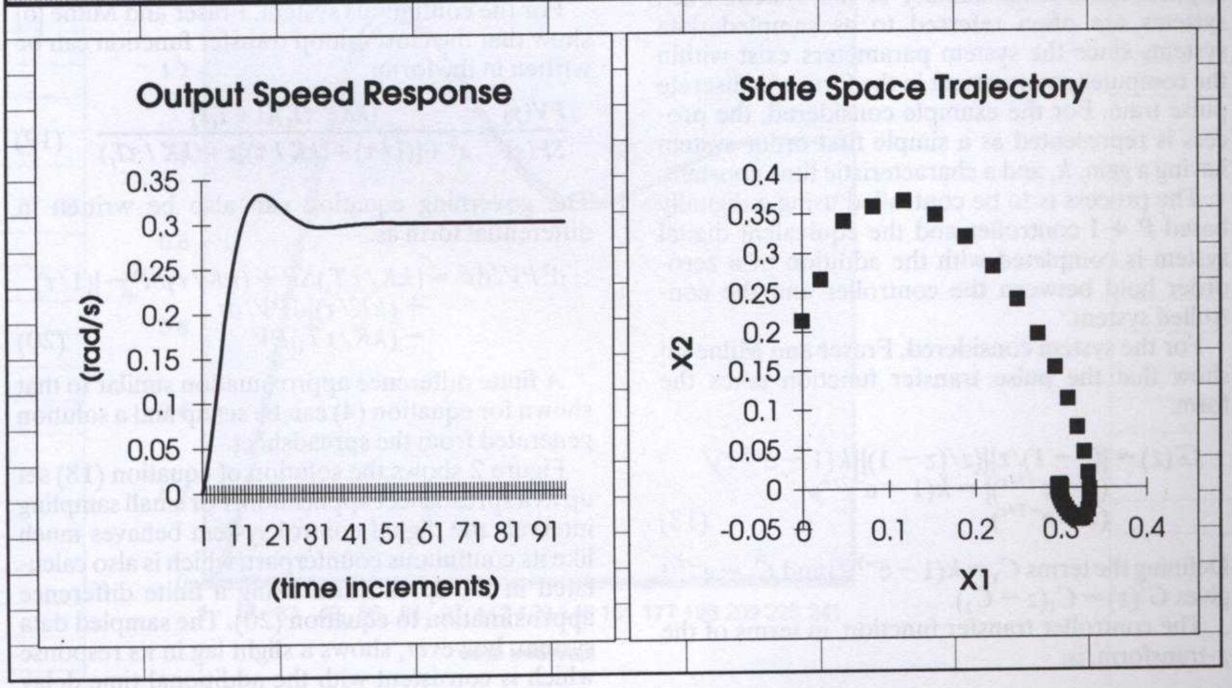

Fig. 1. Second-order system response with PID controller.

ships are automatically adjusted when pasting a copied function to any other cell and this feature provides a very efficient means of assigning the correct formulae to all consecutive cells which represent the solution being marched forward in time.

The system output, PV, or any other parameter of interest, can be selected and plotted as shown in Fig. 1. In this instance the output speed response in the time domain is shown, as is the state space trajectory. Since the system is third order, the state space trajectory should ideally show the variation of the three state variables in a three-dimensional plot. This cannot be done effectively in the spreadsheet; however, the plot of $X2$ against $X1$ still gives a useful indication of the system stability.

With a little experience, the complete simulation on the spreadsheet can be set up very quickly and then used to investigate the effect, for example, of varying the integral time constant. This is done by simply assigning a new value to the cell where the integral time constant is allocated. The spreadsheet will then automatically recalculate the new solution and update the data and the graphical displays.

DIGITAL CONTROL SYSTEMS

Digital control systems are characterized by the use of a computer which performs the dual functions of comparator and controller in the equivalent continuous counterpart. The application of digital control techniques, however, always invokes an additional time delay which can have serious implications on the stability of the system. Such systems are often referred to as sampled data systems since the system parameters exist within the computer environment in the form of a discrete pulse train. For the example considered, the process is represented as a simple first-order system having a gain, k, and a characteristic time constant, τ. The process is to be controlled using a digitally based P + I controller and the equivalent digital system is completed with the addition of a zero-order hold between the controller and the controlled system.

For the system considered, Fraser and Milne [6] show that the pulse transfer function takes the form:

$$G'(z) = [(z-1)/z][(z/(z-1)][k(1-e^{-T/\tau})/(z-e^{-T/\tau})] = k(1-e^{-T/\tau})/(z-e^{-T/\tau}) \quad (13)$$

Defining the terms $C_1 = k(1-e^{-T/\tau})$ and $C_2 = e^{-T/\tau}$ gives $G'(z) = C_1(z - C_2)$.

The controller transfer function, in terms of the z-transform, is:

$$C(z) = U(z)/E(z) = K[1 + (1/T_i)\{Tz/(z-1)\}] \quad (14)$$

The system closed-loop transfer function becomes:

$$\frac{PV(z)}{SP(z)} = \frac{C(z)G'(z)}{1+C(z)G'(z)} = \frac{K[1+(1/T_i)\{Tz/(z-1)\}][C_1/(z-C_2)]}{1+K[1+(1/T_i)\{Tz/(z-1)\}][C_1/(z-C_2)]} \quad (15)$$

Multiplying the top and bottom by $(z - C_2)(z - 1)z^{-2}$ gives, after some manipulation,

$$\frac{PV(z)}{SP(z)} = \frac{KC_1[1+(T/T_i]z^{-1} - KC_1z^{-2}}{1+[KC_1(1+T/T_i)-(C_2+1)]z^{-1}+(C_2-KC_1)z^{-2}} \quad (16)$$

Defining the terms $a = KC_1[1 + T/T_i]$, $b = KC_1$, $c = (C_2 + 1) - KC_1(1 + T/T_i)$ and $d = KC_1 - C_2$, the closed-loop transfer function may be written as:

$$PV(z)/SP(z) = (az^{-1} - bz^{-2})/[1 - cz^{-1} - dz^{-2}]$$

Hence

$$PV(z)[1 - cz^{-1} - dz^{-2}] = SP(z)[az^{-1} - bz^{-2}]$$

or

$$PV(z) = SP(z)[az^{-1} - bz^{-2}] + PV(z)[cz^{-1} + dz^{-2}] \quad (17)$$

Writing equation (17) as an equivalent finite time sequence gives:

$$PV_j = aSP_{j-1} - bSP_{j-2} + cPV_{j-1} + dPV_{j-2} \quad (18)$$

where j denotes the sampling time interval.

