Int. J. Engng Ed. Vol. 14, No. 2, p. 145–152, 1998
Printed in Great Britain.
0949-149X/91 $3.00+0.00

Teaching Mathematics to Engineering Students*

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Some new ideas in teaching mathematics to engineering students and the implementation of these ideas into the teaching of mechanical engineering students at Brighton University are discussed. The importance of explaining to the students why knowledge of mathematics is essential for their future practical work is emphasized. Mathematics is a language for expressing physical, chemical and engineering laws and general equations should be illustrated by practical numerical examples in order to transfer the surface/atomistic approach to learning to the deep/holistic one. Necessary steps in the manipulation of algebraic equations should be highlighted. Formal lecturers should be supplemented by compulsory reading, handouts, elements of small group teaching and formative assessment. The analysis of self-assessment forms completed by students show that they learn physical concepts much easier than mathematical concepts.

INTRODUCTION

AN ENGINEERING student once said, 'Mathematics is when numbers are put into equations'. This statement obviously contains an element of truth. One cannot expect engineering students to perceive mathematics in the same way as professional mathematicians usually do, yet the professional engineer must acquire not only empirical but also abstract understanding of mathematics. It seems that the objective of teaching mathematics to engineering students is to find the right balance between practical applications of mathematical equations and in-depth understanding. In this paper I discuss this balance and some practical ways of achieving it based on my experience of teaching thermofluids to engineering and energy students at Brighton University. The achieved results are discussed based on self-assessment forms completed by the students.

MOTIVATION

It should not be taken for granted that engineering students understand the need to study mathematics in the first place. Although my subject is not mathematics but thermofluids, it inevitably contains a number of mathematical equations which I tried to explain in detail. When, after the first few lectures of the course, I asked my students to complete feedback forms, about 80% of the students complained that my course was too academic. One of the students tried to describe this general mood by writing: We are mostly not academics but practical engineers; we forget what we are told but never forget what we see or discover for ourselves!'

It was clear that I made at least two mistakes in designing my course. Firstly, the theory was indeed not properly balanced with practical applications. Secondly, the need for the theoretical part was not well explained at the beginning of the course. I had to put things right in order to complete the course successfully.

There are obvious 'natural' limits to the depth of the mathematical analysis. If we don't set these limits we can, in theory, end up studying topology (the foundations of mathematics) *ad infinitum*. Our brain may be working very hard, but its direct contribution to the science of engineering would be negligible. On the other hand, there are not so many objects that can be physically touched in modern engineering. For example, one cannot 'touch' the boundary layer of a supersonic aircraft or the inside of a working internal combustion engine. In order to study them one needs to describe them as abstract concepts in terms of mathematical equations.

This means that mathematics is indispensable for the engineering community, but the depth of its study is bound to be limited. The best 'practical' approach to mathematics is to understand it as a language for describing physical and chemical laws. From this point of view understanding an engineering problem means the conversion of this problem into a physical and/or chemical problem, and its formulation in terms of mathematical equations.

Note that the fact that predictions of theory agree with observations does not necessarily mean that the theory is correct. For example, Ptolemy's theory of the heavens was in good numerical agreement with observations over two millennia. This, however, did not prevent it from

* Accepted 25 September 1997.

being wrong. (This idea was taken from the manuscript 'What can we learn from numerical simulations' by R. A. Treumann.) This means that a 'practical' engineer cannot avoid the in-depth study of physics, chemistry and 'practical' mathematics before applying them to engineering problems. One cannot just take a mathematical model as a 'black box' and compare it with experiments. For example, a research engineer can find himself or herself severely hindered if he or she attempts to apply a computational fluid dynamics (CFD) code to the solution of an engineering problem without understanding the underlying physical phenomena and/or the limitations of the code [1]. I believe that this should be the main motivation for studying mathematics for engineering students and it needs to be explained to students properly.

