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The Offset Slider Crank: Kinematic Pseudographic Analysis*

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Kinematics for the offset slider crank are modelled using simple Euclidean plane geometry rule statements written in the software TK Solver for Windows. Co-ordinates of the position, velocity, and acceleration diagrams are generated for any angle of the driving link, for both open and crossed linkage configurations. The cardinal kinematic features of the slider crank are calculated from rudimentary mathematical concepts of co-ordinate geometry, with the physical interpretations being both intuitive, and easy to identify with their diagrammatic depictions. (The term 'pseudographic' refers to a process for determination of kinematic diagram co-ordinates for a complete revolution of the driving link, without the execution of any actual drawings.)

AUTHOR QUESTIONNAIRE

1. The paper discusses materials/software for a course in Dynamics of Rigid Bodies.
2. Students of all engineering departments are taught in this course.
3. Level of the course: Sophomore—2nd year of a 5 year programme.
4. The mode of presentation is through lectures.
5. The material is presented in a regular course.
6. Class or hours required to cover the material: 3 hours of lectures per week plus 3 hours of tutorial/laboratory work for 1 semester.
7. Student homework or revision hours required for the material: about 10 hours per semester.
8. This paper is novel because pseudographics offers an alternative method for the kinematic solution of planar mechanisms. The method is non-vectorial and strongly reinforces the principles of graphical methods for mechanisms.
9. The standard text recommended in the course, in addition to authors' notes is A. Bedford and W. Fowler, *Engineering Mechanics-Dynamics*, Addison-Wesley Publishing Co. Inc (1995).

INTRODUCTION

VELOCITY and acceleration parameters for the slider crank can be derived [1] by successive differentiation of the complex number form of the loop position vector equation. However, the mathematical steps are somewhat tortuous, and it can be difficult to relate intermediate results to physical parameters.

An alternative approach, in vogue in introductory dynamics texts [2], involves vectorial expression of the basic rigid body kinematic relations between pairs of points at each end of the members. Simultaneous solution of sets of scalar equations yield the unknown kinematic variables.

Graphical methods offer a more intuitive analysis, but are usually limited to a single driving link position, with determination of the complete dynamic status requiring a large number of repetitions of the diagram set. Small diagram features may cause significant inaccuracies in the type of drawing shown in Fig. 2(c). A previous work [3], aimed at generating co-ordinates of the position, velocity, and acceleration figures, was circumscribed by a limited control statement and subroutine capacity of an earlier TK software.

This paper presents a method of locating the co-ordinates of the kinematic diagrams for any feasible assembly of the offset slider crank of Fig. 1. The TK Solver [5] model follows the steps in the execution of a graphical solution without any actual drawing—hence the term 'pseudographic' of the title.

Equations are entered in declarative form into the TK Solver Rule Subsheets. Here the rules define the mechanism constraints, but do not specify the algorithms necessary for solution, but rather, the software devises the path. Also, rules do not have to be written in a strict sequence, making TK a very convenient tool for mechanism analysis. However, as with any 'black box' solver, care must be exercised to ensure that the solution is physically apposite, in addition to satisfying the equation set.

MECHANISM ANALYSIS

A model was used to derive numerical values for co-ordinates of the vertices of the position, velocity, and acceleration polygons for both the open and crossed configurations of the offset slider crank. The TK rules of Figs. 4, 5 and 6

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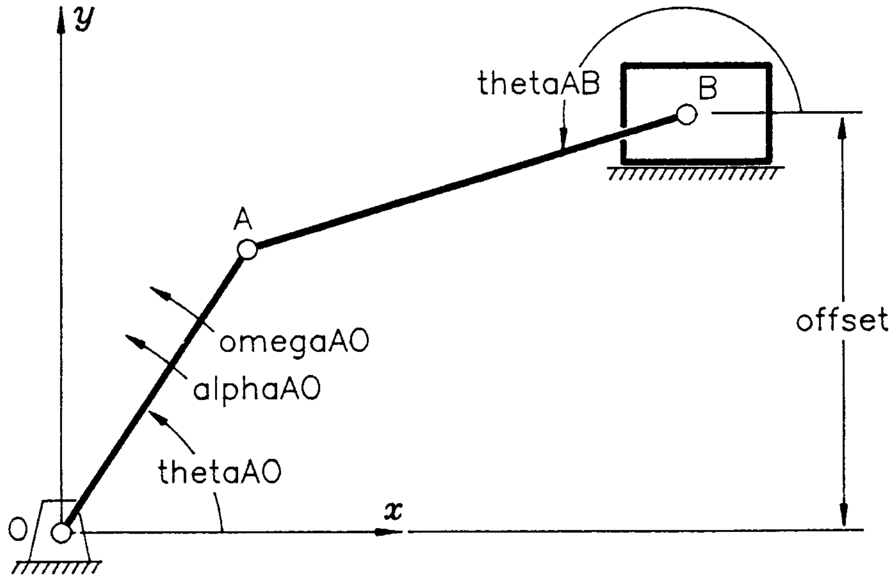


Fig. 1. Offset slider crank in open configuration. The model is solved for nondimensional data as presented in Norton [1], Table P7-2, row c, P. 262. AO = 3; AB = 8; offset = 2. thetaAO = −30; omegaAO = −15; alphaAO = −10.