For the continuous system, Fraser and Milne [6] show that the closed-loop transfer function can be written in the form:

$$\frac{PV(s)}{SP(s)} = \frac{(kK/\tau T_1)[1+T_1 s]}{s^2+[(1/\tau)+(kK/\tau)]s+(kK/\tau T_i)} \quad (19)$$

The governing equation can also be written in differential form as:

$$d^2PV/dt^2 = (kK/\tau T_i)SP + (kK/\tau)\dot{S}P - [(1/\tau) + (kK/\tau)]dPV/dt - (kK/\tau T_i)PV \quad (20)$$

A finite difference approximation similar to that shown for equation (4) can be set up and a solution generated from the spreadsheet.

Figure 2 shows the solution of equation (18) set up as a spreadsheet application. For small sampling intervals the digital control system behaves much like its continuous counterpart, which is also calculated in the spreadsheet using a finite difference approximation to equation (20). The sampled data system, however, shows a slight lag in its response which is consistent with the additional time delay associated with the sampling process. It should be noted that the sampling interval of 0.01 sec for the digital system is also used as the finite time interval in the approximation of equation (20). This time interval is small enough to provide reasonable

Spreadsheet simulation of a digital control system

Process is represented as a first order system, ie. G(s) = k/(1+tau.s)

Controller uses a proportional + Integral strategy, ie. C(s) = K(1+1/Ti.s)

Input system and control parameters		derived parameters	
system gain, k =	1	C1 =	0.00995
time constant, tau =	1	C2 =	0.99005
sampling time, T =	0.01	a =	0.02189
Controller gain, K =	2	b =	0.0199
		c =	1.968159
Integral time, Ti =	0.1	d =	-0.97015

Finite difference solution :-		t = 0-dt	t=0	t=t+dt -->	t=t+dt -->	t=t+dt -->	
	SP	0	1	1	1	1	1
Sampled data control	PV	0	0	0	0.00199	0.005907	0.011685
	time	-0.01	0	0.01	0.02	0.03	0.04
	dSP/dt	0	1	0	0	0	0
Continuous control	PV	0	0	0.0022	0.006334	0.01234	0.020152
	dPV/dt	0	0.22	0.4134	0.600558	0.781274	0.955368

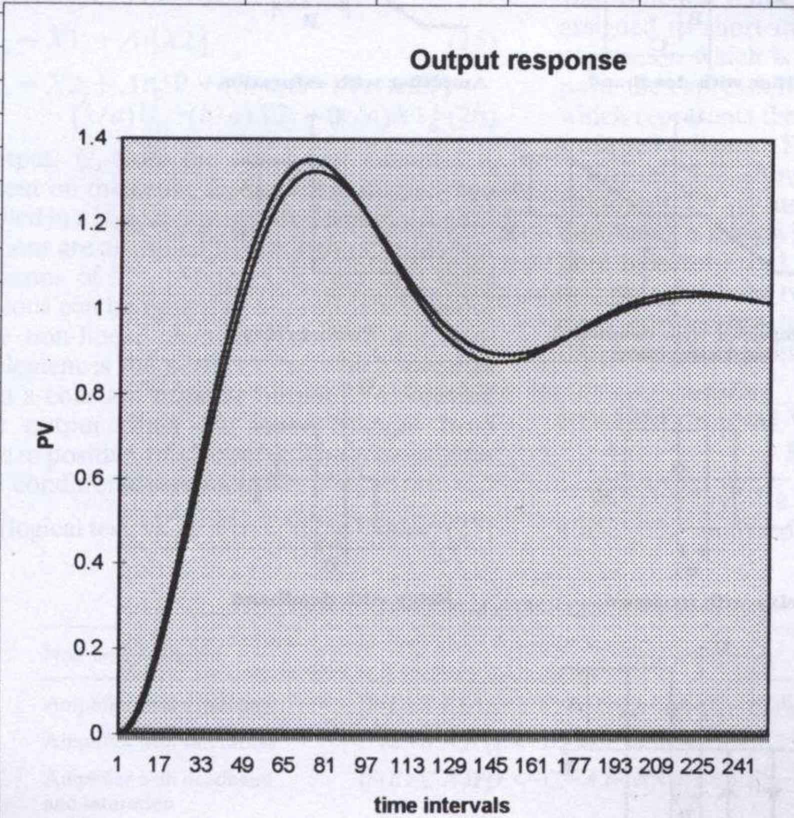

Fig. 2. Sampled data control system with P + 1 controller.

accuracy in the solution of the continuous system parameters. For higher values of the calculation time interval, the accuracy of the predicted continuous control system variables would be reduced. Increasing the time interval, however, does have relevance to the behaviour of the digital control system. The effect is similar to an increase in the time associated with the zero-order hold and its influence on the digital control system can be studied independently from the continuous system. If, for example, the sampling interval is increased to 0.36 sec then the sampled data system becomes marginally stable and this is clearly illustrated in the plots of the output response against time.

NON-LINEAR CONTROL SYSTEMS

For control systems which incorporate some form of non-linear element, the analysis of the system stability can be performed using the error and its rate of change as suitable characteristic state variables. For second-order systems, the plot of the two state variables are often referred to in the literature as the phase plane. Typical non-linearities are depicted in Fig. 3 and all can be described in terms of three, or fewer, variables. These are A, B and C as shown. The process under control is described as a second-order system having the general form:

$$G(s) = k/[as^2 + bs + c] \quad (21)$$

A typical control system including a non-linear element is shown in Fig. 4. The governing relationship is:

$$PV(s) = kU(s)/[as^2 + bs + c] \quad (22)$$

or

$$kU(s) = PV(s)[as^2 + bs + c]$$

Since $PV(s) = [SP(s) - E(s)]$, then:

$$kU(s) = [SP(s) - E(s)][as^2 + bs + c]$$

i.e.

