Int. J. Engng Ed. Vol. 16, No. 1, pp. 39–49, 2000
Printed in Great Britain.

0949-149X/91 $3.00+0.00

The Practical Optimisation of Machine Components*

ANDREI LOZZI and PAUL BRIOZZO
Dept of Mechanical & Mechatronic Engineering, University of Sydney, Australia.
E-mail: andrei@mech.eng.usyd.edu.au

The optimising software built into modern spreadsheets is applied to the design of machine components. This software allows a student or engineer to develop increasingly sophisticated design analysis. The speed of modern PCs is such that searching over 10 or more variables while meeting as many constraints causes only imperceptible delays to the operator. The ease with which variables and equations can be defined within a spreadsheet allows for quick responses and rapid learning cycles. Significantly more elaborate and realistic design solutions can be arrived at than had been possible in the near past. The applications presented in some details and discussed are: preloaded bolted joints which can be either the cheapest or lightest, concentric helical compression springs which occupy the minimum area and gear pairs that require the least amount of material.

INTRODUCTION

THE MATHEMATICAL optimisation of the design of machine components has been difficult to carry out in the past because of the complexity of real design problems and the lack of practical optimising tools. Real-world complexity is a factor because most engineers are trained with simple problems, often solvable by hand and often with obvious solutions. In contrast the real world presents us with orders of magnitude more variables and with situations where the influence of many of these variables is difficult to determine. When we are asked to deal with ten or more variables and as many constraints, hand methods become quite impossible, what seems to be required are user-friendly computer methods.

Many real-world designers optimise their designs subjectively by examining the efforts of others and by positioning their own design within carefully chosen and usually well proven ranges of existing hardware. In other words, they predetermine a lot of their designs by choosing apparently effective combination of values of variables from established designs.

We must note that mathematical or objective optimisation can only be carried out after the invention or conceptualization phase of a design has taken place [1]. We can only mathematically optimise what is well defined, we can hardly optimise what are just concepts. We would also like to make it clear that in this paper we present examples of optimisation that may be a step forward in breadth and practicality but are not of course truly global or completely realistic. Each of the applications shown here have taken only a few days to develop, which give an indication of the potential for real world problems if more information and more time were available to develop better solutions.

* Accepted 6 October 1999.

METHOD

For as long as computers have been available to industry there have also been mathematical packages that could be used with them. Specialists at times applied these packages to find solutions to design problems. Predominantly effective and practical solutions were sought, while at other times mathematical optimal solutions, within necessary constraints, were arrived at. We speculate that usually practicing design engineers have not had the time, the necessary experience or the computer orientation to use these packages. Consequently their uses were limited to relatively rare applications.

Today we have reached a point when spreadsheets having improved rapidly in the past are able to deal with surprisingly elaborate and hitherto difficult problems. Mathematical and programming tools have been incorporated into them with extensive capabilities [2, 3]. We will demonstrate here three applications of modern spreadsheets, such as Microsoft Excel, Quattro Pro and we presume others, to some traditional machine design problems. We will consider the quantum leap in analysis that can be now carried out with these packages. The major hurdles in introducing this approach to students and even experienced engineers appear to be:

1. To show how we may translate geometry and equations into spreadsheet terms.
2. To simplify the problem initially, so that the procedures and outcomes can be verified easily.
3. To increase the sophistication of the analysis

progressively as each step in the optimising process becomes understandable and credible.

This sort of experience should be effective in expanding a student's or an engineer's concept of what is now possible and may even be likely, using current and emerging software.

THE SOFTWARE

When we use a package like a spreadsheet, we may not be informed how it is programmed; for good reasons one is usually only informed on how to use it. In this instance Frontline Systems [4] has developed the optimising software and provides a lot of information for its use on the Internet. Our demonstration begins with the simple application of finding the sides of a rectangular prism of which the volume is fixed at 1 and its surface area is at a minimum. Thus we have the 3 sides being variables (l, w and h), the volume ($V = lwh$) being constrained to 1, and the surface area ($A = 2(lw + lh + hw)$) required to be a minimum. These conditions are entered in an appropriate dialogue box, selected from the menu of the package, in a completely analogous way. This example is about geometry, it has an obvious answer to nearly all the students, it runs in a split second and any errors or improvements are dealt with interactively. The students are then asked to come up with their own examples and to explore the effect of the options that are available, on the outcome.

The solution is arrived at by numerical iteration, but no matter what the method may be now, it may be changed surprisingly quickly in the future. In any circumstance it is important to explore its limitations, and we chose to do this with a range of problems to which we know the answers. The rectangular prism mentioned above, of which we minimised the surface area, is extended progressively into higher dimensions. The area is always calculated as the twice the sum of the product in pairs of all sides and the volume as the product of all the sides, that extend on orthogonal axes in the respective multidimensional space. When dealing with 6 dimension for example, six sides are set as variables, 4 volumes are each constrained to be 1 (the volumes for the 3, 4, 5 and 6 dimensions space) and the 6 dimensional surface area is minimised. This area expression has 15 terms, each a different product of two sides.