Sometimes engineering students complain that they physically cannot perceive mathematical concepts. I believe that in this case the students can be given the following formula:

$$\text{Result} = \text{Ability} \times \text{Work}$$

Even the low ability students can almost always compensate by hard work. This formula was suggested to me by one of my own lecturers in mathematics. It can be generalized to:

$$\text{Result} = \text{Ability}^{x_a} \times \text{Work}^{x_w}$$

where $x_w > x_a$ or even $x_w \gg x_a$.

What this is trying to say is that increasing the amount of work can easily compensate the limited ability of a particular student. Note that in many real-life situations lack of ability is confused with lack of confidence (see [2] for a more detailed discussion on confidence in learning). Obviously in the rare cases when the mathematical ability is close to zero this cannot always be compensated for by hard work. Another factor which can contribute to the result of learning mathematics is the students' orientation to learning [3]. For example, students with personal or intrinsic academic orientation, who enjoy exploring new and challenging material are expected to get better results in mathematics than students with a vocational or social orientation. The subject of mathematics requires higher levels of concentration compared with other subjects, in general, and its immediate relevance to future students' job prospects is not at first evident. Hence, students with vocational orientation do not have much stimulus for this concentration and for them the focus on practical elements of the mathematical parts of the course is particularly important. It is more difficult to accommodate students with social orientation in designing the course without sacrificing scientific and engineering standards.

NUMBERS AND FORMULAE

If engineering students are asked to solve a simple problem of finding the temperature distribution between two parallel plates at temperatures T_1 and T_2 provided that thermal conductivity between these plates is constant, some of them might find it difficult. On the other hand, the same problem can be reformulated in numbers:

> Two parallel plates are kept at temperatures 200°C and 300°C 6 m apart. The thermal conductivity between these plates is 10 Wm^1 K^1. Calculate the temperature at the point which is 3 m from the first plate.

In this case, almost everybody will promptly answer that the temperature is equal to 250°C. The reason for this is very simple. Most engineering students think in terms of numbers rather than in terms of abstract concepts. For this reason, students who experience difficulties with simple analytical calculations, can turn out to be very good in practical applications.

This does not mean that we should avoid dealing with abstract concepts altogether for the reasons already discussed. This means, however, that every new abstract concept needs to be accompanied by plentiful numerical examples.

For example, if one just introduces Wien's law for blackbody radiation ($\lambda T = \text{const}$) and moves on to the next topic, most students just forget it by the next lecture. On the other hand, if a lecturer spends some time illustrating this law by practical examples then it eventually registers. In other words, referring to referential aspects of students' experience, numbers lead the engineering students from surface to deep knowledge (see [4, 5]). Referring to organisational aspects of their experience, numbers help make the transition from an atomistic cognitive approach to a holistic approach, that is, students start understanding the problem as a whole, rather than concentrate on its parts [5, 6]. Note that the reverse process takes place in the mathematical students community: deep and holistic approaches are related in most cases to their concentration on formulae, while surface and atomistic approaches appear at the stage of working with numbers (this observation is based on my own experience as a student, and the discussion of the matter with other students).

ALGEBRAIC EQUATIONS

When I started my course I assumed that the students felt comfortable with algebraic manipulations. After the first few lectures, however, I understood that this assumption does not always hold. None of the students have problems in solving the equation $ax = b$ and obtain the solutions $x = b/a$. However, if the same equation is written in a slightly more complicated way, say, as:

$$\alpha\phi^2 x + \beta x = \gamma$$

then its solution $x = \gamma/(\alpha\phi^2 + \beta)$ causes difficulties among students if written straightaway.

Instead the left hand side of this equation needs to be rearranged as:

$$\alpha\phi^2 x + \beta x = (\alpha\phi^2 + \beta)x = \delta x = \gamma$$

where $\delta = \alpha\phi^2 + \beta$, before its solution $x = \gamma/\delta$ is written. This normally takes just a few minutes, but if this is not done, then for many students the whole lecture may be lost.