$$kU = ad^2SP/dt^2 + bd\,SP/dt + cSP - ad^2E/dt^2 - bdE/dt - cE \quad (23)$$

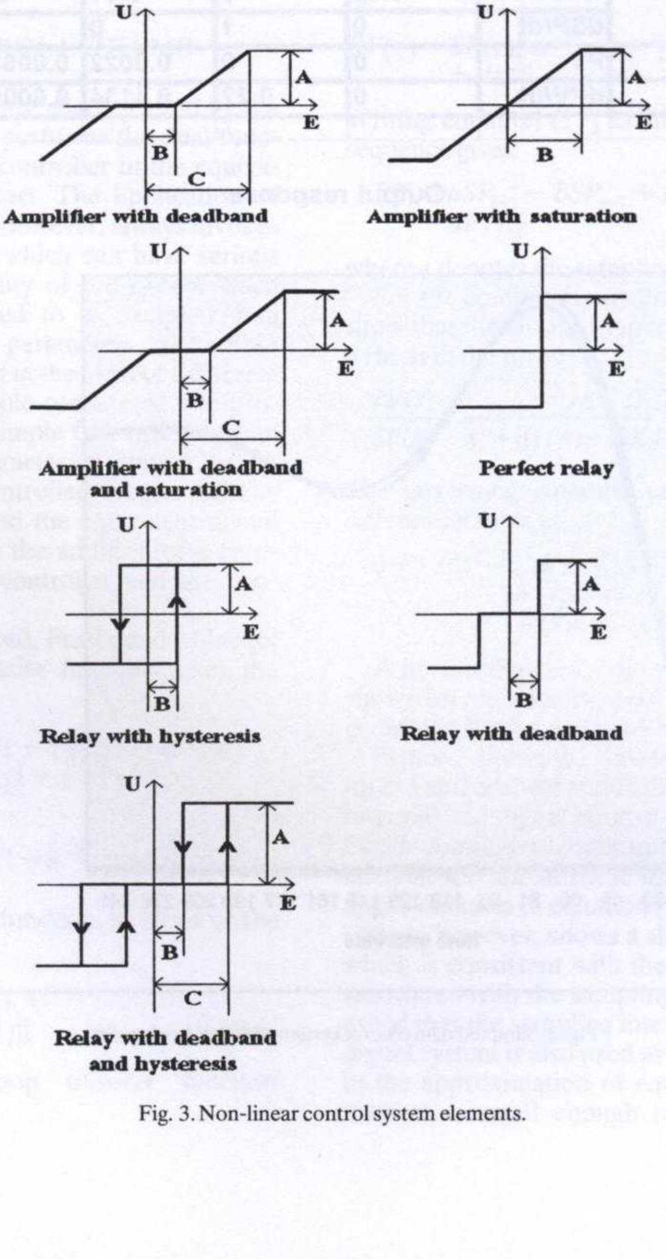

Fig. 3. Non-linear control system elements.

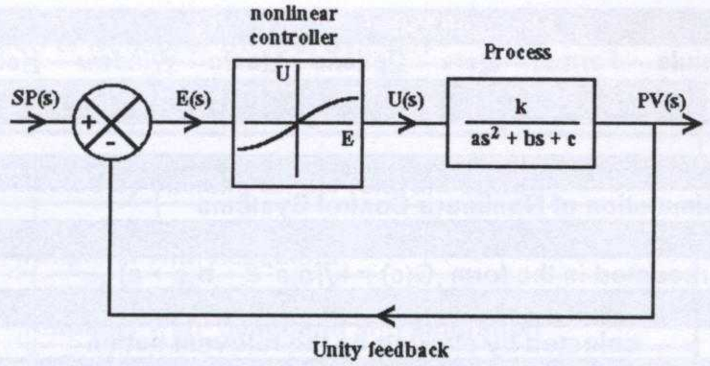

Fig. 4. Schematic representation of a non-linear control system.

If the system is subjected to a step input, then the first and second derivatives of SP can be approximated using equations (11) and (12).

It is perfectly reasonable to consider the step input only, since any system which is stable to a step input will also be stable to any other form of input. Hence:

$$d^2E/dt^2 = d^2SP/dt^2 + (b/a)dSP/dt + (c/a)SP - (k/a)U - (b/a)dE/dt - (c/a)E \quad (24)$$

Equation (24) may be solved using the state variable representation in discrete form, where $X1 = E$ and $X2 = dE/dt$. This results in:

$$X1_{j+1} = X1_j + \Delta t[X2_j] \quad (25)$$

$$X2_{j+1} = X2_j + \Delta t[\ddot{SP} + (b/a)\dot{SP} + (c/a)SP_j - (k/a)U_j - (b/a)X2_j - (c/a)X1_j] \quad (26)$$

The output, U, from the non-linear elements is dependent on the error, E, and these outputs can be handled in a logical expression. The usual logical expressions are all available within the spreadsheet in the forms of 'IF', 'AND', 'OR', etc. and these expressions can be nested as appropriate to simulate the non-linear element. The simplest non-linear element is the perfect relay which switches between a constant negative output to a constant positive output when the error changes from negative to positive, or vice versa. The general form of an IF conditional statement is:

IF(logical test, value if true, value if false) (27)

To simulate the perfect relay, therefore, the relevant expression is:

$$\mathrm{IF}(E>0, A, -A) \quad (28)$$

More complex relationships can be simulated by nesting suitable IF or AND arguments as appropriate. The present expressions used to simulate the non-linear elements are given in Table 1.

The solution of equations (25) and (26) are shown as a spreadsheet application in Fig. 5. The particular system parameters are entered in the normal way, but the selection of any particular non-linear element is performed by clicking on the relevant function button. These function buttons are assigned to short macros which copy the logical expression which is held in column 'H' and then paste the expression to all cells located on the row which represents the controller output, U, starting from cell reference F31.

Figures 6–8 show respectively the non-linear responses for an amplifier with saturation and a deadband, a relay with hysteresis, and a relay with both hysteresis and a deadband. The limit cycles associated with the two non-linear elements having a hysteresis characteristic are well represented in the phase plane plots.