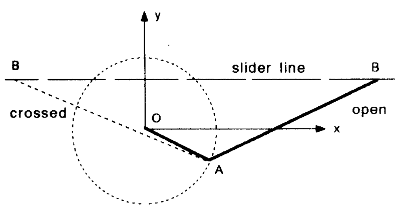
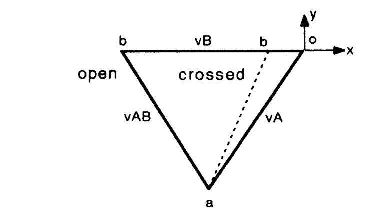
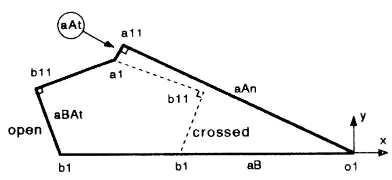
Kinematic Polygon	Vertex	x	y	Location per TK model
	0	ø*	ø*	Linkage origin O placed at origin of coordinates.
	A	xA	yA	Locus of A is circle of radius AO, centre O.
(a) Position	B	xB	yB*	Locus of B is horizontal line yB = offset, xB is at the intersection of this line and a circle of radius AB, centre A.
	o	ø*	ø*	Vertex o at origin of coordinates.
	a	xa	ya	Locus of a is circle of radius vA, centre o, with the line oa being ± 90° out of phase with the position line OA.
(b) Velocity	b	xb	ø*	Slider line is horiz. so yb=0; xb is at intersection of the line yb = 0, and the line ab (slope m2) ⊥ to the position line AB (slope m1).
	o1	ø*	ø*	Vertex o1 at origin of coordinates
	a11	xa11	ya11	Locus of a11 is circle of radius aAn, centre o1, with the line o1a11 being 180° out of phase with the position line OA.
	a1	xa1	ya1	a1 distanced aAt from a11 with the line a11a1 being ± 90° out of phase with the position line OA.
	b11	xb11	yb11	b11 distanced aBAn from a1 with the line a1b11 parallel to the position line BA.
(c) Acceleration	b1	xb1	ø*	Slider acceleration is horizontal so yb1 = 0; xb1 is at the intersection of the line yb1 = 0 and the line b11b1 (slope m2).

Fig. 2. The labels mimic those of Morrison [4], an early mechanisms text with emphasis on graphical methods; primes and double primes are supplanted by the single numeral 1 and the double numeral 11. Input co-ordinates appear with an asterisk. (a) Position diagram, open and crossed branches. Upper case letters denote the end of links. (b) Velocity diagram, open and crossed. Lower case letters denote the ends of velocity vectors, e.g., velocity of pin b relative to pin A = ab = vBA. (c) Acceleration diagram, open and crossed branches. Lower case letters, with single and double numerals, denote the ends of acceleration vectors, e.g. tangential acceleration of pin B relative to pin A = b11b1 = aBAt.

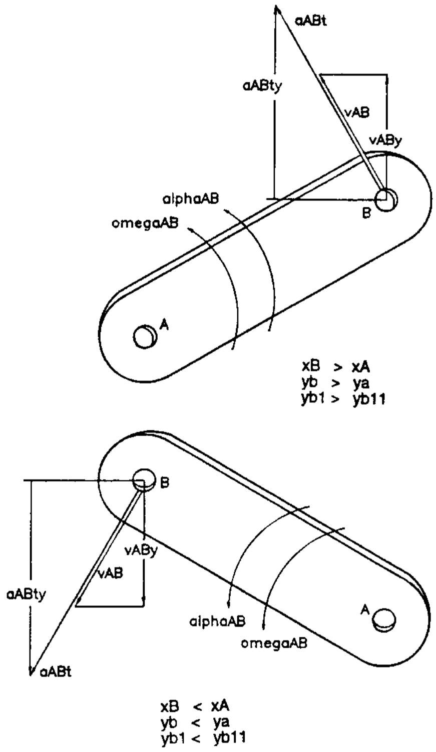


Fig. 3. The signs associated with angular velocity/acceleration of the link AB are determined by comparing values of the appropriate co-ordinates of the kinematic polygons. Anticlockwise rotation is taken as positive, so if one, *and only one*, of the pair of conditions in each of the lines below is untrue, then the rotational velocity/acceleration is negative (clockwise).

if $xB > xA$ and $yb \geq ya$ then $omegaAB \geq 0$
if $xB > xA$ and $yb1 \geq yb11$ then $alphaAB \geq 0$

handle the problem in the same manner, and in the same order, as a uni-positional graphical solution.