The software as installed behaved flawlessly but, took a discernable and increasing amount of time, until we reached 13 dimensional spaces. Here we have 78 terms in the objective function (the area), 10 constraints (the volumes from the 3^{rd} to the 13^{th} dimension) and 13 variables (the sides). No apparent singularity, but nevertheless a great deal of calculations are called for. The software adjusts the given variables starting from the last in the list that is typed in, and if the objective function reaches the required result within the precision stipulated, the operations may cease before the first variable is adjusted to the same degree as the last. When minimizing the area in 13 dimensional space, using the default options provided, the first 3 variables were hardly altered (i.e. l, w and h), whereas all the others had reached the value of 1 (precisely). If we desire a reasonably even adjustment of all the variables, we can change the order of the variables and rerun the application, or increase the precision called for. The difficulty was easily overcome.

The operator caused all the problems that arose, for example the area was typed in as a constraint, variable names were given to cells that were empty, and so on. In all, this is a very salutary demonstration of what can be done, particularly in real-life problems where the functions are well behaved and singularities are few. We must underscore the point that optimising calculations of this magnitude, as a rule, just cannot be done by hand or subjectively in one's head. Furthermore the tools available here are interactive and easy to use, providing effective, testing and learning opportunities.

EXAMPLE: BOLTS AT BASE OF CYLINDER

Figure 1(a) is a cross-section through the base of a hypothetical cylinder of an internal combustion engine. The pressure in this cylinder is assumed to cycle sinusoidally between a maximum and a minimum value, billions of times in its service life. The load cycle is intentionally simplified so that the geometric aspects may be explored and we may limit the spreadsheet to one page. We need a bolted joint at the bottom of that cylinder that will ensure that it will not fail under fatigue and that it is a good design from a cost and weight perspective.

The solution here is provided in two steps. The first is a relatively simple approach where only the bolt diameter is varied, all the other parameters are selected by the designer. This represents a design that is subjectively 'optimized' by the designer leaving only one variable to be mathematically determined. The second is more elaborate and more demanding, where the bolt diameter, number of bolts, grip length and bolt grade are all allowed to vary, together with the flange height and width. It may be noticed that in every step in these spreadsheet applications there are shortcomings and limitations. Here as it is always the case, improvements may be continued *ad infinitum*.

The simple solution

We predetermine all aspects of the bolted joint except for the bolt diameter and with it the flange width. The analysis used is well known and detailed in a number of texts that students and engineers have been using for quite a number of years [5–7]. Figure 2 shows a Goodman diagram for a bolt being subjected to a steady preload and an alternating load. A bolt represented by point P

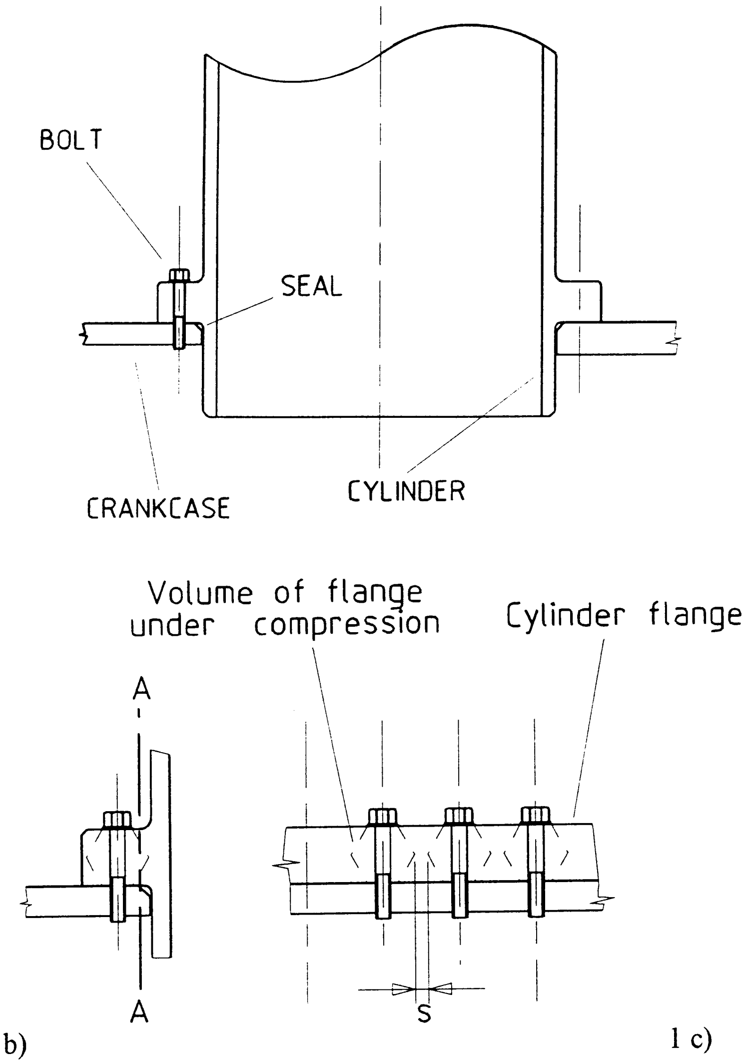

Fig. 1. (a) Section through a simplified cylinder base and crankcase, attached together by a preloaded bolted joint. (b) Enlargement of the bolt and flange shown on (a), showing section AA through which the average shear stress in flange is calculated. (c) Section through flange along bolt centerlines showing the volumes of flange material under compression by individual bolts and their separation S.

