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Calculus Reform to Mechanics Reform*

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This paper advocates the inclusion of computational software in courses in statics and dynamics. This removes the numerical burden of solving simultaneous linear equations and transcendental equations that arise repeatedly in these courses. These programs allow full use of vector algebra and calculus, numerically and symbolically. Currently most dynamics problems are quasi-static in nature and do not require the students to study the motion. Computational software can be used to solve the nonlinear differential equations that arise. Use of this software allows emphasis to be placed on modeling, constraints, formulation of the equations and design parameters.

INTRODUCTION

A CURRENT movement to change undergraduate mathematics education is called 'Calculus Reform'. The motivation for this movement can best be summarized by the following quote from a calculus book by Bradley and Smith [1]:

> 'The major issue driving calculus reform is the poor performance of students trying to master the concepts of calculus. Much of this failure can be attributed to ways in which students study and learn mathematics in high school. The high school mathematics textbooks published over the last twenty years have increasingly focused on reducing mathematics to a series of small repeatable steps. Process is stressed over insight and understanding. In these same books we often find problems matched to worked-out sample problems, which encourages students to memorize a series of problem-solving algorithms. Unfortunately, this reduces mathematics to taxonomy; students show up for calculus generally willing to work hard; for many, this translates into working hard at rote memorization! The task of getting students *to think conceptually* falls to the teachers of calculus . . .'

If the word calculus was replaced by mechanics, we see that much that drove calculus reform might serve as the same motivation to drive reform of undergraduate mechanics education. Notice, that as in calculus, no one is proposing revolution only reform. Recently a mechanics teacher wrote the following in a review of an introductory dynamics text manuscript:

> 'Why are people choosing to write so much on a subject that does not have the half-life of other engineering material? How many people are doing research in basic mechanics? What substantially new material or approach is ever adopted? All the books are variations on a theme. That is not to say that the delivery of the material cannot be improved. The computer opened up all sorts of possibilities.'

The current leading undergraduate texts were originally written 15 to 30 years ago and have not changed during that period. This is not to say that the presentation of the subject is flawed but that it is inconsistent with the current tools available to the undergraduate student. We feel that these tools allow emphasis to be placed on the fundamentals of modeling and problem formulation and remove most of the computational burdens. One major addition in calculus reform is the use of graphical calculators and modern computational software packages in the instruction of calculus adding both conceptual insight and removing the necessity to use 'a bag of tricks' to reduce the problem to a tractable form. Currently student editions of such software packages as *Mathcad*, *Matlab* and *Mathematica* are available for about $100. These software packages can be used by the student throughout his or her education. Much of calculus reform is based upon the utilization of software packages of this nature. At Michigan State University and Virginia Polytechnic Institute, the engineering students are introduced to these computational programs during their freshman year. We will illustrate the use of computational programs in statics and dynamics by examples of three separate applications: solution of simultaneous equations, direct vector calculations and numerical integration of equations of motion.

SOLUTION OF SYSTEMS OF EQUATIONS

Statics and dynamics involve solution of simultaneous linear and sometimes nonlinear equations. Many of the methods introduced are solely to reduce the computational efforts required to solve these equations. The student is taught to take moments about support points to eliminate these

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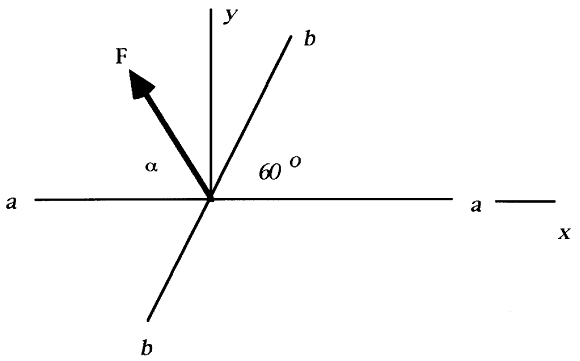


Fig. 1. Resolve force into two components.

unknown reactions in some of the equations of equilibrium. The student frequently becomes confused as to the number of linearly independent equations of equilibrium that can be written. D'Alembert's principle is introduced in dynamics to enable the student to simplify the system of linear equations. However, many hand calculators and all computational software packages include subprograms to solve linear systems of equations. Most of these require a matrix formulation of the equations and computation of the inverse of the coefficient matrix. Many students have seen this approach in high school and those who have not, accept the notation and obtain solutions without a course on linear algebra or matrices. Consider the following problem found in a first block of homework problems from a statics text.