STABILITY ANALYSIS IN THE FREQUENCY DOMAIN

The stability of a continuous control system is commonly analysed in terms of the frequency

Table 1. Simulation of control system non-linearities

Non-linear element	Logical simulation
Amplifier with deadband	$\mathrm{IF}(E>B, E*A/(C-B), \mathrm{IF}(E<B, E*A/(C-B), 0))$
Amplifier with saturation	$\mathrm{IF}(E>B, A, \mathrm{IF}(E<-B, -A, E*A/B))$
Amplifier with deadband and saturation	$\mathrm{IF}(E>C, A, \mathrm{IF}(E<-C, -A, \mathrm{IF}(\mathrm{AND}(E<B, E>-B), 0, E*A/(C-D))))$
Relay with hysteresis	$\mathrm{IF}(E>B, A, \mathrm{IF}(E<-B, -A, \mathrm{IF}(E_j - E_{j-1}>0, -A, A)))$
Relay with deadband	$\mathrm{IF}(E>B, A, \mathrm{IF}(E<-B, -A, 0))$
Relay with hysteresis and deadband	$\mathrm{IF}(E>C, A, \mathrm{IF}(E<-C, -A, \mathrm{IF}(\mathrm{AND}(E<B, E>-B), 0,$ $\mathrm{IF}(\mathrm{AND}(E>0, \mathrm{ABS}(E_j) - \mathrm{ABS}(E_{j-1})<0, A,$ $\mathrm{IF}(\mathrm{AND}(E<0, \mathrm{ABS}(E_j) - \mathrm{ABS}(E_{j-1})<0, -A, 0)))))$

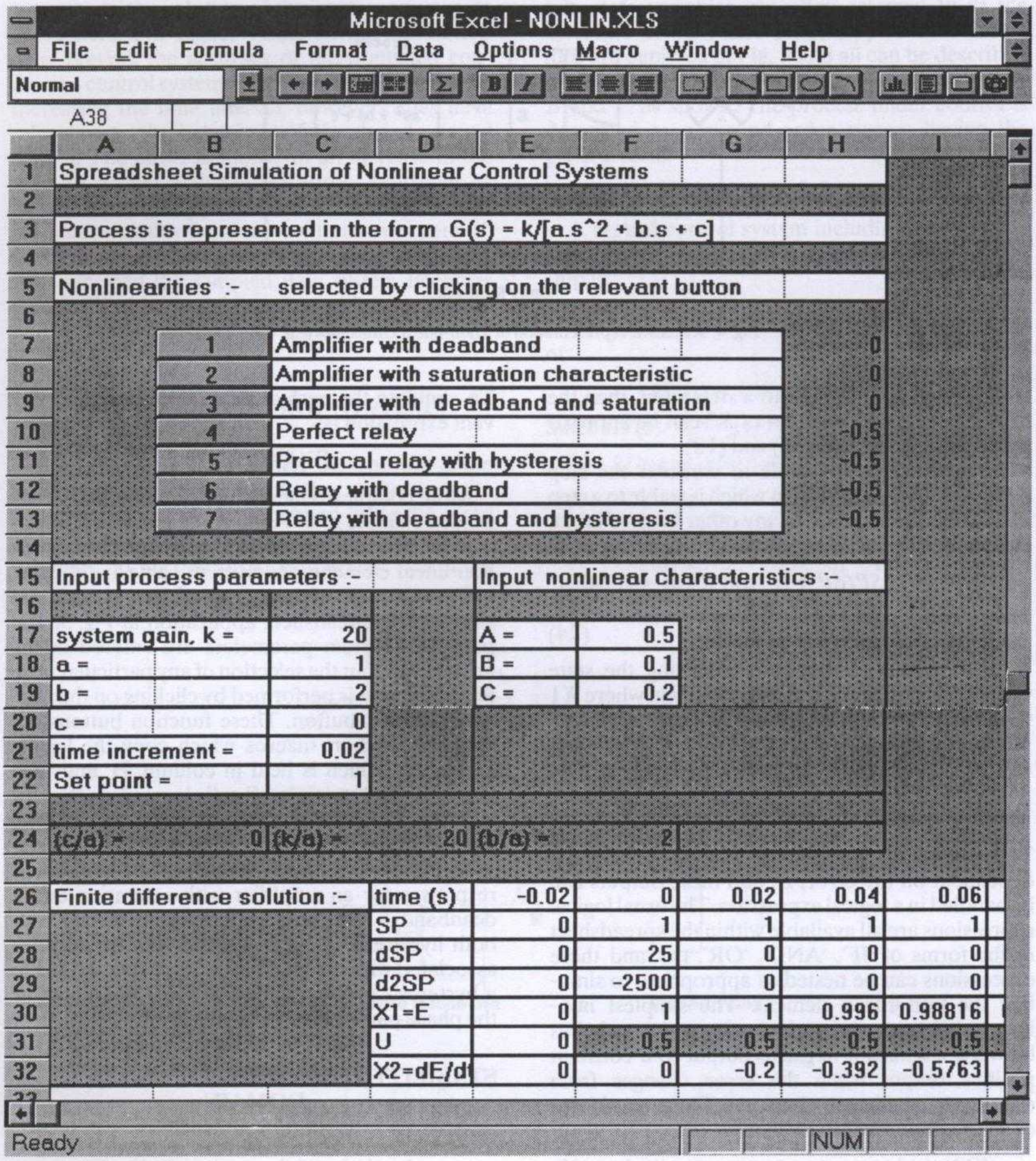

Fig. 5. Spreadsheet layout for non-linear control systems.

response and represented graphically as Bode or Nyquist plots. In general terms the transfer function will contain a variety of elements which could include an overall gain, K, a simple integration term or any combination of first- or second-order terms in either the numerator or the denominator. Second-order terms can be represented in the form given as equation (1), with $k = 1$. The modulus of the amplitude ratio of any second-order term is given as:

$$M = 1/\sqrt{[(1-(\omega/\omega_n)^2)^2 + (2\zeta\omega/\omega_n)^2]} \quad (29)$$

The phase angle is given as:

$$\phi = -\tan^{-1}[(2\zeta\omega/\omega_n)/(1-(\omega/\omega_n)^2)] \quad (30)$$

Any second-order term may also be denoted in the form given as equation (21) with suitable values assigned to the parameters a, b, c and the gain, $k = 1$. For the general second-order term, c will have a

Spreadsheet Simulation of Nonlinear Control Systems

Process is represented in the form G(s) = k/[a.s^2 + b.s + c]

Nonlinearities :- selected by clicking on the relevant button

1	Amplifier with deadband	0
2	Amplifier with saturation characteristic	0
3	Amplifier with deadband and saturation	0
4	Perfect relay	-2
5	Practical relay with hysteresis	-2
6	Relay with deadband	-2
7	Relay with deadband and hysteresis	0

Input process parameters :-		Input nonlinear characteristics :-	
system gain, k =	100	A =	2
a =	1	B =	0.3
b =	3	C =	2.3
c =	0		
time increment =	0.01		
Set point =	1		