Another problem with algebraic equations is the notation. Whether we like it or not most students tend to memorise equations in a particular notation. Say, if the distance is indicated as s in one lecture, then this notation should be kept until the end of the course. I tried hard to persuade students to understand the structure of the equations rather than to memorize the notation (adopt deep rather than surface learning) but I had little success with most of the group. My conclusion is that notation needs to be unified to avoid any confusion especially among the students who are at a novice level of skill acquisition [7].

Finally, any sloppiness in the presentation of algebraic equations must be avoided by all means. Students do not easily recognize even the most obvious printing mistakes and become stuck. On many occasions they tend to memorize and reproduce wrong equations. The best solution to the problem is to avoid sloppiness altogether. If a mistake is found after the lecture it needs to be explicitly admitted afterwards and not glossed over. The lecturer's handwriting is also very important. One example in my experience is when one of my students copied the angle of attack of an aeroplane as 80 instead of 8° As a result, he effectively dropped out of that particular lecture. I believe that the best way to tackle the handwriting problem is to print formulae using Latex software and show them to students using transparencies.

DIFFERENTIAL EQUATIONS

Differential equations, even relatively simple ones, seem to be a stumbling block for many students. My experience suggests that the simplest way of tackling this problem is to avoid it altogether by guessing the solution rather than solving the equation.

For example, if we take the equation:

$$\mathrm{d}^2y/\mathrm{d}x^2 = 0$$

then it should be just proven that:

$$y = C_1 x + C_2$$

is its solution by direct substitution of this expression for y into the original equation. Another approach to the problem is to rewrite the original equation as:

$$\mathrm{d}z/\mathrm{d}x = 0$$

where $z = \mathrm{d}y/\mathrm{d}x$. Then it should be explained to the students that the derivative of a constant is zero, so that the solution of the equation for z can be written as:

$$z = C_1$$

Remembering the definition of z, this equation can be rewritten as:

$$\mathrm{d}y/\mathrm{d}x = C_1$$

Then students can be explained that derivative of:

$$y = C_1 x + C_2$$

is equal to C_1. Hence $y = C_1 x + C_2$ is the solution of the original equation.

Based on my experience, this rather lengthy approach to the problem pays off and the students begin to understand more complicated types of differential equations and their application to engineering problems. Note that on many occasions students are not confident in the concept of derivative itself. One teaching method of introducing this concept is based on the discussion of velocity as a 'natural derivative' [8, 9].

VECTORS AND TENSORS

Even simple manipulations with vectors, such as summation and subtraction can cause problems if students are not prepared for them. A one-dimensional problem could be a good starting point. One can consider the problem of calculating the velocity relative to the platform of two passengers in a moving train walking in opposite directions to each other inside this train. This problem can be easily visualised and students can recall their own experience. Students should really become very confident with this simple problem before they move on to the problem of summation and subtraction of vectors in three-dimensional space.

The product of a vector and a scalar and the scalar product of vectors do not cause too many problems if they are explained in detail. The vector product is often a stumbling point. In practice it seems to be more efficient to deal with the components of the vector product rather than with the general equation written in vector form (cf. the Gestalt theory as discussed by Laurillard [10]).

The basic concepts of vector analysis such as gradient, divergence and curl can look rather intimidating to some of the students. They can be introduced if necessary, but it seems better to avoid any general manipulations. Instead, the analysis can be focused on simple limiting (one or two dimensional) cases when the expressions for gradient, divergence or curl can be presented in simple forms.

Tensor analysis is normally excluded from the engineering curriculum altogether. This is regrettable since tensor is an essential and powerful concept for the analysis of many engineering and physical phenomena. General formulation of tensor relations for engineering students seem to be too complicated. Tensors, however, can be introduced as a set of numbers in vector equations.