Example 1

The force F of magnitude 300 N is to be resolved into two components along lines *a–a* and *b–b* as shown in Fig. 1. Determine (by trigonometry) the angle α, knowing that the component of F along line *a–a* is to be 240 N.

Solution.

The x–y coordinates have been added to the figure so that the problem can be solved using the components of the vector equation:

$$F = F_a + F_b$$

The two resulting scalar equations are:

$$-300 \cos \alpha = -240 + F_b \cos 60^\circ$$

$$300 \sin \alpha = F_b \sin 60^\circ$$

There are now two equations for the two unknowns F_b and α. However, these equations are nonlinear and trigonometric identities are required to reduce them to a single quadratic equation. If the problem is solved by trigonometry as suggested by the author, the law of sines and the law of cosines are used to generate two similar equations. In either case, a problem designed to show non-orthogonal components of a vector is now, in the student's mind, how to solve two simultaneous nonlinear equations. It might be argued that this is a useful exercise for the students to learn algebra but it does more to confuse students than enlighten them. Of course, the nonlinearity could be eliminated by giving the angle α and asking for the components in the *a–a* and *b–b* directions which would require solution of two linear equations. Since computational software packages have iterative programs to solve the nonlinear equations, the student could examine both variations of the problem without becoming confused by the algebra.

Even in equilibrium of a particle, solutions require many hand calculations to establish unit vectors and to solve the system of linear equations. *A* typical example of this type of problem is an object supported by three cables.

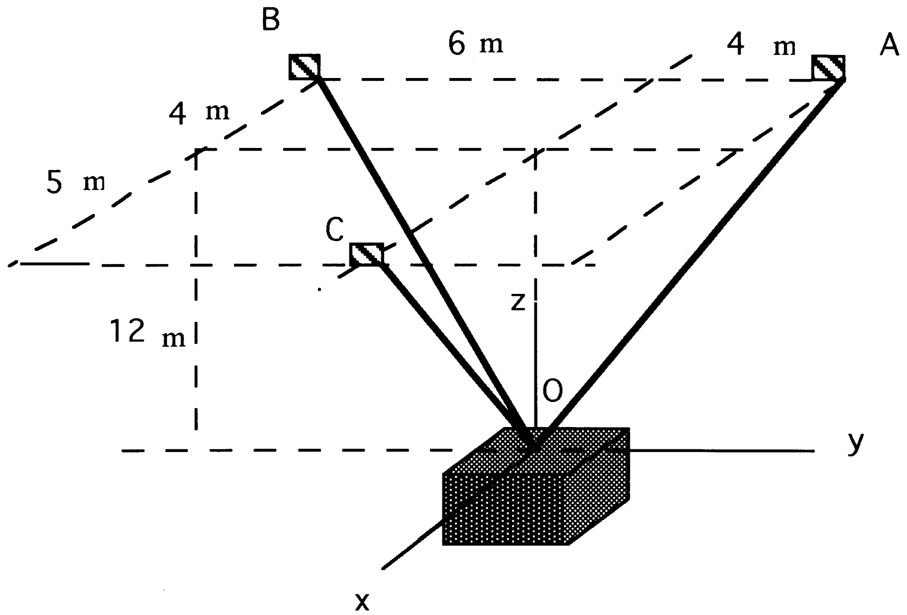


Fig. 2. Scoreboard supported by three cables.

Example 2.

An electronic scoreboard in a gymnasium has a mass of 200 kg and is supported by three cables as shown in Fig. 2. Determine the tension in each of the cables.

