

Vibration-mode Shape Visualization with a Time Average TV Holography System*

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A new experiment of vibration-mode shape visualization for undergraduate and graduate courses of applied optics and structural mechanics is developed. A full-field optical measurement method is used to replace the traditional point-wise one. The experimental method allows real-time studies of resonant vibrations. Computer simulation of resonant vibrations is also shown with live display by a visualization software.

SUMMARY OF EDUCATIONAL ASPECTS OF THIS PAPER

1. This paper presents a new experiment for courses in laser applications and structural mechanics.
2. Postgraduate and undergraduate students of the mechanical engineering department are taught on this course.
3. The course is for senior year students.
4. The experimental methods allow real-time full-field studies of vibration shapes. Also, computer simulation of resonant vibrations is shown with live display by visualization software.
5. The material is used as an experiment or a demonstration tool for subjects of vibration and applied optics.
6. Reference book for presented materials: Robert Jones and Catherine Wykes, *Holographic and Speckle Interferometry*, Cambridge University Press (1989). Research paper for the presented materials: W. O. Wong, K. T. Chan and T. P. Leung, Contrast and sensitivity pattern interferometry: effect of variations of force level, *Optics and Laser Technology*, **29**, (1997) pp. 179–185.
7. Most of the students are impressed by the visual effect of the optical measurement and the computer simulation. It makes learning of vibration and optics more interesting. The use of modern measurement and presentation tools may benefit modern engineering education.

INTRODUCTION

THE USE OF computer simulation and visualization experiment has been reported as an effective means to improve understanding and motivate intuition in the study of structural

vibration [1–3]. Traditional methods use accelerometers and signal analyzers for vibration measurement. While those methods are precise and reliable for measuring modal parameters for structural modeling, they are pointwise, time consuming and contacting. Optical vibrometers are the modern alternative to accelerometers. Though non-contacted, they are point-wise instruments.

Holographic interferometry has been a well-known non-contacting technique for vibration measurement in full-field. It was prevented by dark-room film development practice from wide applications. However, the principle has been employed to develop dry type interferometry, known as TV holography or Electronic Speckle Pattern Interferometry (ESPI) [4]. TV holography is now extended to the development of computer-aided (digital) TV holography which consists of holographic optics, a CCD camera and an image processing system. This new development has enabled a convenient real-time, non-contact and full-field measurement method of vibration mode shape [5, 6]. TV holographic-based methods of measuring vibration have several techniques, viz. time-averaged technique, stroboscopic technique or pulsed laser technique. Time-average mode of TV holography was chosen for the experiment for its simpler and cheaper measurement system.

THEORY OF MEASUREMENT

In Fig. 1 a schematic diagram of the experimental setup of a digital TV holography system arrangement is shown. Theory of time-average TV holography system can be found in texts such as [9]. A brief description of the measurement theory is shown as follows. An expanded continuous wave laser beam is projected onto a thin circular metal plate of optically rough surface with clamped boundary. The imaging lens of the CCD camera collects part of the scattered wave from the surface

