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Electronic Delivery of Mathematics: What is Possible with the Current Technology?*

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The advancement of communication technology is shaping the delivery of our courses. Universities are incorporating more and more on-line materials into the curricula in response to the changing students' needs. In many courses, the distinction between on-campus teaching and distance education is becoming increasingly blurred. Academics are under growing pressure to produce learning support materials that are flexible, innovative, and interactive. Mathematics educators are seriously disadvantaged: the electronic communication of mathematical content via the World-Wide-Web is rather challenging, time consuming, and the results are usually less than satisfactory. CD-ROM delivery can produce better results, but it still requires a substantial commitment from the academic developer. This paper discusses the possibilities and limitations within the current technological environment for presenting mathematical content in a flexible and interactive format, and introduces the mathematics courseware Epsilon—Calculus and Linear Algebra as an example of a multimedia product that can be developed with a modest budget. Results of a students survey are included.

INTRODUCTION

MANY EDUCATORS agree that the paradigm of instruction has to change from standardisation to customisation, from a focus of presenting material to a focus on making sure that learners' needs are met [6]. The current approaches to support learning revolve around instructional support systems which provide learning stimuli to encourage active engagement and support optimal learning [4]. Governments and universities [5] are calling for the introduction of innovative and flexible course delivery, taking advantage of the many opportunities offered by the current technology to diversify learning materials.

Academics are under growing pressure to produce learning support materials that are innovative, flexible, and interactive. Many are sceptical about the needs for flexible learning and the benefits of incorporating new technologies in the development of curricula. Mathematics instructors, and more generally, instructors in any technically based discipline, face the additional challenge of having to use technology that is not ready for the easy electronic communication of mathematical content—any development involving mathematics is time consuming and requires sophisticated technical skills.

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MATHEMATICS ON THE WORLD-WIDE-WEB

The standardisation of graphics formats and the HTML language makes the World-Wide-Web an ideal medium for the exchange of textual and graphical information across all platforms. Unfortunately, HTML does not support the encoding of mathematical expressions, and there is no widely used standard for the embedding and rendering of mathematical expressions and objects.

However, despite the limitations, mathematicians still manage to create pages for the WWW with mathematical content, but they do so with much work and poor results. The most widely used method for displaying mathematical expressions consists of embedding these as images. The main drawback is that the size and style of the mathematical expressions and the text do not match; also, the transmission of documents with many equations can be very slow. There are other methods which use special HTML coding and font sets, software, viewers, Java scripts, HTML/LaTeX converters, and plug-ins. None of these methods is completely reliable, and they require specialist technical expertise on the part of the developer, and special software on the part of the user. A list of the currently used methods with further information is available at: http://forum.swarthmore.edu/typesetting/web.choices.html.

The methods mentioned above are only for displaying mathematical objects. However, learning mathematics involves more than reading mathematics. If an electronic tool is to support and enhance the learning of mathematics, this should allow for interactive activities which involve the input, output, evaluation of mathematical expressions, and the provision of customised and informative feedback.

Fortunately there is some good news on this front. Several software companies and academics from around the world have joined forces to form the WWW Consortium (W3C) mathematics working group to discuss the problem of communicating and preserving mathematics with the evolving technologies. This group has recently developed a markup language called MathML which describes mathematical expressions in terms of their content, rather than in terms of what they should look like on the screen or printed; the language would be easy to use and would allow conversion to and from other mathematics formats such as LaTeX, and the various syntaxes used by computer algebra systems. The output capabilities would include print media (including braille), graphics displays, speech synthesisers, and any computer algebra system input. Some important progress has already been made in this respect.

The markup language MathML consists of a number of XML tags which can be used to mark up an equation; it is a low-level format for describing mathematics as a basis for machine-to-machine communication. Its first version was released in April 1998.

The current widely available browsers do not support MathML, but this markup language should eventually be rendered by browsers without the use of plug-ins. In the meantime, there are commercially available applets and plug-ins which can render MathML in place in a browser, as well as the W3C's browser and authoring tool, Amaya. Detailed information about MathML, Amaya, and the W3C is available at http://www.w3.org/.