in Fig. 2(a) has an excess of fatigue strength, because P is below the Goodman line. This excess fatigue strength is referred to here as the 'Goodman condition' and by the symbol *Esa*, shown on the second last line of the spreadsheet – Table 1. We have set up the analysis so that the bolt diameter varies to arrive at the diameter for which the excess fatigue strength is minimised to

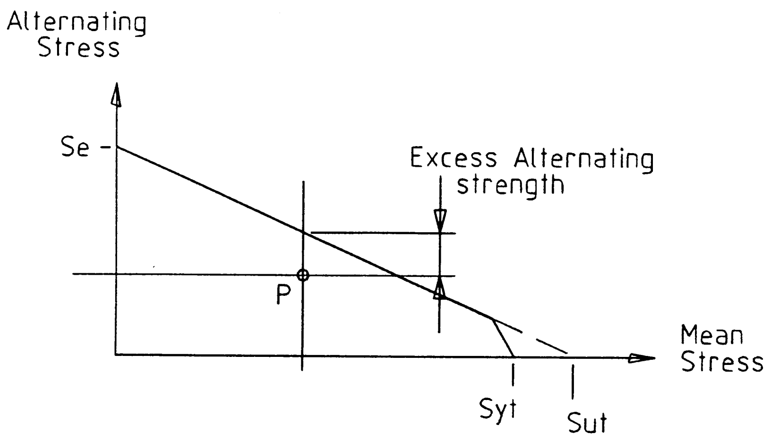

Fig. 2. Goodman diagram similar to that of Fig 8–19 of Shigley [5], showing a stress condition P with excess alternating strength.

Table 1. Optimising spreadsheet for the bolted joint shown on Fig. 1, for which the bolt diameter is varied to arrive at a stress condition of the bolts on the Goodman line.

BOLTED JOINT OPTIMISATION Des II 97

For bolts at the base of a cylinder in a diesel engine, ID 120, max alternating press 12 Mpa.

SELECTED VALUES:

Variable	**Symbol**	**Value**	**Equation**
Bolt grade	BG	14.8	
Y modulus of flange N/mm^2	Em	207000	Sep 400
Y modulus of bolt N/mm^2	Eb	207000	tka 0.8 surface finish
Preload	pr	0.8	fkb 1 size
Bolt Sut (N/sq mm)	Su	1400	fkc 0.897 reliability
Bolt Syt (N/sq mm)	Sy	1120	fke 0.667 stress conc
Cylinder yield strength (MPa)	CSy	250	fkd 1 temperature
Reliability	kc	1	Se = Sep*fka*fkb*fkc*ficd*fkf
Cone angle (degrees)	Ac	30	
Loss of area due to thread	Al	0.2	
External force N	Ptot	135716.803	
Number of bolts	n	12	
Grip length (mm)	1	15.00	
VARIABLES:			
Bolt diameter (mm)	Db	5.93	
PROPERTIES:			
Bolt stem area	Ab	27.62	PI Db Db/4
Thread stress area (sq mm)	As	22.09	Ab (1-Al)
Bolt preload N	Fi	19796.83	pr Sy As
Bolt stiffness N/mm	Kb	381133.18	Ab Eb/l
Flange stiffness N/mm	Km	1228393.61	Km (Ac, l, Em,)
Stiffness ratio	Rk	0.24	Kb/(Kb+ Km)
Force/bolt N	P	11309.73	Ptot/n
Alternating stress (N/sq mm)	sa	60.61	P Rk/(As 2)
Mean stress (N/sq mm)	sm	956.61	Fi/As + sa
Endurance limit (N/sq mm)	Se	191.36	Sep fka ficb fkc fkd fke
New bolt force N	Fb	22474.96	Fi + Rk P
New flange force N	Fm	−11165.22	(1–Rk) P– Fi
Goodman condition	Esa	0.00	ABS (sa + Se sm/Sm–Se)
Max external force N	Pmax	25939.186	Fi (1 + Kb/Km)

practically zero (or possibly where it reaches a required factor of safety). The stress conditions for the bolt will then be a point on the Goodman line. As a result we see from Table 1 that if we wish to use 12 bolts, they should be of the following specifications or better: 6mm dia, 15mm grip length, grade 14.8 and preloaded to 80% of their yield strength, to produce a safe joint in fatigue.

If the designer were to change the grip length or bolt grade we would just get different working solutions. The relative effectiveness of any of these joints, when compared to competitor's joints relies on the designer having made appropriate judgements for all of the predetermined values. But, we have no idea how close this joint is for example to the cheapest or most compact joint.