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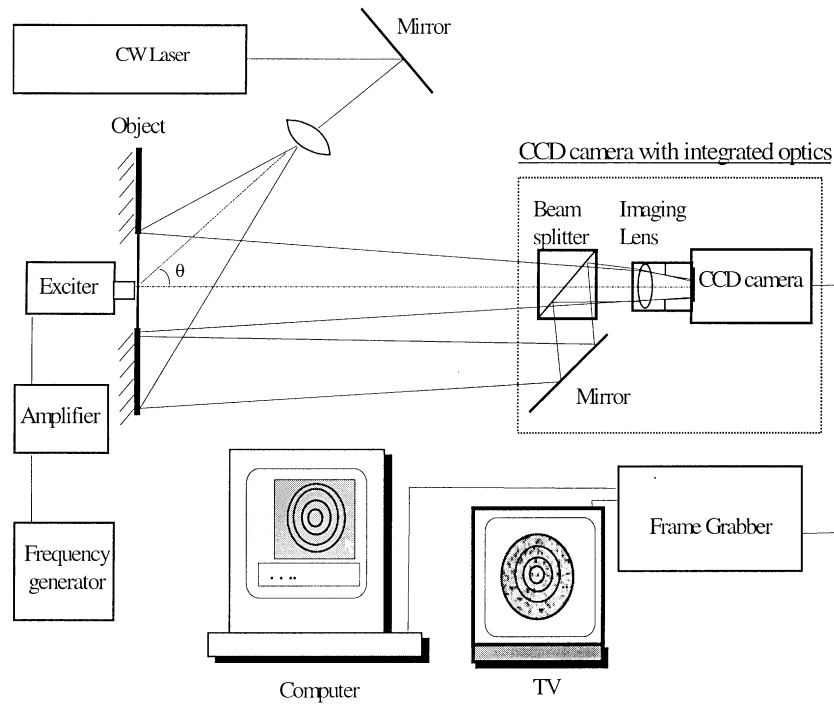


Fig. 1. Schematic drawing of the time-average TV holography setup.

of the object and imaged onto the CCD sensor. This forms the object beam. A part of the rigid support of the object scattering the laser light is also imaged onto the CCD sensor via a mirror, a beamsplitter and the imaging lens of the camera. This forms the reference beam. A speckled reference beam [7, 8] is used so that the setup is simpler and less sensitive to ambient vibrations than the traditional TV holography system with smooth reference beam. The output of the CCD sensor is the intensity values of the interference between the object and the reference waves. It may be written as [9]:

$$I_s = I_r + I_o + 2\sqrt{I_r I_o} \cos(\phi_o - \phi_r) \quad (1)$$

where all the variables depend on the pixel position on the CCD sensor; I_r is the reference-beam intensity; I_o is the intensity of object's scattered waves; ϕ_o and ϕ_r are the phases of the object and reference light beam respectively. If the object is deformed and a point P on the object surface has an out-of-plane displacement w , the object beam will contain an additional phase change

$$\Delta = \frac{2\pi}{\lambda} w(1 + \cos \Theta) \quad (2)$$

where Θ is the angle between the illumination and observation directions of the object. The total intensity at the CCD camera then becomes:

$$I_d = I_r + I_o + 2\sqrt{I_r I_o} \cos(\phi_o - \phi_r + \Delta) \quad (3)$$

If the object is vibrating at the n^{th} natural

frequency ω_n , the out-of-plane displacement w in equation (2) may be written as:

$$w = w_o \cos(\omega_n t + \phi) \quad (4)$$

where ϕ is the vibration phase and w_o is the vibration amplitude at point P. At resonant vibration, the vibration phases of the surface points will be either in-phase or 180° out-of-phase. For normal illumination and viewing, $\Theta \approx 0^\circ$, equation (2) may be rewritten as:

$$\Delta = \frac{4\pi w_o}{\lambda} \cos(\omega_n t + \phi) \quad (5)$$

If the vibration frequency ω_n is much higher than the grabbing rate of the imaging system,

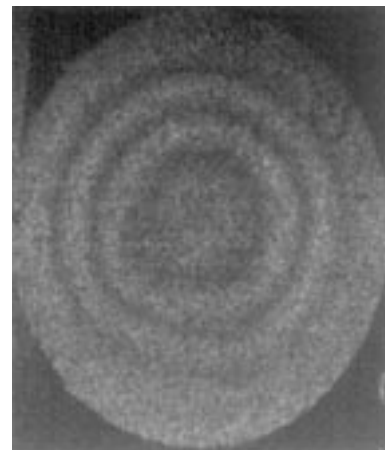


Fig. 2. Time-average fringe pattern of a circular plate with the clamped boundary vibrating at its 1st resonance.