The three kinematic diagrams for a particular slider crank with a driving link position of 30° below the horizontal are shown in Fig. 2. With nine given input values, an additional thirteen co-ordinates completely define these figures.

Position, Fig. 2(a)

The locus of pin A is circular, with the co-ordinates (xA, yA) defined by the angle thetaAO and the link length AO. The pin B is constrained to move horizontally with a vertical displacement, yB, equal to the slider offset; xB is found by the simultaneous solution of the straight line equation yB = offset, and the equation for the circle centered at A, with a radius equal to the length of the connecting link AB.

Velocity, Fig. 2(b)

The locus of vertex a is circular, with a radius equal to the speed of pin A, vA. The co-ordinates (xa, ya) are defined by the fact that the velocity vector for point A is ±90° out of phase with the spatial line OA. The pin B has a horizontal velocity, so yb is zero; xb is located by noting that velocity of point B relative to point A is represented by the line ab perpendicular to the link AB. Thus the slope, m2, of line ab is determinable, as are three of four co-ordinates, allowing xb to be found. The velocity of pin B is represented by the line ob.

St	Input	Name	Output	Unit	Comment
	'open	branch			declaration of open or crossed configuration
	+3	AO			length of link AO
	+8	AB			length of link AB
	+2	offset			slider offset
	-15	omegaAO			angular velocity of link AO
	-10	alphaAO			angular acceleration of link AO
L	-30	thetaAO			angle of link AO
L		xB	+9.79		coordinate of B
L		vB	-41.46		velocity of pin B
L		omegaAB	+5.42		angular velocity of link AB
L		aB	-709.1		acceleration of pin B
L		alphaAB	-29.03		angular acceleration of link AB

Fig. 4. TK Solver variable Sheet. The symbolic variable ['open] in the first line dictates that the model is to execute a solution for the open configuration; any other numerical entry in this cell runs the crossed branch.

S	Rule
	call position(thetaAO;xA,xB,thetaAB,m2)
	call velocity(thetaAO,xA,xB,thetaAB,m2;vB,omegaAB)
	call acceleration(thetaAO,xA,xB,thetaAB,omegaAB,m2;aB,alphaAB)

Fig. 5. TK Solver Rule Sheet. The call command invokes the appropriate Function Subsheet—variables to the left of the semicolon are argument variables; those to the right are result variables.

```
Comment:                 position function rules
Parameter Variables:     AB,AO,offset,branch
Argument Variables:      thetaAO
Result Variables:        xA,xB,thetaAB,m2
S  Rule
   yB=offset  ;yB
   (xA,yA)=(AO*cosd(thetaAO),AO*sind(thetaAO))  ;xA,yA
   if (branch='open) then xB-xA=+sqrt(AB^2-(yB-yA)^2) else xB-xA=-sqrt(AB^2-(yB-yA)^2)  ;xB
   thetaAB=atan2d(yA-yB,xA-xB)  ;thetaAB
   m1=tand(thetaAB)  ;m1
   m2*m1=-1  ;m2
```
(a)

```
Comment:                 velocity function rules
Parameter Variables:     omegaAO,AO,AB
Argument Variables:      thetaAO,xA,xB,thetaAB,m2
Result Variables:        vB,omegaAB
S  Rule
   vA=omegaAO*AO  ;vA
   (xa,ya)=(vA*cosd(thetaAO+90),vA*sind(thetaAO+90))  ;xa,ya
   yb=0  ;yb
   if or (thetaAB=180,thetaAB=360) then xb=xa  ;xb
   if and (thetaAB<>180,thetaAB<>360)then ya-yb=m2*(xa-xb)  ;xb
   vAB=sqrt((xa-xb)^2+(ya-yb)^2)  ;vAB
   vB=xb  ;vB
   if and (xB>=xA,ya>=0) then vAB =-omegaAB*AB  ;omegaAB
   if and (xB<=xA,ya<=0) then vAB =-omegaAB*AB  ;omegaAB
   if and (xB>=xA,ya<=0) then vAB =omegaAB*AB  ;omegaAB
   if and (xB<=xA,ya>=0) then vAB =omegaAB*AB  ;omegaAB
```
(b)

Fig. 6. TK Rule Function Subsheets for the determination of the variables for (a) position, (b) velocity, (c) acceleration. Each line generates a value for the variable shown to the right of the double quote mark. The signs associated with the angular velocity and acceleration of the link AB are determined in the nest of four conditional (and if) equations in (b) and (c).