The elaborate solution

We will move in one step from the above procedure to one where the number of bolts, their diameter, grade and grip length are all variables, together with flange dimensions. Figures 1 (b) and (c) show sections across the flange and along the flange through the bolt centerlines. A decision has to be made as to how close the bolts are allowed to come to each other. A modern means of estimating flange stiffness uses the truncated hollow cones (frustums) shown in these figures. Here we have decided that each bolt would compress an individual volume of the flange, that means that the spacing *S* in Fig. 1(c) has to be larger than zero. This condition determines the maximum number of bolts, given as *nx* on Table 2. The cone angles of these frustums, *Ac*, together with the bolt grip length *l,* determine the minimum width and thickness of the flanges.

Table 2 is similar to Table 1, except that the number of variables is increased to 4 and the constraints to 6. It was decided *a priori* that the number of bolts was to be 4 or more, their diameter a minimum of 3 mm, their tensile strength between 600 and 14000 N/mm^2, and the alternating shear stress in the flange had to be below a maximum – *sasx.* If we do not constrain the shear stress in the flange, the flange thickness would plummet to zero. This is the outcome of searching for the cheapest or lightest joint, as

Table 2. Similar to Table 1 except that each bold diameter, length, number of bolts and bolt grade are all allowed to vary to arrive at the cheapest of lightest solution.

BOLTED JOINT OPTIMISATION Des II 97

For bolts at the base of a cylinder in a diesel engine,ID 120, max alternating press 12 Mpa.

RELATIVELY FIXED VARIABLES			
Bolt grade	BG	6.88	
Y modulus of flange N/mm^2	Em	207000	Sep 400
Y modulus of bolt N/mm^2	Eb	207000	fka 0.8 surface finish
Preload	pr	0.8	fkb 1 size
Bolt Sut (N/sq mm)	Su	600	fkc 0.897 reliability
Bolt Syt (N/sq mm)	Sy	480	fke 0.6666 stress concentration
Cylinder yield strength (MPa)	CSy	250	fkd 1 temperature
Reliability	kc	1	Se = Sep*fka*fkb*fkc*fkd*fke
Cone angle (degrees)	Ac	30	
Loss of area due to thread	Al	0.2	
External force N	Ptot	135716.802	
VARIABLES and CONSTRAINTS:			
Number of bolts	n	33.39	4 < n < nx
Bolt Diameter (mm)	Db	3.54	Db >= 3
Grip length (mm)	l	12.39	sas < sasx
Bolt ult tensile strength	Su	1400	600 < Su < 1400
PROPERTIES:			
Bolt stem area	Ab	9.8397	PI Db Db/4
Thread Stress Area (sq mm)	As	7.8718	Ab (1-Al)
Bolt preload N	Fi	3022.8	pr Sy As
Bolt stiffness N/mm	Kb	164363	Ab Eb/l
Flange stiffness N/mm	Km	647998	Km (Ac, l, Em,)
Stiffness ratio	Rk	0.20233	Kb/(Kb+ Km)
Force/bolt N	P	4064.83	Ptot/n
Alternating stress (N/sq mm)	sa	52.24	P Rk/(As 2)
Mean stress (N/sq mm)	sm	436.24	Fi/As +sa
Endurance limit (N/sq mm)	Se	191.36	Sep fka fkb fkc fkd fke
New bolt force N	Fb	3845.20	Fi + Rk P
New flange force N	Fm	219.63	(1–Rk) P– Fi
Goodman condition	Esa	0.010	ABS (sa+Se sm/Sm–Se)
Max external force N	Pmax	3789.49	Fi (1 +Kb/Km)
Alt shear stress in flange N/mm^2	sas	12.500	(Fb–Fi)/(PI 1.5 Db l)
Max alt shear stress N/mm^2	sasx	12.5	CSy/8
Max no of bolts	nx	33.39	Lb/w
OTHER DIMENSIONS:			
Cylinder min thickness (mm)	Tcyl	2.49	sqrt(3/4) 12 60/CSy
Width of flange (mm)	w	12.46	1.5 Db+ l/sqrt(3)
Length of flange at bolts mm	Lb	416.15	2 PI (60+ w/2)
Volume of flange (cubic cm)	Vm	64.28	PI l (ww+ 2 w 60)/1000
Volume of bolts (cubic cm)	Vb	8.72	(n PI Db Db/4) (l+ 4 Db)
Total volume	Vt	73.00	Vm+ Vb
COSTS:			
Flange cost ($)	SSm	4.50	Vm 0.07
Bolt cost ($)	SSb	1.47	Vb 0.14 (Su Su/100000– Su/100+ 3.6)
Drilling and handling cost ($)	SSd	76.78	10+ 2n
Total cost	SSt	82.74	

either condition by itself would eliminate the flange if at all possible. The maximum allowed alternating shear stress is here applied to the average stress at section *AA* of the flange, shown on Fig 1(b). In this example it has been arbitrarily set to be less than 1/8th of the flange shear strength, *CSy*. A more realistic criterion is desirable but not applied here for the sake of simplicity. At the bottom of the table the volume of the bolts and flange materials are calculated and below that the cost is arrived at. The cost of the bolted joint here takes into account the price of flange steel and bolt steel for different grades, using an equation that interpolates between bolt prices established locally.