For example, instead of writing the Ohm's law in a tensor form

$$\mathbf{j} = \hat{\sigma}\mathbf{E} = \begin{Bmatrix} \sigma_{xx} & \sigma_{xy} & \sigma_{xz} \\ \sigma_{yx} & \sigma_{yy} & \sigma_{yz} \\ \sigma_{zx} & \sigma_{zy} & \sigma_{zz} \end{Bmatrix} \mathbf{E}$$

one can write the expression for the components of electric current density, $\mathbf{j}$, as:

$$\left.\begin{aligned} j_x &= \sigma_{xx}E_x + \sigma_{xy}E_y + \sigma_{xz}E_z \\ j_y &= \sigma_{yx}E_x + \sigma_{yy}E_y + \sigma_{yz}E_z \\ j_z &= \sigma_{zx}E_x + \sigma_{zy}E_y + \sigma_{zz}E_z \end{aligned}\right\}$$

It can be explained to the students that the components of the tensor $\hat{\sigma}$ are just coefficients in the latter system of equations. In other words, in order to see the whole tensor we concentrate on its individual elements (as in the case of the vector product, here we again effectively deal with the Gestalt theory).

TEACHING METHODS

Formal lectures

In spite of all criticism of formal lectures (see [4]) they still remain the main teaching method used. Formal lectures alone are not particularly effective in teaching mathematics because of a number of reasons. Mathematical courses are built in such a way that if a student misses a key concept at the beginning of the lecture, the rest of the lecture can be lost for him or her. Besides, every student has his or her own pace of acquiring mathematical knowledge. The pace of the lecture can be too slow for some students and too quick for others. Finally, formal lectures can deprive students from using their initiative, encouraging surface/atomistic rather than deep/holistic learning (see [11] for the discussion on this subject).

The obvious way of overcoming many of these problems is to give students motivation [5] although in practical terms this can often be difficult to achieve. Alternatively, these problems can be resolved if formal lectures are used in conjunction with other teaching methods. These may include compulsory reading of the recommended textbooks, detailed handouts, small group teaching and formative assessment. All these methods, which I have implemented into various parts of my course, will be briefly discussed in the following subsections.

Compulsory reading

Recommending a particular book as complementary reading often proves to be inefficient. Many students ignore this recommendation and try to base their learning exclusively on the lecture notes. Others start reading but become stuck somewhere and eventually drop the book altogether. It seems that the most efficient way ahead is to link specific sections of the recommended book with the corresponding parts of the lecture course. This can be explicitly indicated in handouts as compulsory reading for a particular topic. Part of the lecture time can be spent on giving an overview of the book material and highlighting the most important concepts. I adopted this approach and it was well received by the majority of students. The main problem with this reading was unavailability of books in the library and their high cost for students in the bookshops.

Handouts

Handouts are particularly relevant for teaching mathematics. They save a lot of students' time which would otherwise be spent on copying formulae from the whiteboard. Also they prevent unavoidable mistakes during this process. In most cases I used the handouts as printed copies of overheads (4 or even 8 overheads can be copied onto a double-sided A4 page). They also included some extra material, not presented in standard textbooks for compulsory reading.

There are two main pitfalls with handouts. Firstly, overly detailed handouts discourage some students from active participation in the lecture. This can even make them disruptive. Secondly, the handouts may sometimes restrict the flexibility of the lecture.

These problems were avoided when the handouts presented the highlights of the lecture (key formulae) rather than a detailed presentation of the contents of the lecture.

Small-group teaching

Various methods of small-group teaching have been extensively discussed (e.g. [11, 12]). Unfortunately the scope of application of these methods for teaching mathematics to engineering students is rather limited in practice. In what follows I will briefly describe just three methods, based on my own recent experience.