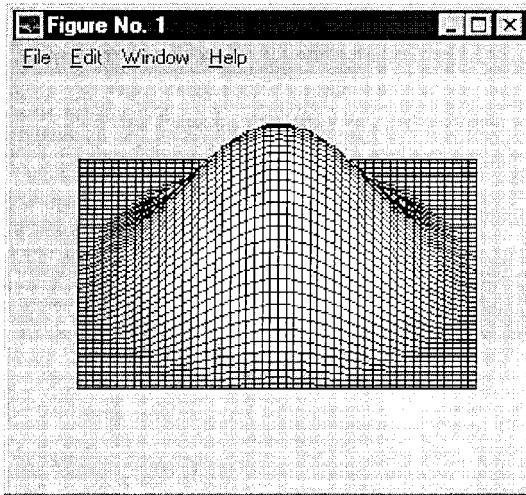


Figure 3a

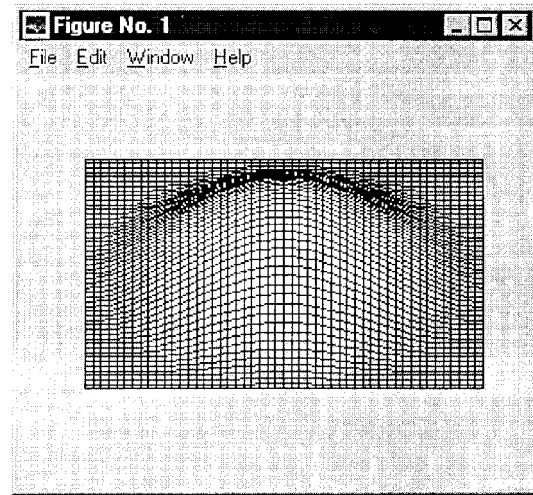


Figure 3b

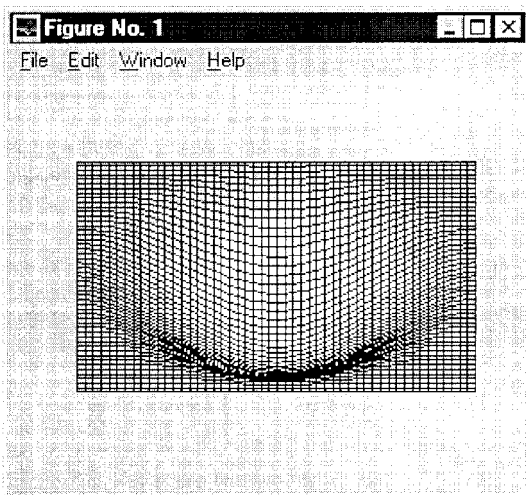


Figure 3c

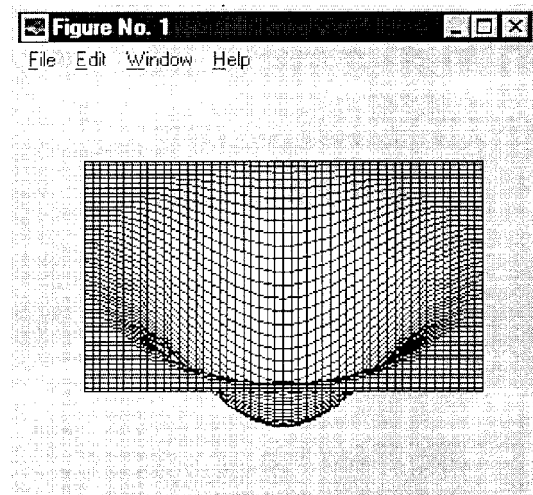


Figure 3d

Fig. 3. Snapshots of a computer-generated movie animating the vibration of a circular plate with clamped boundary vibrating at its 1st resonance.