Solution.

The unit vectors along the cables may be obtained by finding position vectors from the origin 0 to the cable attachments at A, B and C. A unit vector along each cable can then be determined. Since each cable must be in tension, the unit vectors must be directed from point 0 to the point of attachment. A vector from point O to point A along the cable A is:

$$A = -4\hat{\imath} + 4\hat{\jmath} + 12\hat{k}(m)$$

Therefore a unit vector in that direction may be obtained by dividing A by its magnitude:

$$|A| = \sqrt{(-4)^2 + (4)^2 + (12)^2}$$

$$|A| = 13.27\,(\text{m})$$

$$\hat{a} = -0.302\hat{\imath} + 0.302\hat{\jmath} + 0.905\hat{k}$$

In a similar manner, vectors from O to B and O to C can be determined:

$$B = -4\hat{\imath} - 6\hat{\jmath} + 12\hat{k}\,(\text{m})$$

$$C = 5\hat{\imath} + 12\hat{k}\,(\text{m})$$

The unit vectors in these two directions are:

$$|B| = 14.0 \quad \hat{b} = -0.286\hat{\imath} - 0.429\hat{\jmath} + 0.857\hat{k}$$

$$|C| = 13.0 \quad \hat{c} = 0.385\hat{\imath} + 0.923\hat{k}$$

If T_A, T_B and T_C are the tension vectors in the three cables, the equation of equilibrium is:

$$T_A + T_B + T_C + W = 0$$

The scalar equations are obtained from the unit vectors $\hat{a}$, $\hat{b}$ and $\hat{c}$ and are:

$$-0.302T_A - 0.286T_B + 0.385T_C = 0$$

$$0.302T_A - 0.429T_B = 0$$

$$0.905T_A + 0.857T_B + 0.923T_C = 1962$$

Solution of this system of three equations yields:

$$T_A = 723\,\text{N} \quad T_B = 509\,\text{N} \quad T_C = 945\,\text{N}$$

Although this is a typical equilibrium example, the solution of the system of equations involves a large amount of hand calculations that adds little to the student's understanding of the concept of equilibrium. The numerical work can be reduced by use of computational software as shown in Computational Window 1.

Computational Window 1

Position vectors from the origin to each of the attachment points are:

$$A := \begin{pmatrix} -4 \\ 4 \\ 12 \end{pmatrix} \quad B := \begin{pmatrix} -4 \\ -6 \\ 12 \end{pmatrix} \quad C := \begin{pmatrix} 5 \\ 0 \\ 12 \end{pmatrix}$$

These values must be entered from the space diagram of the problem but now the numerical computation can all be done use computational software. If the student realizes that a component of the original vectors is in error, the entry can be corrected and the remaining work will be updated. The unit vectors along the cables are:

$$a := \frac{A}{|A|} \quad b := \frac{B}{|B|} \quad c := \frac{C}{|C|}$$

$$a = \begin{pmatrix} -0.302 \\ 0.302 \\ 0.905 \end{pmatrix}$$

$$b = \begin{pmatrix} -0.286 \\ -0.429 \\ 0.857 \end{pmatrix}$$

$$c = \begin{pmatrix} 0.385 \\ 0 \\ 0.923 \end{pmatrix}$$

The scalar equations of equilibrium can be written in matrix notation as:

$$CO := \begin{pmatrix} -0.302 & -0.286 & 0.385 \\ 0.302 & -0.429 & 0 \\ 0.905 & 0.857 & 0.923 \end{pmatrix}$$

The coefficient matrix of the unknown tensions.