A special position, occurring when the link AB is horizontal, and thus the slope of ab becomes indeterminate, is dealt with by noting that for this configuration, xb = xa.

Link AB has an angular velocity omegaAB, with the associated sign determined by one of four conditional (if and) statements appearing at the end of the velocity portion of the TK Rule Function Subsheet, and explained in Fig. 3.

Acceleration, Fig. 2(c)

The locus of vertex a11 is circular, with a radius equal to the normal acceleration of pin A, aAn, with the co-ordinates of a11 being (xa11, ya11). This normal acceleration is 180° out of phase with the spatial line OA.

The distance of point a1 from point a11 corresponds to the tangential acceleration of pin A, aAt, with the line a11a1 being ±90° out

```
Comment:                 acceleration function rules
Parameter Variables:     AO,AB,omegaAO,alphaAO
Argument Variables:      thetaAO,xA,xB,thetaAB,omegaAB,m2
Result Variables:        aB,alphaAB
S  Rule
   aAt=alphaAO*AO  ;aAt
   aAn=omegaAO^2*AO  ;aAn
   aBAn=omegaAB^2*AB  ;aBAn
   yb1=0  ;yb1
   (xa11,ya11)=(aAn*cosd(thetaAO+180),aAn*sind(thetaAO+180))  ;xa11,ya11
   (xa1,ya1)=(xa11+aAt*cosd(thetaAO+90),ya11+aAt*sind(thetaAO+90))  ;xa1,ya1
   (xb11,yb11)=(xa1+aBAn*cosd(thetaAB),ya1+aBAn*sind(thetaAB))  ;xb11,yb11
   if or (thetaAB=180,thetaAB=360) then xb1=xb11 else yb1-yb11=m2*(xb1-xb11)  ;xb1
   aB=xb1  ;aB
   aBAt=sqrt((xb1-xb11)^2+(yb1-yb11)^2)  ;aBAt
   if and (yb1>=yb11,xB>=xA) then aBAt=alphaAB*AB  ;alphaAB
   if and (yb1<=yb11,xB<=xA) then aBAt=alphaAB*AB  ;alphaAB
   if and (yb1<=yb11,xB>=xA) then aBAt=-alphaAB*AB  ;alphaAB
   if and (yb1>=yb11,xB<=xA) then aBAt=-alphaAB*AB  ;alphaAB
```
(c)

Fig. 6. Continued.

of phase with the position line OA. The appropriate sign is determined by the sense of the angular acceleration.

The line a1b11 of the acceleration diagram represents the normal acceleration of pin B relative to pin A, aBAn, with an inclination parallel to the spatial line BA of slope m1, i.e.

$$\text{a1b11} = \text{aBAn} = (\text{omegaAB})^2\text{AB}$$

With the angular speed of link AB determined in the velocity section of the model, the vertex b11 is thus located.

The pin B has a horizontal acceleration, so yb1 is zero, and xb1 is located by noting that the tangential acceleration of point B relative to point A is represented by the line b11b1, perpendicular to the link AB. Thus the slope, m2, of line b11b1 is known, as are three of four co-ordinates, allowing xb1 to be found. The acceleration of pin B is represented by the line o1b1.

The sign associated with alphaAB, the angular acceleration of link AB, is determined by one of four conditional (if and) statements appearing at the end of the acceleration portion of the TK Rule Function Subsheet, and explained in Fig. 3.

CONCLUSION

Pseudographic kinematic analysis, in conjunction with a convenient software package such as TK Solver, offers a viable alternative to traditional vectorial methodologies.

An advantage of the method is that it progresses in a sequence that mimics the uni-positional graphical solution. The model presented here can be readily adapted to a variety of boundary conditions. So, for example, if the slider is nominated as the driving member, then the behaviour of the link AO can be found by simply designating the velocity and acceleration of the slider as input variables, while the rotational angular velocity and acceleration of link AO become output variables, i.e., TK can 'backsolve'.

Peripheral benefits arise from easy access to values of kinematic parameters that may be generated as output lists in TK. Textbook problems in planar kinematics often focus on solutions for a particular position of the driving member, whereas with the short, and relatively straightforward schema presented here, data are derived for all input link positions.

REFERENCES

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