the intensity distribution recorded by the CCD camera would be the average intensity value expressed by:

$$\begin{aligned}
 I_{avg} &= \frac{1}{T} \int_0^T I_d dt \\
 &= \frac{1}{T} \int_0^T \left\{ I_o + I_r + 2\sqrt{I_o I_r} \right. \\
 &\quad \left. \times \cos \left[\phi_o - \phi_r + \frac{4\pi w_o}{\lambda} \cos(\omega_n t + \psi) \right] \right\} dt \\
 &= I_o + I_r + 2\sqrt{I_o I_r} \cos(\phi_o - \phi_r) J_0 \left(\frac{4\pi w_o}{\lambda} \right)
 \end{aligned}
 \tag{6}$$

A common process of fringe generation in



Fig. 4. Time-average fringe pattern of a circular plate with clamped boundary vibrating at its 2nd resonance.

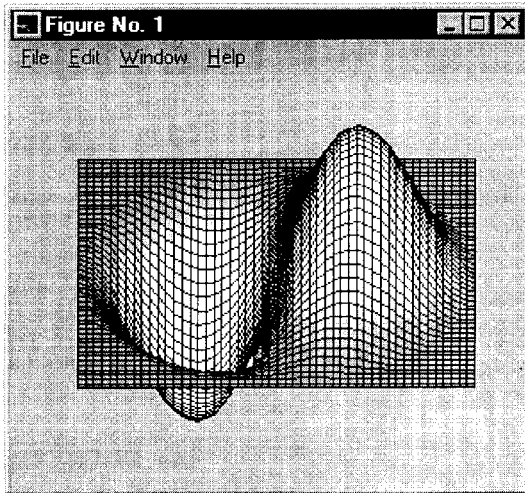


Figure 5a

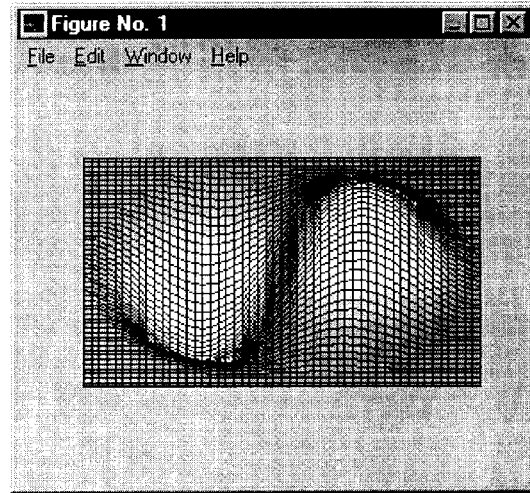


Figure 5b

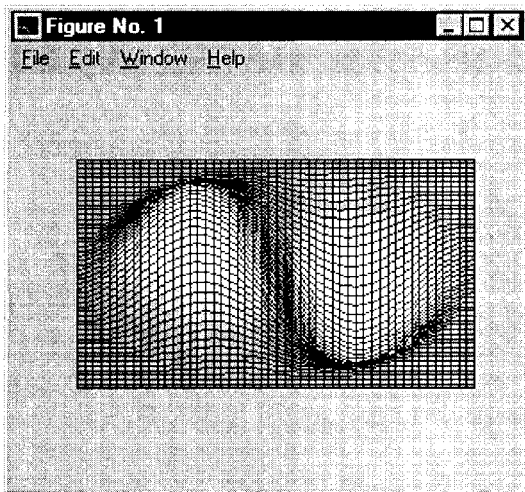


Figure 5c

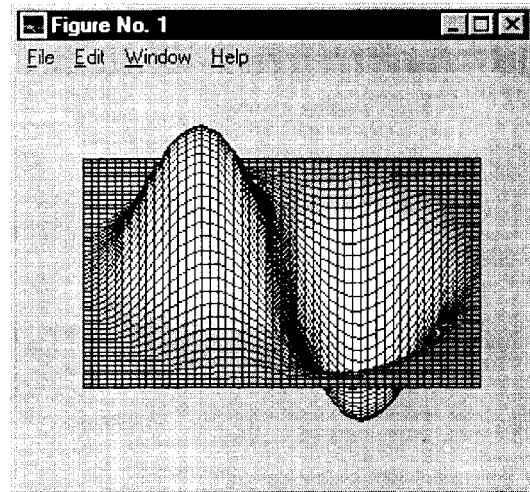


Figure 5d

Fig. 5. Snapshots of a computer-generated movie animating the vibration of a circular plate with the clamped boundary vibrating at its 2nd resonance.