The weight is the only term on the right-hand side of the equilibrium equations.

$$W := \begin{pmatrix} 0 \\ 0 \\ 9.81 \cdot 200 \end{pmatrix}$$

$$\begin{pmatrix} T_A \\ T_B \\ T_C \end{pmatrix} := CO^{-1} \cdot W$$

$$T_A = 722.655 \quad T_B = 508.722 \quad T_C = 944.77$$

DIRECT VECTOR METHODS

The computational software was used in the previous example to determine unit vectors along cables. Vectors can be added, subtracted or the magnitude of the vector can be determined directly by the use of these software programs. The use of vectors is minimized in some current texts because the calculations required are lengthy and two-dimensional problems can be solved using trigonometric methods. Moments are determined using Varignon's theorem avoiding numerical effort required to take the vector product. However the computer can easily be used as a 'vector calculator'. The student can then determine moments with ease, create unit vectors to represent forces along cables, etc. In plane dynamics, a 'direct vector' solution to determine angular velocity or acceleration of a rigid body can be obtained by use of Rodrigue's Formula [2] or extensions of it:

$$\bar{\omega} = \frac{\bar{r}_{B/A} \times \bar{v}_{B/A}}{\bar{r}_{B/A} \cdot \bar{r}_{B/A}}$$

$$\bar{\alpha} = \frac{\bar{r}_{B/A} \cdot \bar{a}_{B/A}}{\bar{r}_{B/A} \cdot \bar{r}_{B/A}}$$

If an instantaneous center of rotation is desired and the angular velocity of the rigid body and the velocity of a point A on the body are known, the location of the instantaneous center C is:

$$\bar{r}_{C/A} = \frac{\bar{\omega} \times \bar{v}_A}{\bar{\omega} \cdot \bar{\omega}}$$

These examples introduce methods of inverting the cross-product:

$$A \times B = \mathrm{C}$$

when C and A or B are known and the remaining vector is unknown and A and B are orthogonal. Students will readily use this approach rather than expand into scalar form if computational software is available. This is illustrated in the Computational Window 2 using Mathcad.

Computational Window 2

Consider that the position and velocity and acceleration of two points, A and B, on a rigid body in plane motion are known and the angular velocity and acceleration and the position of the instantaneous center of rotation are desired.

$$r_A := \begin{pmatrix} 1.6 \\ 1.5 \\ 0 \end{pmatrix} \quad R_B := \begin{pmatrix} 2 \\ 1.8 \\ 0 \end{pmatrix}$$

$$v_A := \begin{pmatrix} 3 \\ 0 \\ 0 \end{pmatrix} \quad v_B := \begin{pmatrix} 2.4 \\ 0.8 \\ 0 \end{pmatrix}$$

$$a_A := \begin{pmatrix} 3 \\ 2 \\ 0 \end{pmatrix} \quad a_B := \begin{pmatrix} 1.1 \\ 1.2 \\ 0 \end{pmatrix}$$

$$r_{BA} := r_B - r_A$$

$$v_{BA} := v_B - v_A$$

$$a_{BA} := a_B - a_A$$

$$\omega := \frac{r_{BA} \times v_{BA}}{r_{BA} \cdot r_{BA}}$$

$$\alpha := \frac{r_{BA} \times a_{BA}}{r_{BA} \cdot r_{BA}}$$

$$r_{CA} := \frac{\omega \times v_A}{\omega \cdot \omega}$$

$$\omega = \begin{pmatrix} 0 \\ 0 \\ 2 \end{pmatrix} \quad \alpha = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \quad r_{CA} = \begin{pmatrix} 0 \\ 1.5 \\ 0 \end{pmatrix}$$

SOLUTION OF DIFFERENTIAL EQUATIONS

Many dynamics texts avoid solution of linear or nonlinear differential equations, that are easily handled by computational software packages. Examine any of the current texts and note the number of dynamics homework problems which are prefaced by 'at the instant shown', 'in the position shown' or 'the cable suddenly breaks', 'the wire is cut'. The problem is then reduced to a sort of quasi-static formulation. The problem is limited in this manner so that the student is not confronted with the fact that the acceleration of the rigid body will, in general, be a function of position, velocity and time. It is difficult to see how this contributes to the understanding of the dynamics of the motion.

As another example of the use of computational software, consider the damped oscillations of a simple pendulum. This problem can be presented when dynamics of plane motion is discussed and is easily accepted by the students although the solution requires numerical integration of a nonlinear differential equation.