(c/a) =	0	(k/a) =	100	(b/a) =	3

Finite difference solution :-

time (s)	-0.01	0	0.01	0.02	0.03
SP	0	1	1	1	1
dSP	0	50	0	0	0
d2SP	0	-10000	0	0	0
X1=E	0	1	1	0.99	0.9704
U	0	1	1	0.99	0.9704
X2=dE/dt	0	0	-1	-1.96	-2.8716

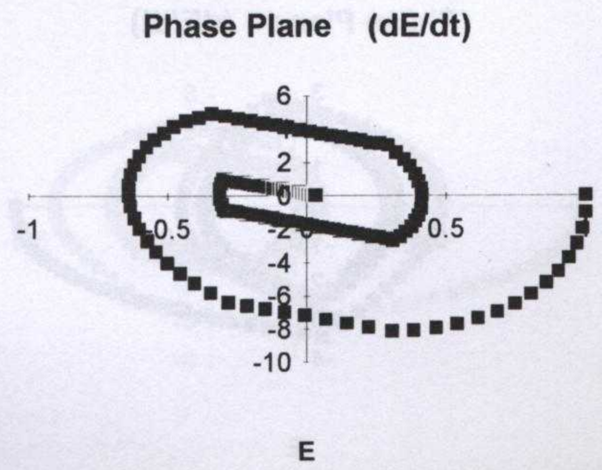

Fig. 6. Control system amplifier with saturation and deadband characteristics.

Spreadsheet Simulation of Nonlinear Control Systems							
Process is represented in the form G(s) = k/[a.s^2 + b.s + c]							
Nonlinearities :-	selected by clicking on the relevant button						
1	Amplifier with deadband					0	
2	Amplifier with saturation characteristic					0	
3	Amplifier with deadband and saturation					0	
4	Perfect relay					-0.5	
5	Practical relay with hysteresis					-0.5	
6	Relay with deadband					-0.5	
7	Relay with deadband and hysteresis					-0.5	
Input process parameters :-			Input nonlinear characteristics :-				
system gain, k =	20		A =	0.5			
a =	1		B =	0.1			
b =	2		C =	0			
c =	0						
time increment =	0.02						
Set point =	1						
(c/a) = 0	(k/a) = 20		(b/a) = 2				
Finite difference solution :-		time (s)	-0.02	0	0.02	0.04	0.06
		SP	0	1	1	1	1
		dSP	0	25	0	0	0
		d2SP	0	-2500	0	0	0
		X1=E	0	1	1	0.996	0.98816
		U	0	0.5	0.5	0.5	0.5
		X2=dE/dt	0	0	-0.2	-0.392	-0.57632

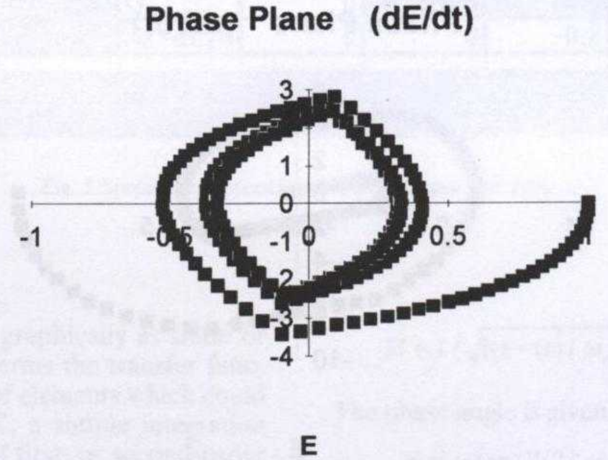

Fig. 7. Control system relay with hysteresis characteristic.

Spreadsheet Simulation of Nonlinear Control Systems

Process is represented in the form G(s) = k/[a.s^2 + b.s + c]

Nonlinearities :- selected by clicking on the relevant button

1	Amplifier with deadband	0
2	Amplifier with saturation characteristic	0
3	Amplifier with deadband and saturation	0
4	Perfect relay	-0.5
5	Practical relay with hysteresis	-0.5
6	Relay with deadband	-0.5
7	Relay with deadband and hysteresis	-0.5

Input process parameters :-

system gain, k =	20
a =	1
b =	2
c =	0
time increment =	0.02
Set point =	1

Input nonlinear characteristics :-

A =	0.5
B =	0.1
C =	0.2

(c/a) =	0	(k/a) =	20	(b/a) =	2

Finite difference solution :-

time (s)	-0.02	0	0.02	0.04	0.06
SP	0	1	1	1	1
dSP	0	25	0	0	0
d2SP	0	-2500	0	0	0
X1=E	0	1	1	0.996	0.98816
U	0	0.5	0.5	0.5	0.5
X2=dE/dt	0	0	-0.2	-0.392	-0.57632

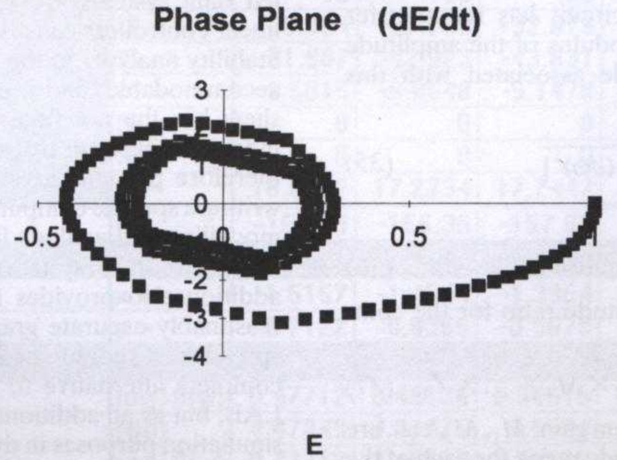

Fig. 8. Control system relay with hysteresis and deadband characteristics.

numerical value of unity while $a = 1/\omega_n^2$ and $b = 2\zeta/\omega_n$. The expression for the modulus of the amplitude ratio then becomes:

$$M = 1/\sqrt{[(c-a\omega^2)^2+(b\omega)^2]} \quad (31)$$

and

$$\phi = -\tan^{-1}[(b\omega)/(c-(a\omega^2)] \quad (32)$$

If the parameter a is set equal to zero, while $c=1$ and b is assigned some finite value, then the general expression becomes equivalent to a first-order term. The modulus of the amplitude ratio and the phase angle become respectively:

$$M = 1/\sqrt{[(1+(b\omega)^2]} \quad (33)$$

and

$$\phi = -\tan^{-1}[(b\omega)] \quad (34)$$

Equations (33) and (34) therefore reduce to the correct forms for any first-order term.