time-average speckle pattern interferometry is to subtract two speckle patterns to yield correlation fringes which correspond to the vibration amplitude of an object undergoing steady-state vibration. The advantage of the subtraction process is the removal of the self-interference terms which enables direct observation of the zeroth-order Bessel fringes.

A common way to generate a fringe pattern to depict a contour map of the vibration amplitude is to subtract the time-averaged data from a reference frame recorded with the object at rest. the resultant image will have the intensity described by subtracting equation (6) from equation (1):

$$I_s - I_{avg} = 2\sqrt{I_o I_r} \cos(\phi_o - \phi_r) \left[1 - J_o \left(\frac{4\pi w_o}{\lambda} \right) \right] \quad (7)$$

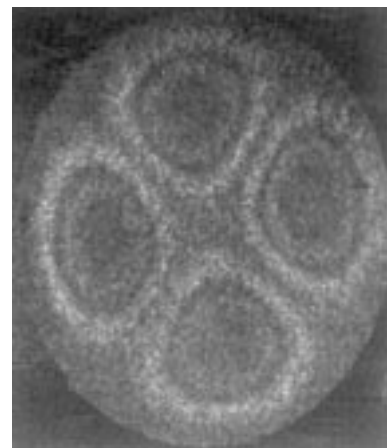


Fig. 6. Time-average fringe pattern of a circular plate with clamped boundary vibrating at its 3rd resonance.