OTHER ALTERNATIVES

As pointed above, although a mathematics markup language already exists, it will take a while before the rendering of pages written in MathML is possible without the use of plug-ins or a special browser. In the meantime mathematics educators around the world keep exploring the different media that are reliable for the flexible electronic delivery of mathematics programs.

The World-Wide-Web is still being used as a text-based communication medium, to provide information about the teaching programs, and to make possible the communication between learners and teachers. The mathematics is usually delivered in the form of worksheets based on computer algebra systems such as Derive, Maple or Mathematica. The major drawback of this approach is that students need to have access to the corresponding commercial software. In addition, although these packages offer almost unlimited opportunities for exploration and discovery, the weaker students still find it very difficult to learn with them.

Many academics are taking advantage of the modern and powerful authoring tools to prepare materials for their courses. These authoring tools are becoming increasingly easier to use; the development of attractive screens with hyperlinks and simple animations can be carried out with very little technical knowledge. Academics are supported by leading instructional designers such as G. Gery, L. J. Briggs, K. L. Gustafson, and M. H. Tillman who have made available to developers their wealth of experience, identifying strategies and techniques for the development of effective computer-based support tools [1, 2, 3].

Unfortunately, these authoring tools have not been developed with the mathematicians' needs in mind. The display of mathematical expressions is possible only as graphics; mathematical input, its evaluation, and the provision of customised and informative feedback requires extensive programming. It is possible to use a computer algebra system as a mathematical engine to perform the evaluations, but licensing and copyrights may limit the distribution and usage of any materials relying on these systems.

AN EXAMPLE

Epsilon mathematics courseware

Epsilon Mathematics Courseware—Calculus and Linear Algebra is an example of a low budget development feasible within the current technological environment [7, 8]. *Epsilon* was developed at Monash University [10] as a result of an unsuccessful search for a computer-based resource to support the university's first year engineering and science courses.

Epsilon was developed using the authoring system Toolbook II, and is available on a CD-ROM. Although it is technically possible to open this book with a web browser using the Neuron plug-in, depending on the bandwidth limitations, this could be a rather frustrating experience.

This mathematics courseware uses the principle of guided hypermedia: there is a linear structure that guides users through the different steps, but students also have the choice to navigate freely throughout the book pages. User control of pace is an important feature: longer pieces of information and procedures involving several steps are disclosed step-by-step, and the user decides when to proceed to the next step. Information is displayed only when the user is ready for it. Full solutions to all exercises are also available. The feedback to user inputs consists of simple *right* and *wrong* evaluations; however, the user has access to full solutions in all cases. In exercises involving multiple steps, the evaluation is done at each step.

