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Designing and Testing a Dynamic Vibration Absorber Rig*

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This paper presents the design, fabrication and testing of a dynamic vibration absorber rig that is used by undergraduates to apply vibration control theory to minimize machine vibration. The rig comprises a variable-speed d.c. motor, a V-belt driving a shaft with an unbalanced rotating disc, and a fin supported by two compressive springs. The dynamic vibration absorber, which is another spring mass system, consists of a steel rod with a lumped mass at its end. Qualifying running tests after fabrication reveal a weakness in the vibration absorber system, which often failed due to fracture at the fixed end of the rod. To avoid frequent failures of the dynamic vibration absorber system, the weight of the lumped mass was reduced considerably (by 41%) and no failure has been reported since.

AUTHOR QUESTIONNAIRE

1. The paper describes new training tools or laboratory concepts/instrumental experiments in mechanics of machines or mechanical vibrations or dynamics.
2. The paper describes new equipment useful in a bachelor of engineering degree course in mechanical engineering.
3. Sophomore or junior-level of a 4-year degree programme are involved in the use of the equipment.
4. New aspects of this contribution are (a) a V-belt driven shaft with unbalanced disc, with a fin supported by two carefully selected compressive springs; and (b) a vibration absorber system based on a precise cantilevered lumped mass.
5. The theory and calculations sections, together with Fig. 5, can be incorporated in the teaching of mechanical vibration control.
6. Chapter 5, 'Introduction to Multi-degree of Freedom System', by W. T. Thomson, *Theory of Vibration with Applications*, Prentice-Hall, is used to supplement the material presented here.
7. The concepts presented have been tested in the classroom. The theoretical and experimental results yield a difference of only 3.4%.
8. We have designed and built in-house a test rig, which gives very reliable results.

INTRODUCTION

FOLLOWING a common first-year engineering course, students at the Nanyang Technological University's School of Mechanical and Production Engineering study various core subjects, as well as other prescribed and free electives for the remaining three years. The second-year programme requires each student to perform a total of 16 experiments and three assigned projects, as well as an eight-week In-House Practical Training (IHPT) programme. The IHPT requires the students to take part in practical hands-on work for the design, testing, fabrication, monitoring and control of various industrial products. The third-year course has an integral industrial attachment programme, which requires the students to spend 24 weeks in industry.

The dynamic vibration absorber experiment is one of the 16 experiments that the undergraduate students are required to undertake. The objective of the experiment is to study how the vibration of a machine, particularly at resonance, can be eliminated or minimized by means of a tuned vibration absorber. The vibration absorber is basically a spring mass system which operates at frequencies close to the primary system (or machine) resonance. Earlier work on the selection of vibration absorber parameters was reported by Den Hartog [1] in which it is shown that the response of the absorber is largely dependent on the primary system response. Tuning is achieved by selecting the resonance frequency of the absorber to be equal to the operating frequency of the primary system [2]. Ebrahimi [3] reported more recent work on the optimum design of vibration absorbers. This paper presents the design, fabrication and testing of a vibration absorber experimental rig and the subsequent modification to overcome a fatigue problem. Experimental results using the vibration absorber rig developed demonstrate the feasibility of the design and reinforce the basic vibration absorber theory.

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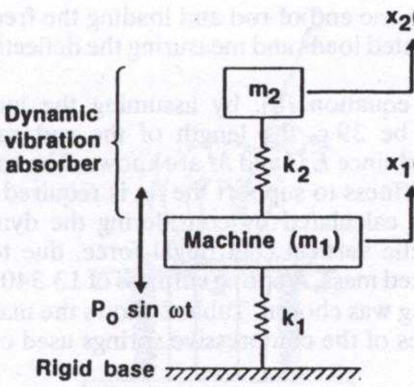

Fig. 1. The two-degree-of-freedom system vibration absorber model.

THEORY

The system being designed and constructed is represented in Fig. 1. The primary machine system is modelled as a spring mass, single degree of freedom system, having a mass m_1 and a stiffness k_1. It is subjected to a harmonic force $P_o \sin \omega t$. A dynamic vibration absorber, which consists of an auxiliary mass m_2 and spring stiffness k_2, is attached to the primary system.