Example 3

The uniform pendulum is released from a horizontal position as shown in Fig. 3. Determine and plot the motion of the pendulum if the motion is retarded by friction at the pin. (The frictional moment always opposes motion).

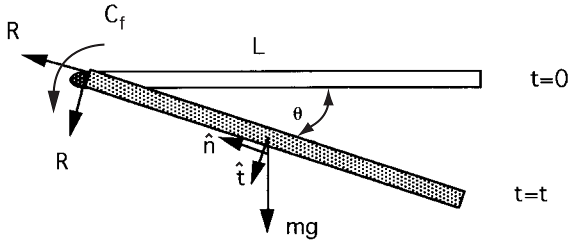


Fig. 3. Pendulum released from rest retarded by friction at the pin.

Solution.

The angular equation of motion about the pivot point is:

$$mg\frac{L}{2}\cos\theta - C_f\,\mathrm{sign}\,(\dot{\theta}) = I_0\ddot{\theta} = \tfrac{1}{3}mL^2\ddot{\theta}$$

$$\therefore\ \ddot{\theta} = \frac{3}{2}\frac{g}{L}\cos\theta - M_f\frac{\dot{\theta}}{|\dot{\theta}|}$$

$$\text{where } M_f = C_f\frac{3}{mL^2}$$

The difficulty now is that the nonlinear differential equation must be integrated. Large oscillations of a pendulum are presented in most intermediate or advanced texts using elliptic integrals. In this case the nonlinear differential equation is integrated using Euler's method of numerical integration. The values L = 1 m and M_f = 0.8 N-m have been used in Computational Window 3.

As a final example of need to upgrade these courses, consider the 'computer problem' contained in one of the leading statics texts.

Example 4

A 100-kg crate is supported by the rope-and-pulley arrangement shown in Fig. 4. Write a computer program which can be used to determine, for a given value of β, the magnitude and direction of the force F which should be exerted on the free end of the rope. Use this program to calculate F and a for values of β from 0 to 30° at 1° intervals.

Solution.

There is no need to write a FORTRAN or *C* program for this problem as it can be solved easily by any commercial computational software package. The equilibrium equations in the horizontal and vertical directions are:

$$-2F\sin\beta + F\cos\alpha = 0$$

$$2F\cos\beta + F\sin\alpha - 100(9.81) = 0$$

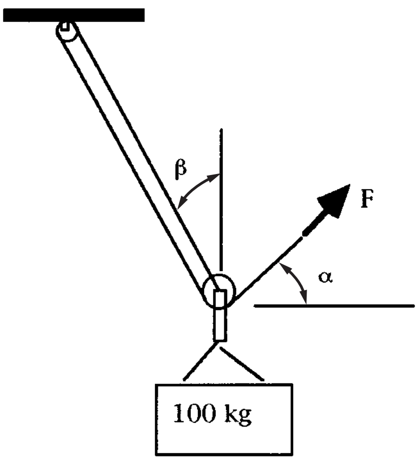


Fig. 4. Pulley system.

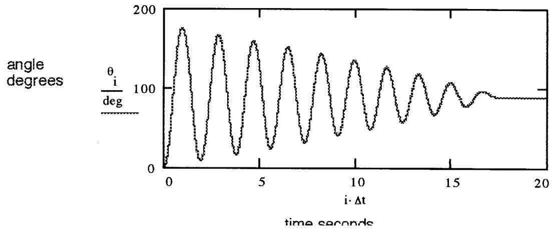
Computational Window 3

$$i := 0\ldots 2000 \quad \Delta t := 0.01 \quad L := 1$$

$$M_f := 0.8 \quad g := 9.81$$

$$\begin{pmatrix}\theta_0\\ \omega_0\end{pmatrix} := \begin{pmatrix}0\\ 0\end{pmatrix}$$

$$\begin{pmatrix}\theta_{i+1}\\ \omega_{i+1}\end{pmatrix} := \left[\begin{matrix}\theta_i + \omega_i\cdot\Delta t\\ \omega_i + \left(\dfrac{3}{2}\cdot\dfrac{g}{L}\cdot\cos(\theta_i) - M_f\dfrac{\omega_i}{|\omega_i|}\right)\cdot\Delta t\end{matrix}\right]$$