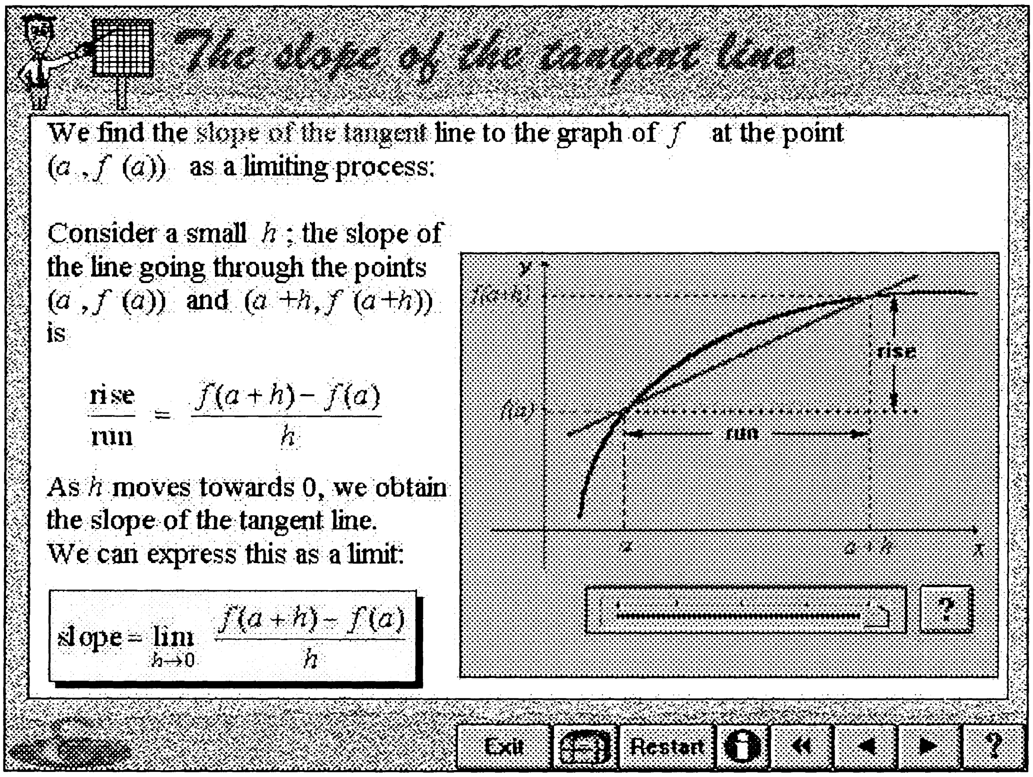

Fig. 1.

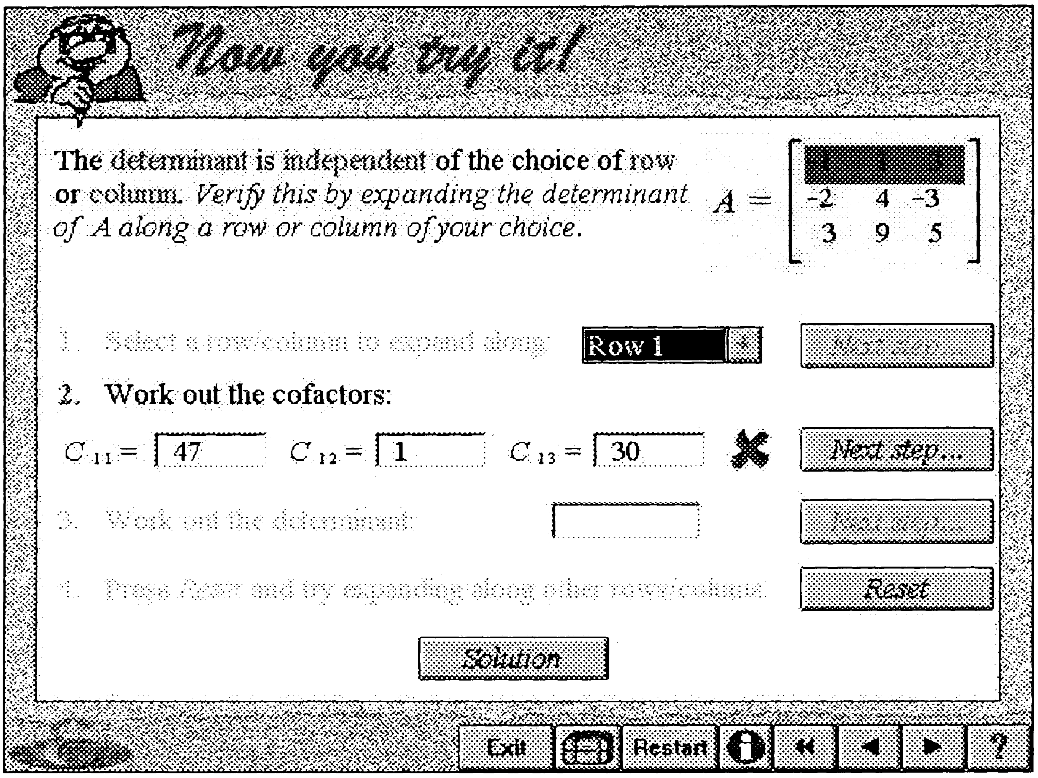

Fig. 2.

Figure 1 shows a typical information screen with an animation; while Fig. 2. depicts a screen where the learner is guided in the calculation of a determinant.