The equations of motion are

$$m_1\ddot{x}_1 = -k_1x_1 - k_2(x_1 - x_2) + P_o \sin \omega t \quad (1)$$

$$m_2\ddot{x}_2 = k_2(x_1 - x_2) \quad (2)$$

To obtain the steady-state vibration displacements X_1 and X_2, let

$$x_1 = X_1 \sin \omega t \text{ and } x_2 = X_2 \sin \omega t. \quad (3)$$

Then, from equations (1)–(3)

$$\frac{X_1}{P_0/k_1} = \frac{1-\dfrac{\omega^2}{\omega_2^2}}{\left[1+\dfrac{k_2}{k_1}-\dfrac{\omega^2}{\omega_1^2}\right]\left[1-\dfrac{\omega^2}{\omega_2^2}\right]-\dfrac{k_2}{k_1}} \quad (4)$$

$$\frac{X}{P_0/k_1} = \frac{1}{\left[1+\dfrac{k_2}{k_1}-\dfrac{\omega^2}{\omega_1^2}\right]\left[1-\dfrac{\omega^2}{\omega_2^2}\right]-\dfrac{k_2}{k_1}} \quad (5)$$

where

$$\omega_1 = \sqrt{\frac{k_1}{m_1}}, \omega_2 = \sqrt{\frac{k_2}{m_2}}$$

From equation (4), it may be seen that for $X_1 = 0$, then $\omega = \omega_2$. Therefore if the machine, before the addition of the vibration absorber, operates near its resonance, we have $\omega^2 \approx \omega_1^2$. And if the vibration absorber is designed such that $\omega^2 = k_2/m_2 = k_1/m_1$, the amplitude of vibration of the machine, while operating at its original resonant frequency, will be zero.

Also, equation (5) gives

$$X_2 = -P_o/k_2 \quad (6)$$

The natural frequencies of the two-degree-of-freedom system are found when X_1 and X_2 are infinite, and since both X_1 and X_2 have the same denominator, these natural frequencies can be obtained by equating the denominator of equation (4) or (5) to zero, i.e.

$$\left[1+\frac{k_2}{k_1}-\frac{\omega^2}{\omega_1^2}\right]\left[1-\frac{\omega^2}{\omega_2^2}\right]-\frac{k_2}{k_1}=0 \quad (7)$$

The natural frequency of the vibration absorber, ω_2, which is basically a cantilever rod with a lumped mass at its tip, may be calculated by Rayleigh's method [4]. This is given by the equation below:

$$\omega_2 = \sqrt{\frac{stiffness}{inertia}} = \sqrt{\frac{3EI}{\left(M+\dfrac{33m}{140}\right)L^3}} \quad (8)$$

where M and m are the masses of the lumped mass and steel rod, respectively, and L, E and I represent the length, Young's modulus and moment of inertia of the rod, respectively.

CALCULATIONS

Unbalanced mass

The unbalanced mass, m_e, is attached to the rotating disc, and the resultant centrifugal force has a vertical sinusoidal component. If ε is the radius of the mass from the centre of the rotating disc, then the exciting force, P_o, is given by

$$P_o = m_e\varepsilon\omega_2^2 \quad (9)$$

When the motor frequency is equal to the vibration absorber's natural frequency, $\omega = \omega_2$, then from equation (6) we have

$$X_2 = -\frac{m_e\varepsilon\omega_2^2}{k_2}$$

or

$$m_e\varepsilon = -m_2X_2 \quad (10)$$

Equation (10) shows that the amount of unbalance, $m_e\varepsilon$ is equal to the product of the auxiliary mass, m_2, and the displacement amplitude, X_2. Therefore the amount of the unbalance is determined from the motion of the auxiliary mass.

Vibration absorber natural frequency

From equation (8) and the material properties data in Table 1, the natural frequency of vibration absorber is

$$\omega_2 = \left[\frac{3(201)(10^9)(3.142)(0.006)^4}{\{0.039 + \frac{33}{140}(0.091)\}(0.41)^3(64)} \right]^{1/2} = 15.27\text{Hz}$$

Table 1. Material properties of the vibration absorber system

Rod length (mm)	410
Rod diameter (mm)	6
Lumped mass (g)	39
Density of steel (kg/m^3)	7860
Young's modulus of steel (GPa)	201

THE VIBRATION ABSORBER RIG DESIGN

Unbalance is recognized to be one of the major sources of vibration in rotating machinery. Excessive rotor unbalance may cause failure to machine components or systems. Therefore, rotor balancing is an important step to achieve smooth operation of rotating machinery [5]. Vibration-severity charts can be used to determine acceptable vibration levels at various operating speeds of rotating machines [6]. However, in the present paper, unbalance is deliberately introduced to the rotor system in order to generate high vibration in the machine system. The energy input to the machine system is a variable-speed d.c. motor, capable of operating up to 3000 rev/min. When the unbalanced rotor is driven by the motor, via a timing belt and pulleys, centrifugal force is generated, which results in sinusoidal forces being transmitted to the bearings and the two supporting compressive springs.