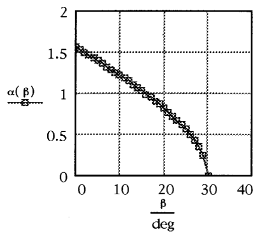


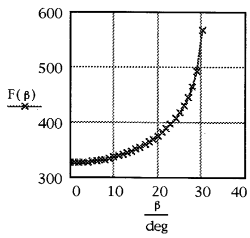
Computational Window 4

$$\beta := 0, 1\cdot\mathrm{deg}\ldots 30\cdot\mathrm{deg}$$

$$\alpha(\beta) := \mathrm{acos}\,(2\cdot\sin(\beta))$$

$$F(\beta) := 100\cdot\frac{9.81}{2\cdot\cos(\beta) + \sin(\alpha(\beta))}$$





The solution is shown in Computational Window 4 including graphs showing the dependency of α and F on the angle β.

Although the examples shown are solved using *Mathcad*, the authors have worked similar problems using *MatLab* and *Mathematica.*

CONCLUSIONS

These examples show that if computational software is included in the undergraduate mechanics courses, the student can apply the theory to a much larger group of applications. Numerical work is reduced without loss of understanding and the results can be graphed and presented in a finished form. When vectors and later calculators were introduced in the undergraduate courses, there was resistance from some instructors who thought the tools obscured the engineering. We have found the opposite to be the fact, and feel that the undergraduate mechanics courses should be updated to reflect the changes in computational methods.

If the undergraduate courses are to be changed to utilize computational tools, new texts need to be available or the current texts revised. The authors are currently preparing texts for statics and dynamics with supplements for *Mathcad*, *Matlab* or *Mathematica* software programs. The supplements support the text material and require only minimal additional classroom instruction. These materials have been classroom tested and the approach was readily accepted by the students. Although, as with calculators, the numerical effort to solve any undergraduate mechanics problem is greatly reduced using computational software, only twenty five percent of the problems in the texts require computational aids.

When we first introduced this approach two years ago, we knew we would have to develop new evaluation methods. Currently there are ten problems in each statics and dynamics class that are solved using computational software and are graded equivalent to one hour exam. The regular exams do not require computational aids although most of the students will solve the linear system of equations using matrices on their calculators. The problems on the exams are similar to the traditional problems and can be solved by hand. In dynamics, the student is asked to find the acceleration for any time or position but not asked to solve the nonlinear differential equations. General solutions are developed in this manner. Although initially the student feels deprived of numbers, they quickly adapt to these problems and realize that less careless mistakes are made.

The examples presented in this paper do not include other powerful applications of the computational software such as the use of the symbolic processors in establishing general solutions. Those instructors who have been teaching mechanics since the 1960's have watched it progress from the tools of the draftsman, to the use of trigonometry, to calculators, programmable calculators, graphing calculators, and now to powerful computational software packages. The need to integrate these packages into the undergraduate mechanics courses is great. The following quotes show that we are already late in meeting this responsibility.

> 'Today engineers are using off-the-shelf sophisticated software to help solve complicated engineering problems.' [3]
> 'In March 1993, ASME's Council on Engineers, Board on Engineering Education and M.E. Department Heads ... recommended that undergraduates be exposed to mathematics software early in the mechanical engineering program.' [4]
> 'Those who hire or work with recent college graduates should expect them to arrive with a working knowledge of computer mathematics.' [4]

It is our responsibility to educate students and prepare them for work and/or graduate school. Both of these paths will require a working knowledge of computational software. Hopefully when this responsibility is met, nobody will write again that our subject 'does not have the half-life of other engineering material'.

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