If a and c are set equal to zero, while b is set equal to unity, then the general function describes a simple integrator. The modulus of the amplitude ration becomes $1/\omega$, which is correct. The phase angle, however, reduces to the form $\phi = -\tan^{-1}[\omega]$, which is not correct. Since a simple integrator always imposes a phase lag of $-90°$, irrespective of the frequency, then a check must be made for the input conditions, $a = c = 0$ and $b = 1$. The phase angle is then simply assigned the correct value of $-90°$, irrespective of the input signal frequency.

It is possible that the numerator of the transfer function could also contain both first- and second-order terms, although in practice this is relatively uncommon. However, allowance is made in the spreadsheet layout for the transfer function to contain one first-order term in the numerator. This would be representative of a phase advance circuit, used perhaps to improve the stability of the system without compromising the overall gain. The simplest phase advance circuit has the transfer function $(1 + ds)$. The modulus of the amplitude ratio and the phase angle associated with this function are:

$$M = \sqrt{[1+(d\omega)^2]} \quad (35)$$

and

$$\phi = +\tan^{-1}[d\omega] \quad (36)$$

The modulus of the amplitude ratio for the complete system is given as:

$$M' = K \times M_1 \times M_2 \times M_3 \times \ldots \times Z_1 \quad (37)$$

where K is the overall system gain, M_1, M_2, etc., are the modulus of the amplitude ratios for each of the individual denominator terms in the transfer function, and Z_1 represents the modulus of the amplitude ratio for the single first-order term appearing in the numerator. The phase angle for the complete system is:

$$\phi' = \phi_1 + \phi_2 + \phi_3 + \ldots \quad (38)$$

where ϕ_1, ϕ_2, etc., are the phase angles of the individual elements which make up the system.

To plot the Nyquist diagram on the spreadsheet, the polar representation (M', ϕ') has to be converted into an equivalent Cartesian representation. This is done through the transformations $x = M' \cos(\phi')$ and $y = M' \sin(\phi')$.

The same data can also be represented as a Bode diagram, where the gain, expressed in decibels, and the phase angle are plotted against the frequency on a logarithmic scale. These parameters are duly calculated in the spreadsheet which is laid out as shown in Fig. 9. Other inputs consist of the frequency range and the number of calculation points required. This allows for control of the resolution and range in the graphical displays. The transfer function considered in the example is:

$$C(s)G(s) = K(1+0.1s)/[s(1+0.2s)(1+0.3s)(1+0.05s+0.002s^2)]$$

with $K = 7.5$.

The graphical output is shown in Fig. 10, where it can be seen, particularly in the Nyquist plot, that the system is stable. Removal of the first-order term in the numerator, by setting $d = 0$, renders the system unstable, as evidenced in the graphical output shown in Fig. 11.

CONCLUSIONS

The spreadsheet represents a valuable additional tool for modelling and analysis applications in control engineering. The educational benefits in using a spreadsheet, as opposed to a customized control system package, are that the principles of the modelling techniques can be fully highlighted. This is an essential factor in the complete educational development of the trainee engineer studying industrial control systems. Continuous and sampled data systems are easily simulated and non-linear controllers can also be effectively modelled. Stability analyses in the frequency domain can be accommodated, and it is evident that the spreadsheet has the potential for applications to much more complex control systems. The spreadsheet therefore presents a very powerful alternative to writing a specific computer program to perform the modelling and analysis functions normally associated with control system design. The spreadsheet in addition also provides the convenient facility of reasonably accurate graphical presentations. The spreadsheet should not, however, be regarded as a complete alternative to packages such as MATLAB, but as an additional and convenient tool for simulation purposes in dynamic system modelling.

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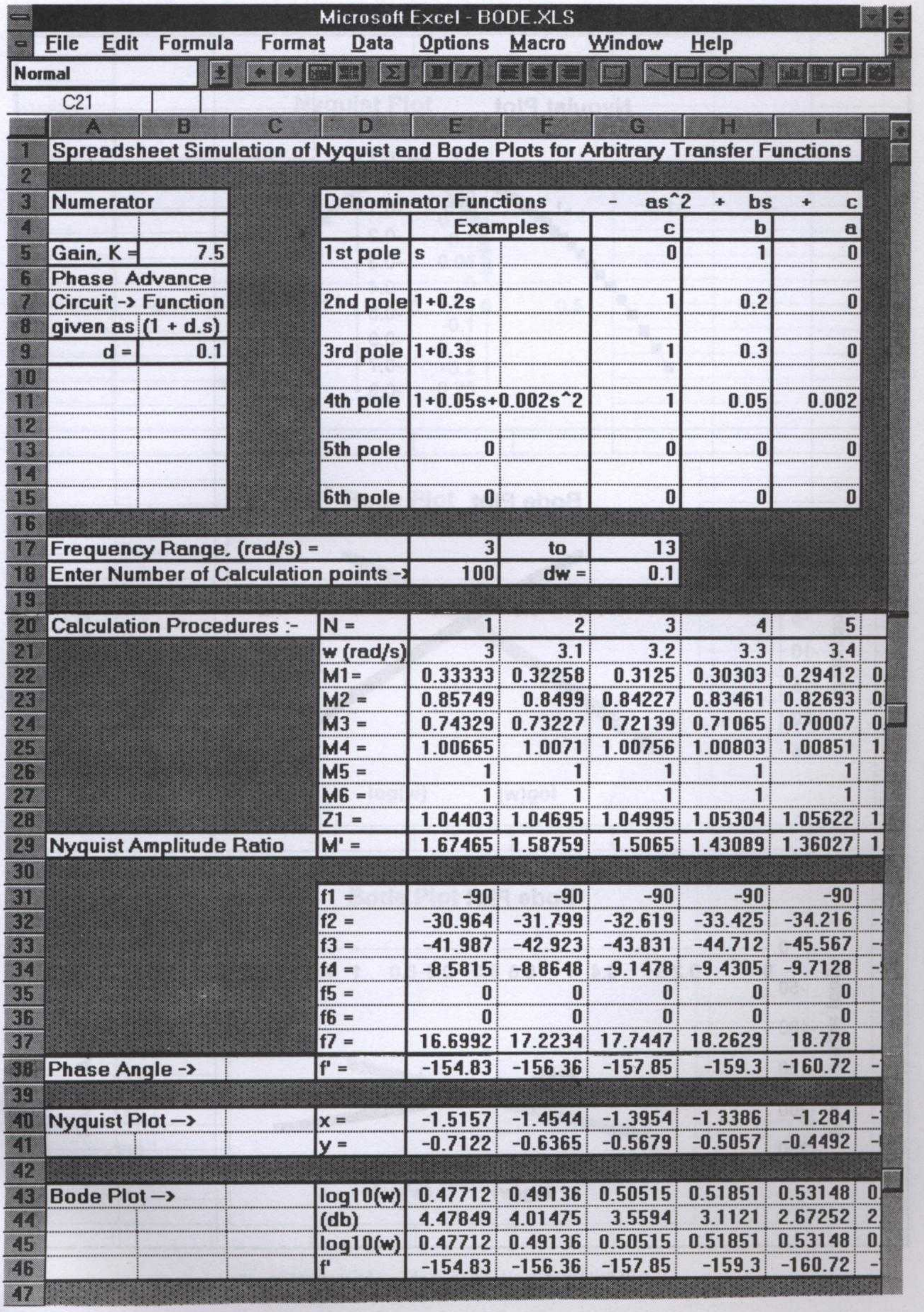
Microsoft Excel - BODE.XLS