Individual learning under supervision. The idea of introducing this method came along when I planned the work of the third year students in Computational Fluid Dynamics (CFD) laboratory on learning how to solve problems using CFD packages. The backgrounds of the students varied considerably. Some of them had spent a year working with CFD packages and were very confident in using them. Others had no idea about these packages at all. Any centralized tuition of this group of students, say via asking them to perform operations with the code simultaneously, would have been inappropriate. Instead, I have prepared detailed handouts, describing the tasks expected to be performed by the students, and allowed them to work at their own pace. I stayed in class during the whole period of students' work

Table 1

Assessment of the CFD laboratory	Number of students	Percentage
Useful	6	32
Above average	6	32
Average	2	10
Below average	2	10
Unhelpful	3	16

but intervened in this work only when I was asked to do so. When there were several students seeking for help simultaneously then I was helped by other students who were strong in the subject. There were no general breaks during the three hours session, but the students were allowed to leave the room and to return at any time provided that they do not produce noise. At the end of the work they prepared the reports which were individually assessed.

Since this approach to teaching was not traditionally used, students were asked for their feedback comments. Their answers are summarised in Table 1.

The questionnaires were anonymous, but from the following discussions with students an impression was gained that the unhelpful verdict was made by the most experienced students who thought that they did not gain much of this laboratory. In future I plan to use the same approach to teaching the second year students when their computer experience is about at the same level. Also I plan to introduce more advanced tasks as options to challenge the strongest students.

Individual tuition. Some sort of individual tuition is inevitable because of the students' different abilities and backgrounds. The best way of doing this is to encourage students to come with their problems to the lecturer's office. Sometimes 5–10 minutes discussion can put students back on track. This time spent with students can also be used to receive additional feedback from the class. Another way of promoting individual tuition is to come to the classroom a bit earlier, and not to be in a hurry to leave it. This encourages some students to approach to the lecturer with their problems.

Fishbowl. It is sometimes useful to invite one of the students to the whiteboard to solve a particular problem. This turns into individual tuition in front of the class. The class, however, almost never remains passive. As a result a productive dialogue is established with the whole group. We can get immediate feedback from the class and this also keeps students alert, since anybody can be called to the whiteboard. Fishbowl can be practised in conduction with so called 'quiet' time when everybody is encouraged to think the problem through in silence.

Consultation hour. The idea behind the consultation hour was to provide an extended version of individual tuition. During this hour I had no preplanned activities and just stood in front of the class taking questions from the audience. Students were encouraged to enter the room and leave it at any time during this hour provided that they kept quiet. For example, there was no point for somebody to stay in the room while I answer a question which is well understood by him or her. Originally this consultation hour was designed to help the weakest part of the class. In practice, however, it was attended mainly by the most hard working students.

Formative assessment. The main problem with assessments in British Universities is that they are in most cases summative rather than formative [14–17]. Students sometimes have little or no idea of why they receive a particular mark. Moreover, they often do not have a clear understanding of what is expected from them at the exams.

I have attempted to overcome these problems by introducing spot tests for my first year students. There were three spot tests during the term. Most of them were suggested during the last 20 minutes of the lectures. Students were asked to solve a problem in class and hand the solution over to me for assessment. They were allowed to use books and any notes, but they were not allowed to communicate between themselves. Sometimes the suggested problems were taken from their homework. This, as well as the fact that spot tests were offered without warning, was intended to encourage students to work consistently during the term rather than just before the exams. If students missed spot tests without good reason then they received zero mark. The averaged mark for the spot tests contributed to the final mark for the subject. The main benefit in the spot tests, in my view, was not in this mark, but in the individual comments which I made in students' papers. In these comments I tried to explain in detail what was wrong and how this could be improved. My aim was to justify each individual mark using the same marking principles as in actual exams. Since this work was very time consuming, some cut-corners were inevitable and hence this aim was only partially achieved. This, perhaps was reflected in the feedback forms from the students when they were asked to comment on the usefulness and fairness of this assessment. The students' replies are summarised in the Tables 2 and 3.