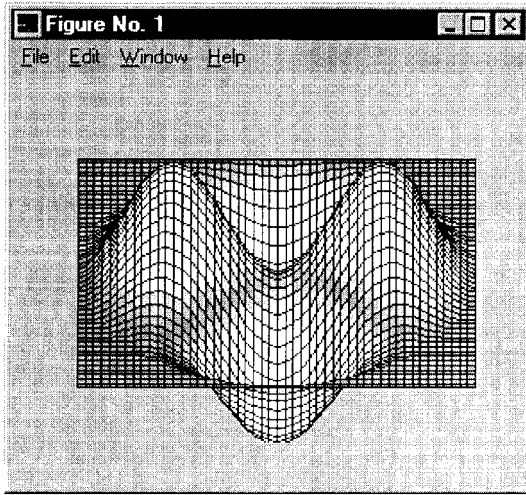


Figure 7a

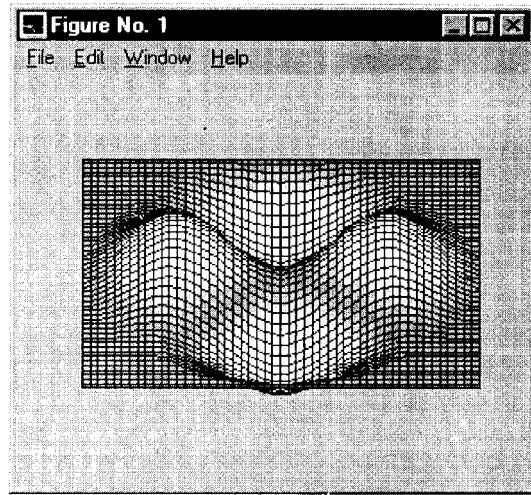


Figure 7b

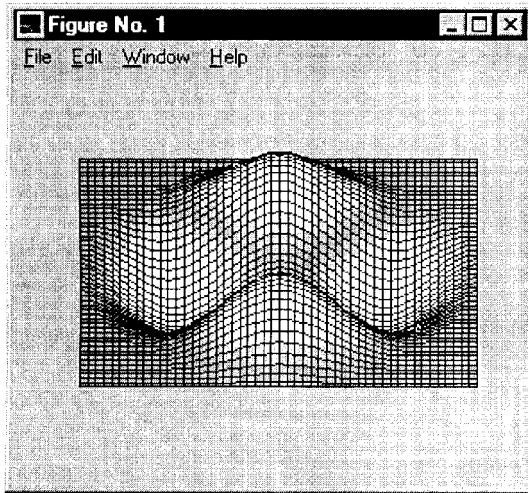


Figure 7c

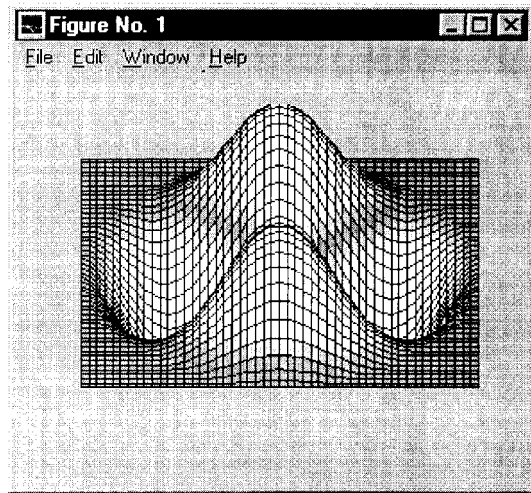


Figure 7d

Fig. 7. Snapshots of a computer-generated movie animating the vibration of a circular plate with clamped boundary vibrating at its 3rd resonance.

Consider a frame subtraction between two time-average speckle patterns of the object at two different force levels with the vibration amplitudes:

$$w_1(x, y, t) = r_1 \varphi_n(x, y) \cos \omega_n t \quad (8a)$$

$$w_2(x, y, t) = r_2 \varphi_n(x, y) \cos \omega_n t \quad (8b)$$

where r_1 and r_2 are amplitudes determined by the force levels used, and φ_n is the n^{th} normalized mode shape. the resultant image will have the intensity described by

$$I_{avg1} - I_{avg2} = 2\sqrt{I_o I_r} \cos(\phi_o - \phi_r) \times \{J_o[r_1 \varphi_n(x, y)] - J_o[r_2 \varphi_n(x, y)]\} \quad (9)$$

It can be seen that equation (7) can be treated as a special case of equation (9) when r_1 is zero. As shown in [5], the fringe pattern described by equation (9) has higher fringe contrast and measurement sensitivity than the one described by equation (7). Any continuous system may be used for the experimental demonstration. A circular plate was chosen as the testing specimen because the visualization result is more impressive and the mode shape is easily predictable. From plate theory [10], the equation of motion of a thin vibrating plate may be written as:

$$\frac{Eh^3}{12(1-\nu^2)} \left(\frac{\partial^4 w}{\partial x^4} + 2 \frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4} \right) + \rho h \frac{\partial^2 w}{\partial t^2} = 0 \quad (10)$$

where E , ν , h , ρ and w are the Young's modulus, Poisson's ratio, thickness, mass density and traverse deflection of the plate respectively. x and y are the spatial variables and t is the temporal variable.

