

Enhancing the Teaching of Engineering Differential Equations with Scientific Notebook*

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Scientific Notebook is a combined word-processing and computer-algebra package operating with a Maple engine. It is powerful, user-friendly and affordable. It can be used in the classroom or in the computer laboratory to formulate a problem, construct a solution, and plot the results, in a short span of time. This saving of time enables one to concentrate more on the interpretation of the solution and to promote links to related mathematical topics. We show how Scientific Notebook can be used in the teaching of differential equations in engineering calculus classes. We illustrate two worksheets provided and report the results of a student evaluation survey.

INTRODUCTION

THE AVAILABILITY and accessibility of ever more sophisticated hand-held graphics calculators and inexpensive user-friendly computer-algebra software has revolutionised the teaching of undergraduate mathematics in various discipline areas. Applications that were previously considered computationally too difficult to be demonstrated in the classroom or set as assessment tasks are now becoming routine, allowing more time to be devoted to real-world examples rather than the mechanics of calculation.

In this paper we will discuss how a particular software package, Scientific Notebook, is being integrated into the teaching of undergraduate engineering mathematics units at Edith Cowan University. This takes the form of both student and instructor use.

ABOUT SCIENTIFIC NOTEBOOK

Scientific Notebook (see [3] and [4] for reviews of this package) is a combined word-processing and computer-algebra system, operating with a Maple engine. It is this feature, together with its very affordable price, that makes Scientific Notebook an excellent teaching and learning tool at the undergraduate level. Further technical details can be found on the Scientific Notebook website (<http://scinotebook.tcisoft.com>), where the potential user is able to download a free 30-day time-locked version of the package. Scientific Notebook's pull-down menus make it very easy to use and students are able to become familiar with this package in a short time. In our case we

found that one two-hour 'hands-on' workshop was sufficient to enable students to be well on the way to becoming independent users. User-friendliness is of particular concern to us since the majority of our students are studying mathematics as a service subject, with no intention of progressing on to mathematical research. Hence its very simplicity gives Scientific Notebook a huge advantage in our context over more comprehensive computer algebra packages such as Maple and Mathematica. These students do not have space in their mathematics units to learn the syntax and techniques needed to master the use of such sophisticated packages while at the same time trying to grasp the new mathematical concepts covered in their unit content. Such an advantage is sometimes lost on professional academics who are themselves Maple or Mathematica experts and who may be looking only at the power of such packages.

Scientific Notebook has some additional advantages also. Its extensive word-processing capabilities allow the development of mathematical results to be accompanied by a commentary explaining the assumptions and principles behind each calculation. Thus course notes, possibly incorporating interactive Internet links (see [2]), can be delivered to the students in a textbook-style format (see [1]), whether presented as printed handouts, as overhead displays, or even on a computer disc. Conversely, students can use Scientific Notebook to complete and submit pre-assigned work in a format that is equally professional. In fact, Scientific Notebook comes with a very detailed Help file incorporating a General Reference Library, which is almost a textbook in itself. It also comes with an associated Exam Builder that is a useful aid for the teacher. For the most part the mathematical syntax employed is identical to the conventional mathematical notation used in the classroom.

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Neither lecturer nor student needs to learn a specialised computer language to perform standard calculations. Writing down the usual symbols in the usual order can perform almost all the relevant operations. Moreover, complicated multi-step calculations can generally be executed interactively by the user, line by line if desired.

SCIENTIFIC NOTEBOOK IN ENGINEERING MATHEMATICS

We shall illustrate the use of Scientific Notebook here with examples from the area of differential equations. We report on a pilot study with two Scientific Notebook workshops, one on first-order differential equations and the other on second-order differential equations, given to a group of second-year students in a calculus unit. We provide here some details of the workshop notes with some examples and exercises used, together with a student evaluation summary. Although Scientific Notebook offers solution procedures both for ordinary differential equations (ODEs) and for partial differential equations (PDEs), we will confine our attention to ODEs. Scientific Notebook offers four solution procedures. Two of these, namely Laplace which works for linear differential equations with constant coefficients, and Exact which is slightly more versatile, are for the exact solution of an ODE. The others are the numerical procedure Numeric and the procedure Series using Taylor series. All of these can be found in the Solve ODE menu. The accompanying help-file gives a brief list of various notations that enable the package to recognise a differential equation, and one has to be sure to use one of these. For instance, a differential equation such as:

$$\frac{1+y^2}{y} dy = \cos x dx, \quad 0 \leq x \leq 10$$

must be rewritten as:

$$\frac{dy}{dx} = \frac{y \cos x}{1+y^2}, \quad 0 \leq x \leq 10$$

before attempting to use any of the ODE subroutines.

One of the main advantages of this package is the relative ease of graphing solutions. The following example concerning the general solution of a simple separable ODE illustrates this. This example is from one of the worksheets referred to earlier.

Example 1

Find the general solution of the initial value problem

$$\frac{dy}{dx} = -4xy^2, y(0) = y_0$$

and plot the particular solutions corresponding to $y_0 = 1, 0.5, 1.5, 2$.

1. First enter the differential equation and the initial condition as the rows of a 2×1 matrix as follows. Choose: Insert, Matrix (or alternatively the <insert graphic> button) to open the Matrix dialog box. Fill in the Dimensions as indicated in Fig. 1 and then click OK.