From the design specification, the harmonic force P_o due to the unbalanced mass can be calculated from equation (9). Next, the stiffness of vibration absorber may be determined by the requirement that it be tuned to the frequency of primary system excitation, i.e. $\omega = \omega_2$. In addition, the Young's modulus of the steel rod in Table 1 was determined experimentally. This was conducted by clamping one end of rod and loading the free end by graduated loads and measuring the deflection at the tip.

From equation (8), by assuming the lumped mass to be 39 g, the length of the rod can be calculated since E, I and M are known. The correct spring stiffness to support the fin is required next. This was calculated by considering the dynamic load of the vertical centrifugal force, due to the unbalanced mass. A spring stiffness of 13 340 N/m per spring was chosen. Table 2 shows the material properties of the compressive springs used on the rig.

Table 2. Material properties of the compressive spring

Wire diameter (mm)	4.0
Normal length (mm)	130.0
Outer diameter (mm)	29.0
Spring rate (N/m)	13,340.0
Material	Stainless steel

Most of the parts were fabricated in-house using mild steel, while other components such as the d.c. motor, V-belt, compressive springs and anti-vibration mounts were supplied from the external sources. During testing, it was found that the entire rig structure had a tendency to move along the floor when vibrating under resonance. To overcome this problem, a much heavier base frame structure was designed and installed.

In practice, fine tuning of the vibration absorber will be necessary. In the present method of absorber design, it is essential that the forcing frequency be constant. When excited at frequencies other than the frequency to which it is tuned, the absorber introduces an additional degree of freedom to the system. Thus, there will be two natural frequencies, one on either side of the original natural frequency. If the natural frequency of the primary system is less than the forcing frequency, it is preferable to tune the absorber to a frequency slightly less than the forcing frequency to avoid the resonance that lies above the primary system's natural frequency. Similarly, if the natural frequency of the primary system is above the

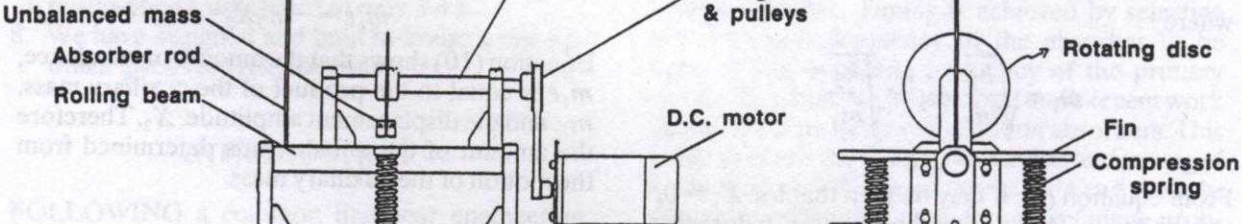

Fig. 2. The dynamic vibration absorber experimental rig.

forcing frequency, it is worth tuning the absorber to a frequency slightly higher than the forcing frequency.

RESULTS AND DISCUSSION

Figure 2 shows the assembly drawing of the dynamic vibration absorber rig. It comprises a variable-speed d.c. motor, a V-belt driving a shaft with an unbalanced rotating disc and a fin supported by two compressive springs.

When the motor is switched on, the rotating speed of the unbalanced disc is measured by a non-contact tachometer, whereas the dynamic displacement of the rolling beam is measured by a transducer, linked to a vibration meter. As the speed is further increased, the corresponding displacement is recorded. When the speed of the rotating disc almost reaches the natural frequency of the primary system, a violent and objectionable vibration was observed. Figure 3 shows the vibration mode of the primary system. The test was repeated with the vibration absorber fitted to the rolling beam and the dynamic displacement against the motor speed was recorded. It was observed that at the natural frequency of the primary system, the primary system has virtually zero displacement, whereas the dynamic absorber vibrated violently. Figure 4 shows the vibration mode observed at resonance that reinforces the vibration absorber theory, discussed in the theory section.

However, due to the frequent fractures of the absorber steel rod, it was proposed to reduce the lumped mass of the absorber system from 39 to

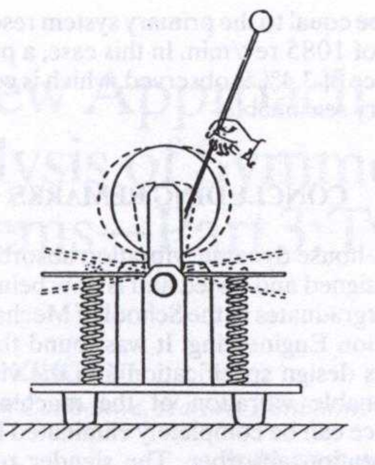

Fig. 3. Vibration mode of the primary system before introducing the absorber.