Since the feedback forms were anonymous, and students avoided writing specific comments I can only guess about the reasons behind their comments. I strongly suspect that those students who found my assessment unhelpful were the weakest students who submitted totally wrong solutions in which case I usually wrote no comments at all. Note that there were relatively few students who found my assessment fair. I believe that this attitude could be overcome if I worked out more

Table 2

Assessment of the spot tests	Number of students	Percentage
useful	6	25
Above average	8	33
Average	3	13
Below average	2	8
Unhelpful	5	21

rigorous assessment criteria used both in spot tests and at the exams. I plan to do this.

SELF-ASSESSMENT

The success of the mathematics teaching can be gauged by some form of assessment. The traditional assessment, via examinations, may be not particularly informative, since the students have a right not to take the mathematically demanding questions (see also the discussion of this question by Carter [15]). I find that more information about the students' knowledge could be obtained via self-assessment rather than external assessment. The main restriction of this method is that it has to be applied to a group of students as a whole rather than to individual students. One cannot expect honest answers from the students unless the feedback forms are anonymous.

When selecting questions for self-assessment forms a model of skills acquisition is needed. One of these models, known as Dreyfus model, considers 5 levels of skills acquisition: novice, advanced beginner, competent, proficient and expert [7]. Although this model can be very useful for studying professional growth in the whole range from novice to expert, it seems to be of limited usefulness for the analysis of students' skills acquisition when their knowledge is not expected to rise above 'competent'.

I have adjusted to my needs the self-assessment form suggested by Walker [18] and applied it to the assessment of students' knowledge of a number of concepts in my course, including mathematical concepts. The items assessed by the first year students are given in Table 4. The items assessed by the third year students were slightly different from the ones shown in Table 4, although most of the concepts (e.g. derivative, integral) remained the same. The assessment forms were given to the students at the beginning and at the end of the course.

Table 3

Assessment of the spot tests	Number of students	Percentage
Fair	4	17
Above average	9	39
Average	6	26
Below average	2	9
Unfair	2	9

Table 4

Item	*A (%)*	*B (%)*	*C (%)*	*D (%)*
Derivative	0 (4)	4 (13)	48 (50)	48 (33)
Partial derivative	4 (8)	4 (13)	48 (46)	44 (33)
Integral	0 (0)	0 (4)	37 (58)	63 (38)
Divergence	8 (25)	44 (25)	32 (33)	16 (17)
Gradient	0 (4)	0 (14)	22 (23)	78 (59)
Curl	31 (59)	46 (29)	23 (8)	0 (4)
Vector	0 (0)	0 (8)	19 (50)	81 (42)
Tensor	31 (42)	50 (46)	8 (8)	11 (4)
Logarithm	4 (4)	0 (14)	40 (64)	56 (18)
Dynamic viscosity	0 (21)	7 (62)	41 (13)	52 (4)
Kinematic viscosity	0 (42)	22 (37)	41 (17)	37 (4)
Static pressure	0 (4)	8 (46)	48 (46)	44 (4)
Dynamic pressure	0 (8)	11 (50)	52 (38)	37 (4)
Total pressure	0 (4)	0 (38)	41 (50)	59 (8)
Force	0 (0)	0 (4)	22 (38)	78 (58)
Velocity	0 (0)	0 (4)	22 (39)	78 (57)
Non-Newtonian fluid	8 (42)	19 (50)	35 (8)	38 (0)
Density	0 (0)	4 (9)	33 (48)	63 (43)
Turbulence	0 (17)	7 (33)	56 (42)	37 (8)
Archimedes' principle	0 (42)	30 (21)	44 (25)	26 (12)
Pascal's law	0 (17)	15 (42)	67 (37)	18 (4)
Reynold's number	0 (42)	7 (42)	52 (12)	41 (4)
Euler's equation	11 (67)	30 (25)	48 (4)	11 (4)
Bernolli's equation	0 (13)	11 (20)	33 (54)	56 (13)
Darcy's law	0 (75)	15 (21)	35 (0)	50 (4)
CFD	45 (92)	29 (4)	13 (0)	13 (4)

The numbers in brackets are the percentages of the first year students who ticked columns A (I have never heard of this), B (I have heard of this but do not know what it means), C (I have some idea of what this means) and D (I know this and could explain to someone else) at the beginning of the course. The numbers next to them are the percentages of students who ticked those boxes at the end of the course. There are some fluctuations due to absences on the assessment days.