File Edit Formula Format Data Options Macro Window Help

Normal

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	A	B	C	D	E	F	G	H	I
1	Spreadsheet Simulation of Nyquist and Bode Plots for Arbitrary Transfer Functions								
2									
3	Numerator			Denominator Functions			- as^2	+ bs	+ c
4					Examples		c	b	a
5	Gain, K =	7.5		1st pole	s		0	1	0
6	Phase Advance								
7	Circuit -> Function			2nd pole	1+0.2s		1	0.2	0
8	given as	(1 + d.s)							
9	d =	0.1		3rd pole	1+0.3s		1	0.3	0
10									
11				4th pole	1+0.05s+0.002s^2		1	0.05	0.002
12									
13				5th pole	0		0	0	0
14									
15				6th pole	0		0	0	0
16									
17	Frequency Range, (rad/s) =				3	to	13		
18	Enter Number of Calculation points ->				100	dw =	0.1		
19									
20	Calculation Procedures :-			N =	1	2	3	4	5
21				w (rad/s)	3	3.1	3.2	3.3	3.4
22				M1 =	0.33333	0.32258	0.3125	0.30303	0.29412
23				M2 =	0.85749	0.8499	0.84227	0.83461	0.82693
24				M3 =	0.74329	0.73227	0.72139	0.71065	0.70007
25				M4 =	1.00665	1.0071	1.00756	1.00803	1.00851
26				M5 =	1	1	1	1	1
27				M6 =	1	1	1	1	1
28				Z1 =	1.04403	1.04695	1.04995	1.05304	1.05622
29	Nyquist Amplitude Ratio			M' =	1.67465	1.58759	1.5065	1.43089	1.36027
30									
31				f1 =	-90	-90	-90	-90	-90
32				f2 =	-30.964	-31.799	-32.619	-33.425	-34.216
33				f3 =	-41.987	-42.923	-43.831	-44.712	-45.567
34				f4 =	-8.5815	-8.8648	-9.1478	-9.4305	-9.7128
35				f5 =	0	0	0	0	0
36				f6 =	0	0	0	0	0
37				f7 =	16.6992	17.2234	17.7447	18.2629	18.778
38	Phase Angle ->			f' =	-154.83	-156.36	-157.85	-159.3	-160.72
39									
40	Nyquist Plot -->			x =	-1.5157	-1.4544	-1.3954	-1.3386	-1.284
41				y =	-0.7122	-0.6365	-0.5679	-0.5057	-0.4492
42									
43	Bode Plot -->			log10(w)	0.47712	0.49136	0.50515	0.51851	0.53148
44				(db)	4.47849	4.01475	3.5594	3.1121	2.67252
45				log10(w)	0.47712	0.49136	0.50515	0.51851	0.53148
46				f'	-154.83	-156.36	-157.85	-159.3	-160.72
47									

Fig. 9. Spreadsheet layout for analysis in the frequency domain.