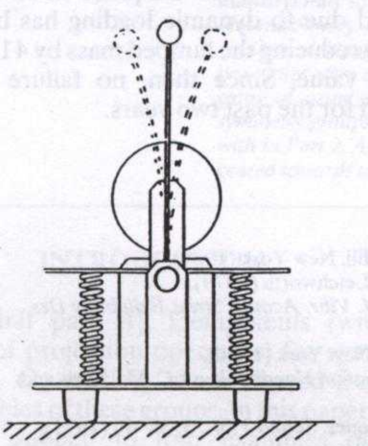

Fig. 4. Vibration mode of the primary system after introducing the absorber.

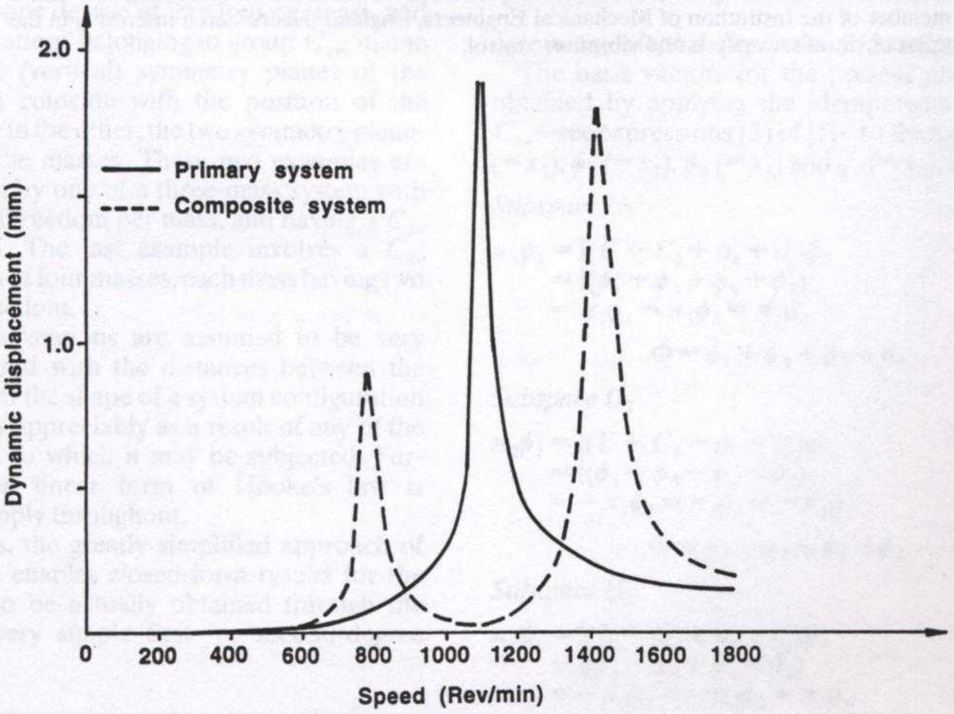

Fig. 5. Dynamic displacement vs. speed of machine.

23 g. With the use of the reduced lumped mass, the natural frequency of the absorber would increase and this was calculated and found to be 17.81 Hz. Tests were carried out and the dynamic displacement of the primary system was plotted against the operating speed of the machine. This was followed by running the test with the absorber system fitted, known as the composite system. Figure 5 shows the displacement vs. speed curves, with and without the vibration absorber. It can be seen that the primary system resonance occurred at 1085 rev/min. This works out to be 18.16 Hz. It will be seen that the absorber was initially tuned to 17.81 Hz, which is slightly lower than the primary system resonance. However, for the composite system, the displacement at this resonant speed was reduced from 1.8 to 0.025 mm, confirming the effectiveness of the absorber. At the same time, the composite system produced two new resonances, at 780 and 1410 rev/min. It is worth noting that, theoretically, the square root of the product of the composite system frequencies, i.e. 780 and 1410 rev/min should be equal to the primary system resonant frequency of 1085 rev/min. In this case, a percentage difference of 3.4% is observed, which is considered to be very reasonable.

CONCLUDING REMARKS

An in-house dynamic vibration absorber rig has been designed and tested and is now being used by all undergraduates in the School of Mechanical and Production Engineering. It was found that the rig meets its design specification and the violent and objectionable vibration of the machine system resonance can be completely eliminated by the use of a vibration absorber. The slender rod with a lumped end-mass proves to be a very effective absorber. Further, the frequent fractures of the steel rod due to dynamic loading has been overcome by reducing the lumped mass by 41% from its original value. Since then, no failure has been reported for the past two years.

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