As can be seen from this table, the progress in learning new physical concepts such as Archimedes' principle, Bernoulli equation or Darcy law is obvious. For example, most of students had never heard about Darcy law at the beginning of the course, while at the end of the course almost half of students were ready to explain the concept to somebody else. At the same time the students' progress in learning mathematical concepts turned out to be far less visible (this could partly be attributed to the fact that since my subject was thermofluids I could spend only a limited amount of time on discussing mathematical concepts). This confirms my observation from the work in the class. The progress of engineering students' learning of abstract concepts was far slower than the progress in learning the concepts more closely related to real life experiences.

Broadly the same conclusions can be drawn from the self-assessment forms from the third year students. In Table 5 only the results of the mathematical part of the self-assessments form for the third year students are reproduced. Note that

Table 5

Item	*A (%)*	*B (%)*	*C (%)*	*D (%)*
Derivative	0 (0)	5 (0)	32 (33)	63 (67)
Partial derivative	0 (0)	5 (0)	42 (48)	53 (52)
Integral	0 (0)	5 (0)	26 (43)	69 (57)
Divergence	0 (5)	26 (29)	53 (52)	21 (14)
Gradient	0 (0)	26 (9)	53 (43)	21 (48)
Curl	5 (14)	42 (52)	42 (24)	11 (10)
Vector	0 (0)	16 (5)	42 (52)	42 (43)
Tensor	26 (52)	32 (29)	26 (19)	16 (0)
Circulation	5 (0)	32 (50)	53 (45)	10 (5)

occasionally students were confused about the meaning of 'B' and 'C', ticking 'B' when they meant 'C' and *vica versa* (this is one of the drawbacks of this form). Also there was a slight difference between the groups of students who submitted the forms at the beginning and at the end of the term. This might explain the appearance of a student who did not know the meaning of the derivative, partial derivative and integral at the end of the course. The apparent reverse in their level of understanding of the concept of gradient is that at the beginning of the course they thought of the common sense meaning of this term rather than of the vector analysis concept. Note that here the phenomenographic analysis of students' perception of the concepts would be appropriate [5]. In this case the categories A, B, C and D would be replaced by other categories which would be derived directly from students' comments on the concepts (see also [19] for the discussion on this subject). I plan to experiment with this approach in the future.

The progress of the third year students' understanding of mathematical concepts when compared with the first year is obvious but it is not as decisive as one would like it to be.

The results obtained from these self-assessment forms look a bit pessimistic. The existing system of teaching mathematics to engineering students does not prepare specialists who are good enough in working with abstract mathematical concepts. Some thoughts and practical experiments to improve this system have been discussed in this paper. More work, however, is still needed. One of the directions of this work might include the implementation of the elements of active learning into the teaching of mathematics (see [20]). This active learning has much in common with the research seminars linked learning in Russian Universities as discussed in my previous paper [11]. The discussion of this topic, however, is beyond the scope of the present paper.

CONCLUSION

Engineering students need to have it explained to them why knowledge of mathematics is essential for their future practical work. Mathematics should be regarded as a language for expressing physical, chemical and engineering laws. All general equations should be illustrated by practical numerical examples in order to encourage a deep/holistic approach to students' learning. Necessary steps in the manipulation of algebraic equations should be highlighted. Rigorous solutions of differential equations can be replaced by checking that a particular function satisfies this equation. It is beneficial to start studies of vector properties with a simple one-dimensional case using the Gestalt theory. Formal lecturers should be supplemented by compulsory reading, handouts, elements of small group teaching and formative assessment. The analysis of self-assessment forms completed by students show that their progress in understanding physical concepts is much more visible than their progress in understanding mathematical concepts.