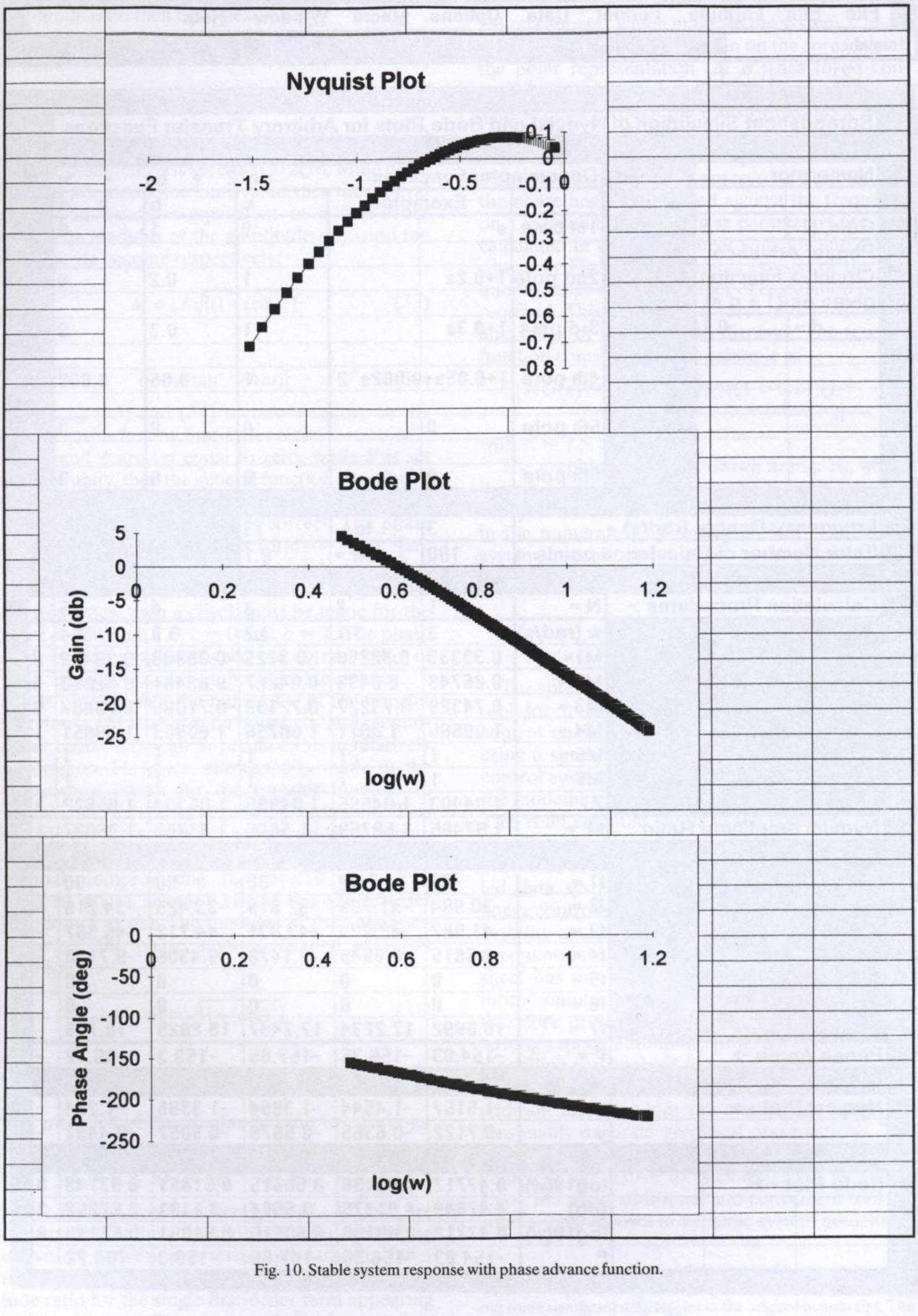

Fig. 10. Stable system response with phase advance function.

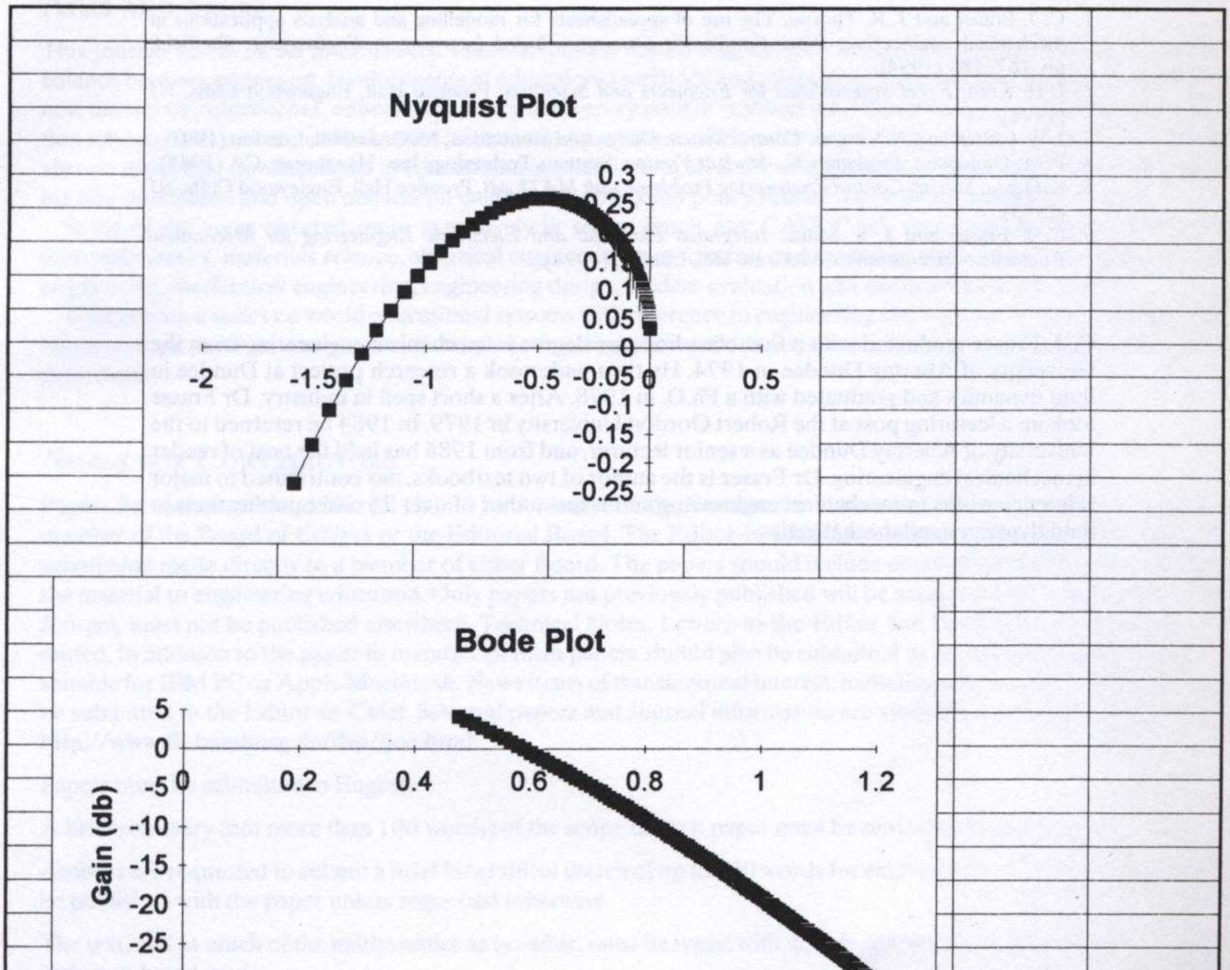

Fig. 11. Unstable system response without phase advance function.

REFERENCES

1. C. J. Fraser and J. R. Thorpe, The use of spreadsheets for modelling and analysis applications in mechanical engineering, *Proc. Conference Computer Aided Learning in Engineering*, Sheffield, pp. 167–181 (1994).
2. I. H. Krall, *Excel Spreadsheets for Engineers and Scientists*, Prentice Hall, Englewood Cliffs, NJ (1991).
3. J. W. Golten and A. Verwer, *Control System Design and Simulation*, McGraw-Hill, London (1991).
4. P. M. Thompson, *Program CC—Student Version*, Systems Technology Inc., Hawthorne, CA (1988).
5. K. Ogata, *Solving Control Engineering Problems with MATLAB*, Prentice Hall, Englewood Cliffs, NJ (1993).
6. C. J. Fraser and J. S. Milne, *Integrated Electrical and Electronic Engineering for Mechanical Engineers—Mechatronics*, McGraw-Hill, London (1994).

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