For a thin circular plate, equation (10) may be rewritten with polar co-ordinates (r, Θ) as:

$$\frac{Eh^3}{12(1-\nu^2)} \left(\frac{\partial^2}{\partial r^2} + \frac{1}{r} \frac{\partial}{\partial r} + \frac{1}{r^2} \frac{\partial^2}{\partial \Theta^2} \right) \times \left(\frac{\partial^2 w}{\partial r^2} + \frac{1}{r} \frac{\partial w}{\partial r} + \frac{1}{r^2} \frac{\partial^2 w}{\partial \Theta^2} \right) + \rho h \frac{\partial^2 w}{\partial t^2} = 0 \quad (11)$$

If the plate has a clamped boundary, the boundary conditions are:

$$w(r_o, \Theta) = \left. \frac{\partial w(r, \Theta)}{\partial r} \right|_{r=r_o} = 0 \quad (12)$$

where r_o is the radius of the plate. Assuming the traverse deflection w has the form:

$$w = w(r, \Theta) = e^{i\lambda t} \varphi(r, \Theta) \quad (13)$$

where λ is the eigenvalue leading to the natural frequencies of the plate and $\varphi(r, \Theta)$ is the eigenfunction corresponding to the mode shape of the object. Substituting equation (13) into equation (11) and solving it with equation (12), the characteristic equation can be shown as [11]:

$$J_m(\lambda) I_{m+1}(\lambda) + I_m(\lambda) J_{m+1}(\lambda) = 0; \\ m = 0, 1, 2, 3, \dots \quad (14)$$

where J_m and I_m are a Bessel function of the first kind and modified Bessel function of the first kind respectively. The eigenvalues can be solved numerically [12]. The first three roots of equation (14) are $\lambda = 3.19622$, 4.61090 and 5.90568 at $m = 0$, 1 and 2 respectively. The natural frequencies can then be calculated as:

$$\omega = \left(\frac{\lambda}{a} \right)^2 \sqrt{\frac{D}{\rho h}} \quad (15)$$

The eigenfunction can be shown as [10]:

$$w(r, \Theta) = \left[J_m \left(\frac{\lambda}{a} r \right) - \frac{J_m(\lambda)}{I_m(\lambda)} I_m \left(\frac{\lambda}{a} r \right) \right] \cos m\theta \\ m = 0, 1, 2, 3, \dots \quad (16)$$

EXPERIMENTAL PROCEDURE

In the experiment, a thin circular steel plate with clamped boundary is used for vibration-mode shape visualization. A 30 mW HeNe continuous-wave laser is used as the light source. With laser light projection as shown in Fig. 1, the image of the object at a static condition is recorded by a CCD camera connected to a 486 microcomputer. The object is excited to vibrate by an electromagnetic shaker. The excitation frequency can be increased automatically by using the sweeping mode of the frequency generator. Interferograms are generated by a real-time frame subtraction method at a video rate of 30 frames per second. They can be stored in a hard disk or displayed on a monitor. Hard copies of the fringe patterns can be printed by a video printer. The fringe patterns in Fig. 2, 4 and 6 are the enhanced fringe patterns for the first three resonant vibrations of the object using equation (9).

In order to enhance understanding of the measurement result, the vibration mode shapes are calculated with equation (16) using a computer program written in the MATLAB software environment. The dynamic motion of the object at resonant vibration can then be calculated and animated by the Movie command of the software. Live display of the animated vibration is shown side-by-side with the TV output of the mode shape measurement. Snapshots of the animated vibration of the first three vibration-mode shapes are shown in Figs 3a-d, 5a-d and 7a-d respectively. The data used in the calculation are $a = 90$ mm, $E = 200 \times 10^9$ N/m², $\nu = 0.29$, $h = 0.3$ mm and $\rho = 7900$ kg/m³. The visualization experiment is being used for the undergraduate and graduate courses of applied optics and structural mechanics.

CONCLUSION

A new experiment of vibration mode shape visualisation is developed. A full-field optical measurement method is used to replace the traditional point-wise one. It is used as an experiment or a demonstration tool for subjects of vibration and applied optics. The experimental method allows real-time studies of resonant vibrations. Computer simulation of resonant vibrations is also shown with live display to help the understanding of the experimental results. The main feature of this tool is the visual effect of the optical measurement and the computer simulation. It makes learning of vibration and optics more interesting. The use of modern measurement and presentation tools may benefit modern engineering education.

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