The objective functions that have been minimised here are the total volume, for the lightest joint, and the total cost of course for the cheapest. Table 3 summarises the results, using

Table 3. Summary of the cheapest and lightest bolted joint, using pradftical dimensions, arrived at from Table 2.

	Cheapest bolted joint	Lightest bolted joint
Cost	$50.60	$96.10
Mass	2020 gm	564 gm
Bolt dia	7 mm	3.5 mm
Grip length	23 mm	12.6 mm
Bolt grade	6.8	14.8
No of bolts	10	35

more practical dimensions than the spreadsheet arrives at. A cost of $2 per bolt, for drilling and handling, has been allowed for. This cost appears generous and yet the cheapest joint has 10 and not 4 bolts. One can easily explore the costs that would drive the joint to the minimum number, or permit more bolts. A surprising aspect of this exercise is the ease with which these results can be achieved and the ease with which one can switch between them. One of these types of joints is typically seen on aircraft engines, while the other on low cost machinery. But, it may well be that some low cost bolted joints are not made as cheaply as they could be and that some aircraft joints may also not be as light as they might be.

EXAMPLE: SPRINGS

Springs are particularly interesting devices. Usually we require them to fit into restricted spaces within machines, while providing a required movement, cope with a maximum and minimum force and give us a desired life expectancy. The variables that describe springs include wire diameter, coil diameter, number of coils, inter-coil space, length, end preparation and material properties. If we are to arrive at a spring design by hand, we begin by evaluating 6 equations to assess one possible spring. Once we have a starting point we change one or more variable at a time. The particular difficulty with springs, as is also the case with gears, is that no one variable has an overriding influence. All variables play significant roles in arriving at a solution. As a consequence the inexperienced is doomed to iterate for a long time and eventually to have to be satisfied with some 'practical' spring, not knowing how much better a solution may have been possible.

Concentric springs

Spring analysis like that for bolts is best approached in manageable steps, but here we will jump to the examination of twin concentric compression springs in one step to save space. These components were particularly chosen to demonstrate the power of the software. The significant difficulty here has been found to be on the part of the user in setting up the spreadsheet to give useful answers. The use of twin springs occurs where there is the need to generate the largest practical force over the longest distance, in the smallest area. This sort of requirements applies to engine valve springs, on the top of most piston engines. Space requirements inside any machine is usually at a premium, and the most compact components are highly sought after. References 5 and 8 were used in the development of this spreadsheet, shown on Table 4.

Fig. 3. A pair of concentric springs of minimum outside diameter but of fixed inside diameter, total stiffness and preloaded length.

As seen on Table 4 we have 6 variables and 9 boundary conditions. We will allow the wire diameters, coil diameters and number of active coils to change together with the initial preload for each spring. In this example we look for the pair of springs that meet the following specifications: compress through 8 mm, be 35 and 40 mm long at preload, fit over the valve guide, have well formed ends, have adequate clearances all around the wires, made from appropriate steel, have the required fatigue life *and* must be of the smallest overall outside diameter. Not the sort of thing one can do easily by hand!

Many examples of concentric engine valve springs have the load divided between them such that the inner spring is about 80% the stiffness of the outer one. We may then be surprised to see that for the optimal solution developed here, the inner spring is only about 10% the stiffness of the outer. When the optimisation search is initiated over the whole range of realistic initial conditions, we find that we can also arrive at the same overall minimum outside spring diameter if the outer spring is about 10% the stiffness of the inner one. Figure 3 shows such a pair of springs. We find that for our solutions the softer spring has many more coils and must be preloaded through a longer distance than the stiffer spring. Here the same factor of safety applies to both springs. It should be noted that

Table 4. Spreadsheet of twin concentric springs of minimum outside diameter, representative of engine valve springs.

Optimised Spring Selection – twin concentric springs
Preloads and stiffness for both springs are variables

FIXED QUANTITIES:

Name	**Value**	**Units**	**Description**	
G	79300.00	N/mm^2	Modulus of Rigidity	
Lfo(req)	40.00	mm	Fully loaded outer spring length	
Lfi(req)	35.00	mm	Fully loaded inner spring length	
k(req)	42.00	N/mm^2	Total spring stiffness	
Fmx(req)	650.00	N	Max total spring force	
Fmn(req)	314.00	N	Min total spring force	
dci(min)	16.00	mm	Min inner spring inside dia	
lft	8.00	mm	Valve lift	
Sse	450.00	N/mm^2	Endurance limit in shear	
Ssui	874.73	N/mm^2	Ssu for inner spring	(0.6 A/power(di, m))
Ssuo	934.27	N/mm^2	Ssu for outer spring	(0.6 A/power(do, m))
A	1790		Factors for chrome vanadium wire for	
m	0.155		Sut of spring wire, Shigley [5] p423	

RELATIVELY FIXED QUANTITIES:

FS(min)	1.20		Factor of Safety
Ni	1.50		End preparation's inactive coils, both springs
s(min)	0.20	mm	Min. fractional space between coils