Acknowledgements—This paper is presented as a special study project for the Postgraduate Certificate in Teaching and Learning in Higher Education. I am indebted to the members of the active learning set Jon Wright, Jacky Bentley, Becky Elmhirst, Mike Hill and especially to the facilitator of this set Liz Beaty for their very useful and helpful comments and suggestions. Special thanks to John Kearsey, my senior colleague, for helping me when I was starting to teach thermofluids to engineering students, and for his continuing support.

REFERENCES

1. R. W. Douglass and J. D. Ramshaw, Perspective: future research directions in computational fluid dynamics, *Trans. ASWE,* **116**, pp. 212–215 (1994).
2. A. Morgan and L. Beaty, The world of learner, in *The Experience of Learning* (ed. F. Marton, D. Hounsell and N. Entwistle), pp. 217–237 (1997).
3. L. Beaty, G. Gibbs and A. Morgan, Learning orientation and study contracts, in *The Experience of Learning* (ed. F. Marton, D. Hounsell and N. Entwistle), pp. 72–86 (1997).
4. G. Gibbs, *Twenty Terrible Reasons for Lecturing,* Oxford Politechnic (1981).
5. F. Marton and R. Säljö Approaches to learning, in *The Experience of Learning* (ed. F. Marton, D. Hounsell and N. Entwistle), pp. 39–58 (1997).
6. L. Svensson, Skill in learning and organising knowledge, in *The Experience of Learning* (ed. F. Marton, D. Hounsell and N. Entwistle), pp. 59–71 (1997).
7. M. Evans, *Developing Professional Knowledge and Confidence,* Falmer Press (1994).
8. B. Aurell, T. Ganelius, W. Kilborn and L. Skoogh, Mathematics for the Upper Secondary School, *Natural Sciences and Technology,* Almqvist och Wiksell, Stockholm (in Swedish) (1970).
9. L. O. Dahlgren and F. Marton, Students' conceptions of subject matter: an aspect of learning and teaching in higher education, *Studies in Higher Education,* **3** (1), 25–35 (1978).

10. D. Laurillard, Styles and approaches in problem solving, *The Experience of Learning* (ed. F. Marton, D. Hounsell and N. Entwistle), pp. 126–144 (1997).
11. G. Gibbs, Improving the quality of student learning, manuscript (1992).
12. S. S. Sazhin, Small group teaching in Russian Universities, *Higher Education Review,* **25** (3), pp. 66–73 (1993).
13. S. Brown, The art of teaching small groups 1, *The New Academic,* **5** (3), pp. 3–5 (1996).
14. I. Hextall, Marking work, in *Exploration in the Politics of School Knowledge,* (Ed. Whitty, G. and Young, M.) Nafferton Books, Driffield, pp. 65–74 (1976).
15. C. Carter, Higher Education for the Future, in *Why and How we Examine,* Blackwell, Oxford, Ch 7, pp. 83–92 (1980).
16. P. Broadfoot, Alternative to public examinations, in *Assessing Educational Achievements,* (ed. D. Nuttall) Lalmer Press, Lewis, pp. 54–77 (1986).
17. S. S. Sazhin, Assessment in Russian universities, *Higher Education Review,* **26** (2), pp. 9–16 (1994).
18. G. Walker, What students think they know, *The New Academic,* **5** (3), pp. 5–7 (1996).
19. E. Taylor, G.. Gibbs and A Morgan, The outcomes of learning from the social science foundation course: students' understanding of price control, power and oligopoly, *Study Methods Group Report No. 9,* The Open University (1981).
20. I. McGill and L. Beaty, *Action Learning*, Kogan Page Ltd (1995).

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