The development process

The first chapters of *Epsilon* were developed as a pilot project with the aim to investigate the authoring tool and to try out a variety of learning stimuli that could support student learning. The intention was to produce an additional resource for students who feel comfortable with computer-based learning, and to make the learning of mathematics more enjoyable, particularly for distance education students whose only resource was print-based material. This resource was not to replace lectures or tutorials, but to complement them. Further university funds were received to complete the version (v1.1) available to the students today.

The content had to be written specifically for screen display: the language used had to be more informal than in printed notes, the amount of information that could be included on each screen was limited and had to be presented in short paragraphs, and the access of information via hyperlinks had to be carefully designed.

Equations are pasted in as bitmaps, but since the user does not have the capabilities of changing the font size, equations always match the text. The learner's input can be in the form of an arithmetic expression including the values of standard functions, but algebraic input cannot be accepted. This is a constraint on the type of exercises that can be evaluated; designing learning activities with meaningful evaluation is a challenging task for the academic developer.

A small and cohesive team was a key in the success of the development of *Epsilon*: two academics with a wealth of experience in first-year teaching and a programmer with sound knowledge of the content and good graphic design skills. The team was also supported by expert instructional design and technical staff from the University.

The question arises whether universities can afford the in-house production of electronic materials. The developers' experience indicates that for large universities, this is not only possible but also desirable. One big advantage is that such materials are tailored, on a continuous basis, to the needs of their particular students. In addition, the materials are available to all on-campus, distance education and off-shore students at a very affordable price. Apart from the academics' time for preparing the content and designing its flow, the only additional cost in the production of *Epsilon* was six months of a programmer's salary. As authoring systems developers resolve the problem of handling mathematical content, the production costs will decrease. Greater efficiency can also be achieved with collaboration between institutions; a positive example of this is the adaptation of *Epsilon* for its use in Germany [9].

Student evaluation

The formative evaluation of *Epsilon* by academics, developers, and school teachers has taken place throughout its development, and it influenced its current content and appearance. Students were surveyed at the end of the first semester of wide use of *Epsilon* across all campuses.

The analysis of the students responses showed that:

- Students clearly liked the user interface: the ease of use, the navigation, and the screen design.
- Students were satisfied with the amount of interactivity and use of animations even though the developers rate these as modest (they are a major factor in the programming costs).
- Students were satisfied with the organisation, display, and flow of information.
- Students were pleased with the language used to present the information in a simple and clear way.
- Distance education students used this resource throughout the semester, while on-campus students tended to use it for revision for preparation of tests and exams.
- Students were pleased with the availability of worked-out solutions to all exercises.
- Many students used this resource because 'it was more fun' to learn this way.
- Many students did not find the audio clips useful. These are used very little, only to provide a motivational introduction to each chapter, and to give the pronunciation of words that may be unknown to the students (these were included with the distance education students in mind).

When asked to make suggestions for improvement:

- Many students wanted to have the content matching more closely their subject syllabus.
- All students would like to have more exercises with solutions.
- A few students suggested to add printing facilities.
- A few students would like to have access to a glossary.

The future of epsilon

A new version of *Epsilon* is currently being developed to be ready for the 1999 academic year and will incorporate most of the suggestions made by the students. The new version will have:

- new content chapters, and reorganisation of the existing ones, matching the syllabus of the first year engineering course;
- a separate module with exercises (and solutions) for each chapter, in addition to the current exercises which are within the content module;
- a separate module with application for each chapter
- a glossary;
- an improved general look and graphic design.

CONCLUSION

The pressure for producing flexible delivery materials does not come only from governments and institutions managers, but also from students who are asking for interactive, flexible, and lively materials to support their learning.

The current World-Wide-Web standard language does not allow for a quality delivery of mathematics courses. Although a standard language for the electronic communication of mathematics has been developed recently, the current widely used browser does not support it yet. In the meantime the most reliable medium for the delivery of electronic mathematics courses is the CD-ROM. It is possible to produce low budget good quality interactive mathematics materials; the keys to success are small cohesive teams and non-commercial collaboration with other institutions.

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Cristina Varsavsky has a strong interest in mathematics education, in particular the effective use of electronic media for improving university teaching and learning. She has worked at Monash University since 1991 mainly with first-year science and engineering students. Further details can be found in her home page at http://www.maths.monash.edu.au/ cristina.