INDEPENDENT VARIABLES:

Inner spring	di	3.76	mm	Wire diameter – inner spring	
	dci	16.00	mm	Inner diameter of coil	
	Nin	4.43		Active coils	
	sin	0.20		Inter coils space/di at Fmxi	
	Fmni	314.00	N	Preload inner spring	variable
Outer spring	do	2.46	mm	Wire diameter – outer spring	
	No	10.23		Active coils	
	so	0.20		Inter coils space/do at Fmxo	

DEPENDENT VARIABLES – INDIVIDUAL:

Inner	Sii	5.26		Spring Index	(dci+ di)/di
	ki	30.00	N/mm	Spring stiffness	(di G)/(8 Nin SIi SIi SIi)
	Lfi	35.00	mm	Length max force -not Orlov	(di+di sin) Nin+di+2 Ni di
	pli	10.47	mm	Preload spring travel	(Fmni/ki)
	Wfi	1.29		Whal Factor	(4Sii-1)/(4Sii-4)+0.615/Sii
	Fmxi	554.00	N	Max load outer spring	(Fmni+lft ki)
Outer	Sio	11.79		Spring Index	(dco+ do)/do
	ko	12.00	N/mm	Spring stiffness	(do G)/(8 No SIo SIo SIo)
	Lfo	40.00	mm	Length max force -not Orlov	(do+do so) No+do+2 Ni do
	plo	0.00	mm	Preload spring travel	(Fmno/ko)
	Wfo	1.12		Whal Factor	(4Sio-1)/(4Sio-4)+0.615/Sio
	Fmno	0.00	N	Preload outer spring	(Fmn Fo_Ft)
	Fmxo	96.00	N	Max load outer spring	(Fmno+lft ko)
	dco	26.52	mm	Inner diameter of coil	(dci+2 di+3)

DEPENDENT VARIABLES – COMBINED:

k	42.00	N/mm	total spring stiffness	(ko+ ki)
Fmx	650.00	N	total max combined force	(Fmxi+Fmxo)
OD	31.43	mm	Outside dia of outer spring	dco+2 do
ko/kreq	0.2857		Outer spring stiffnes/k(req)	(ko/k(req))

STRENGTH AND FATIGUE LIFE:

Inner spring	ssxi	678.79	N/mm^2	Max shear stress	(Wf 8 Fmxi SI)/(PI d d)
	ssni	384.73	"	Min shear stress	(Wf 8 Fmni SI)/(PI d d)
	ssmi	531.76	"	Mean shear stress	(ssx+ssn)/2
	ssai	147.03	"	Alternating shear stress	(ssx–ssn)/2
	Essei	29.41	"	Excess fatig strength	(Sse–ssm Sse/Ssu–ssa)
	Fsi	1.20		Actual FS	(Esse+ssa)/ssa

Table 4. Continued.

Outer spring	ssxo	535.19	N/mm^2	Max shear stress	(Wf 8 Fmxo SI)/(PI d d)
	ssno	0.00	"	Min shear stress	(Wf 8 Fmno SI)/(PI d d)
	ssmo	267.59	"	Mean shear stress	(ssx+ssn)/2
	ssao	267.59	"	Alternating shear stress	(ssx-ssn)/2
	Esseo	53.52	"	Excess fatig strength	(Sse–ssm Sse/Ssu–ssa)
	Fso	1.20		Actual FS	(Esse+ssa)/ssa

OPTIMISED CONDITIONS:

Minimise: OD
Vary: di, Nin, do, No, dci, Fmni, si, so

Constraints: Lfi = 34, Lfo = 40, dci > dci(min), FSii > Fs(min), FSo > Fs(min), si > s(min), so > s(min), k=k(req)

these combinations of springs give only slightly smaller outside diameters than if the stiffness ratio between the springs were fixed to some of the commonly used ratios. But, the authors have observed springs of these proportions in use for example on locomotive suspensions.

It should be noted that this software finds only local optimal solutions, depending on the starting conditions. Global solutions are not determined, and the operator must carry out the search. An interesting comparisons in the search for global solutions may be made with the many examples carried out analytically in [9].

EXAMPLE: GEARS

Figure 4 shows schematically part of a pair of gears in mesh. As the tooth of one gear bears onto the tooth of the other it can transmit torque and angular motion. At point A on Fig. 4, the material of either tooth may fail by crushing if the contact stresses are excessive. Furthermore at points near the base of each tooth, for example point B, the material of each tooth is in tension and if this is excessive that tooth may suffer a tensile failure. References [5, 10] were used in the compilation of this spreadsheet, shown in Table 5.

We note that if the two gears in mesh are of different diameters, there will be four different limiting safe stress conditions. That is, there will be four factors of safety, two for each gear. A principal aim in designing a pair of gears is to choose parameters that will give the same factor of safety against each of these four limiting conditions. Advice can usually be obtained from experienced designers and trade literature on how to search for gear parameters that will not result in any relative weakness or unusable strengths. If

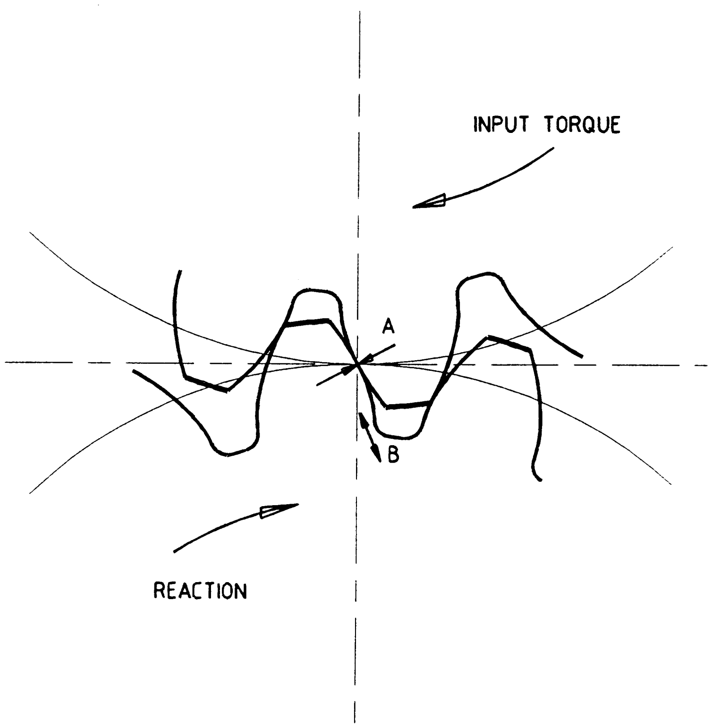

Fig. 4. Schematic diagram of a pair of spur gears in contact, A and B represent points of maximum compressive and tensile stress.

Table 5. Spreadsheet for a spur gear drive in which the number of teeth, pitch circle diameters, strengths and face width are varied to arrive at gear pair of least mass.

OPTIMISED 2:1 GEAR DRIVE

Required parameters:

H	100000.00	W	Motor power – watts	
rpm	1440.00	rpm	Motor speed revs per min	
T	663145.83	Nmm	Motor torque, equa 2–46	(9.5493 HI rpm) 1000
GR	2.00	No.	Gear ratio	
Fst	1.30	No.	Factor of Safety, against tensile failure	
PA	25.00	degrees	Pressure angle	

Minimum material tensile properties:

Sei	164.18	$Nlmm^2$	AGMA fatigue strength, Fig 14-2 p 597	$(-274 + 167$ HB-O.152 $HBA^2)$.
Hbi	170.91	No.	Brinell hardness num.	6.8911000
Sea	147.89	$Nlmm^2$	AGMA fatigue strength, output gear	
Hbo	150.90	No.	Brinell hardness num., output gear	

Pinion & gear specifications:

Dpci	160.26	mm	Pinion pitch circle dia	
Dpco	320.52	mm	Gear pitch circle dia	(Dpci GR)
F	63.73	mm	Face width	
mod	10.61	mm	mm of PCD per tooth	(Dpci/ Ni)

Transmission parameters:

Wt	8275.90	N	Tooth tip tangential force	(T 2/Dpci)
Ni	15.10	No.	No of teeth on input pinion	
Ji	0.29	No.	Geometry factor pinion, Dimaroganas [10]	$(0.39(1\text{–}1.14INi^{0.54}))$
No	30.20	No.	No of teeth on output gear	(Ni GR)
Jo	0.32	No.	Geometry factor gear	$(0.39(1\text{–}1.14/No^{0.54}))$
Vt	12.08	m/sec	Tooth tangential velocity	(Doci rDm PII (60 1000))
Kv	0.50	No.	Velocity factor, hobbed gears	(12.2/(12.2+Vt))
Ka	1.25	No.	Load app. factor, smooth in & light out	
Kmi	1.20	No.	Load distribution factor, Dimaroganas [10]	(F + Dpci/2)
CtoC	240.39	mm	Gear centres distance	(Dpci + Dpco)/2

Tensile stresses and factors of safety:

sti	126.29	$Nlmm^2$	Bending stress in pinion	Equation 14–15 Shigley [5]
sto	113.77	$Nlmm^2$	Bending stress in gear	Equation 14–15 Shigley [5]
Fsti	1.30	No.	Actual FS of pinion	(Sei/sti)
Fsto	1.30	No.	Actual FS of gear	(Seo/sto)

Minimum material compressile properties:

Seci	1080.35	N/mm^2	AGMA surface fatigue strength, Fig 14-3	(26000 + 327 Hbi).6.89.1000
Hbci	400.00	No.	Brinell hardness num. Input gear	
Seco	763.92	$Nlmm^2$	AGMA surface fatigue strength, output gear	
Hbco	259.55	No.	Brinell hardness num., output gear	

Transmission parameters:

Cp	191.00	$Nlmm^2$	Elastic coefficient, steel to steel, table 14-5	
I	0.13		AGMA geometry factor	

Table 5. Continued

Compressive stresses and Factors of Safety:					
sci	831.04	Nlmm^2	Contact stress on pinion	Equation 14–16 Shigley	
sci	587.63	Nlmm^2	Contact stress on gear	Equation 14–16 Shigley	
Fsci	1.30	No.	Actual FS of pinion	(Seci/sci)	
Fsco	1.30	No.	Actual FS of gear	(Secolsco)	
Minimum Mass, Volume and costs:					
Vol	6427.36	cc	Total volume of pinion & gear pair	(PI F (Dpci^2 + Pcdo^2)/4)	
Mass	48.85	Kg	Total mass of pinion & gear pair		
Overview of results:				**Solver conditions:**	
Pinion	Mass	Cent to C	Face width	Minimised:	Vol
15	49	240	64	Variables:	Dpci, F, Hbi, Hbo, Ni
30	49	240	64		Hbcl, Hbco
35	58	253	69	Constraints:	Hbl & Hbci <= 400
40	67	263	74		Fsti & Fsto = Fst
45	77	273	78		Fsci & Fsco = Fst

done without the aid of optimising software this becomes a daunting task. With the sort of spreadsheet that has been used here, it is entirely practical to search for a gear pair that will have the least volume (cheapest in material) while meeting a large number of conditions. These conditions may include that all the four factors of safety (*Fsti, Fsto, Fsci* and *Fsco*) should be equal to some chosen value, such as for example the value of 1.30 used here.

Gears tend to be expensive and critical to the proper operation of the machines into which they are mounted. Consequently the search for optimal gear parameters is reported in the literature more often than for most other machine components [11, 12]. The approach typically presented in the literature is often technically demanding and requires specialised software. The use of modern spreadsheets on the other hand allows engineers to progressively develop elaborate and sophisticated analysis as their understanding and confidence grows.

CONCLUSION

Using an optimising spreadsheet does not reduce the need for a student or engineer to understand what he or she is doing. It in fact gives them a chance to understand a good deal more, while saving them much drudgery. The use of this software provides the opportunity to explore the effect of a wider range of variables and from the responses gain a deeper understanding and arrive at a better design. The spreadsheet used here in its current form does not carry out multi-objective analysis and can only be relied upon to provide local solutions. An important aspect of the options provided is that the user can control the starting point, the elapsed time, number of cycles, the number and order of the variables and has a choice of the means of extrapolating. The user can then explore solutions within realistic and appropriate ranges of variables. We must note that as the optimising model is extended over more features and components the point is quickly reached where it becomes conceptually very demanding to define the problem correctly. A useful attitude that we may take is that we must be in a position to make good use of the next evolutionary or even revolutionary step in the capabilities of the software, because given the size of the market and opportunities within it, it likely to be as impressive as the last. The authors will be happy to make the spreadsheets shown here available to other academics.

REFERENCES

1. G. Pahl and W. Beitz, *Engineering Design*, 2nd ed., London: Springer-Verlag (1995) edited and translated by K. Wallace, L. Blessing and F. Bauert.
2. W.J. Orvis, *Excel for Scientists and Engineers*, 2nd ed., Symbex (1995).
3. J. Walkenbach, *Excel for Windows 95 Power Programming with VBA*, IDG Books (1996).

4. Frontline System Inc. (1999). http://www.frontsys.com/.
5. J. E. Shigley and C. R. Mischke, *Mechanical Engineering Design*, 5th ed., McGraw-Hill (1989).
6. R. L. Norton, *Machine Design, An Integrated Approach*, Prentice-Hall (1998).
7. I. H. Bickford, *An Introduction to the Design of Bolted Joints*, Marcel Dekker (1881).
8. P. Orlov, *Fundamentals of Machine Design, Vol. 5*, MIR Moscow (1980).
9. D. J. Wilde, *Globally Optimal Design*, John Wiley Computer Aided Machine Design, Prentice-Hall (1989).
10. A. D. Dimaroganas, Computer Aided Machine Design, Prentice-Hall (1989).
11. M. Savage, S. B. Lattime, J. A. Kimmel and H. H. Coe, Optimal Design of Compact Spur Gear Sets, *Transactions of the ASME*, **116** (1994) pp. 690–696.
12. H. Wang and H-P. Wang, Optimal engineering design of spur gear sets, *J. Mechanism and Machine Theory*, **29**, 7 (1994) pp. 1071–1080.

Andrei Lozzi: Qantas – Maintenance and design of repairs and modifications of commercial aircrafts. Australian Atomic Energy Commission – development of standard measurements of radiation. University of Sydney – development of specialised wind tunnels and shock tubes; the study of supersonic flow and shock wave reflection. NSW Dept of Road Transport – full size simulation of road crashes; examination of seatbelt joints, testing of child restraints. University of Sydney – teaching machine design; full size simulation of car to truck collisions; farm machinery; finite element simulation of car crashes; circular and non circular gears; pattern recognition in manufacturing.

Paul Briozzo: Elevators P/L – design and development of lift machinery. Contract draftsman – designs of wide range of industrial and military equipment. University of Sydney – lecture and tutor in machine design and manufacturing; design of a wide range of research and testing equipment; the use of finite-element simulation of car to pole crashes.