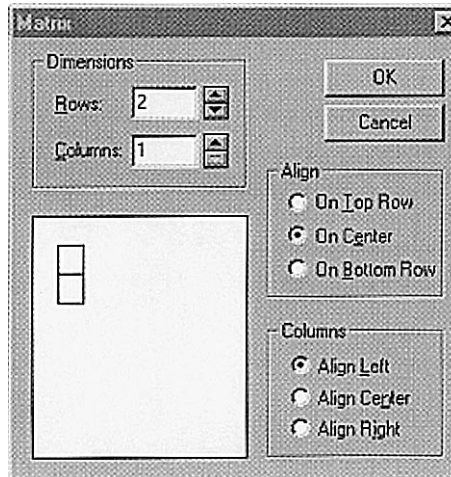


Fig. 1. Matrix dialog box.

Now fill in the upper and lower blank squares with the differential equation and the initial condition respectively to obtain the matrix illustrated in Fig. 2.

$$\begin{array}{|c|} \hline \frac{dy}{dx} = -4xy^2 \\ \hline y(0) = y_0 \\ \hline \end{array}$$

Fig. 2. Matrix for initial value problem.

2. With the cursor immediately after the matrix, choose: Maple, Solve ODE, Exact. You should obtain the following result:

$$\begin{array}{|c|} \hline \frac{dy}{dx} = -4xy^2 \\ \hline y(0) = y_0 \\ \hline \end{array}$$

$$\text{Exact solution is: } y(x) = \frac{1}{2x^2 + \frac{1}{y_0}}$$

3. In order to plot the required particular solutions we first define the function:

$$g(x, y_0) = \frac{1}{2x^2 + \frac{1}{y_0}}$$

To do this, type the function as indicated above,

ensure the cursor is somewhere in the formula, and choose: Maple, Define, New Definition. To check that this definition has been entered in the definitions list choose: Maple, Define, Show Definitions.

- To plot the particular solution corresponding to $y_0 = 1$ first type in the expression $g(x,1)$. Then, with the cursor immediately after this expression, choose: Maple, Plot 2D, Rectangular, to obtain the graph in Fig. 3.

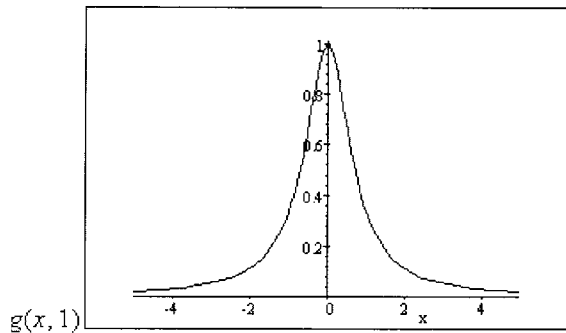


Fig. 3. Particular Solution for $y_0 = 1$.

The graph in Fig. 3 has been obtained using the default domain of $[-5,5]$. If you were not operating with this default domain, or you wish to use a different domain, use the Plot Properties dialog box (see Fig. 4) which is obtained by clicking first on the picture and then on the blue square that appears in the bottom right corner.

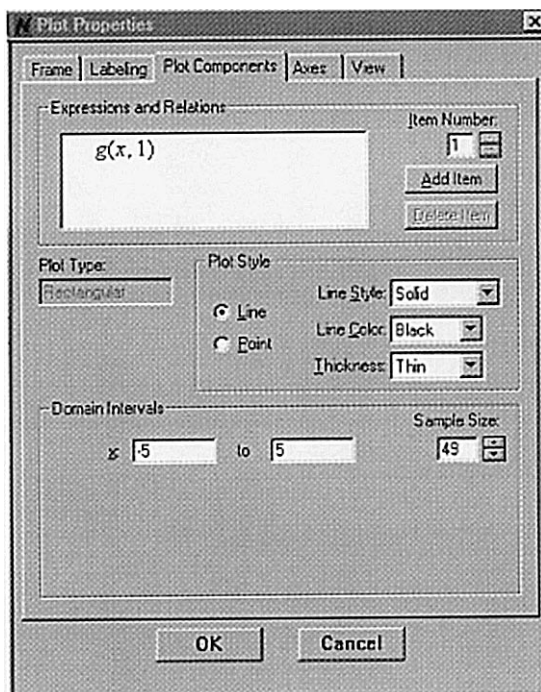


Fig. 4. Plot Properties dialog box.

- To plot the other particular solutions you could simply repeat Step 4. However, if a large

number of plots is required this could be tedious. Also, we wish to obtain all the plots on the same picture. To do this: click on the picture and then on the blue square to obtain the Plot Properties Dialog Box. Make sure you have selected the Plot Components field as illustrated in Fig. 4. Click on Add Item and then type $g(x,0.5)$ in the Expressions and Relations box and change the Line Color to green. Now click on Add Item and then type $g(x,1.5)$ in the Expressions and Relations box and change the Line Color to magenta. Now click on Add Item and then type $g(x,2)$ in the Expressions and Relations box and change the Line Color to cyan. Finally, click OK to obtain the graph in Fig. 5.

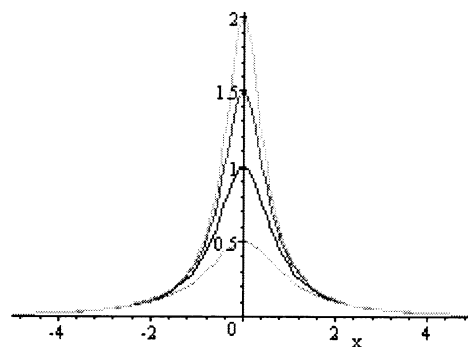


Fig. 5. Particular Solutions for $y_0 = 0.5, 1, 1.5$ and 2 .

The procedure outlined above is readily adopted by students and is particularly useful for cases where the family of solutions of an ODE needs to be examined to determine its global behaviour. A typical example of this is forced harmonic motion. The example we give here was detailed as a guided exercise in the second of our worksheets. In this case we shall fill in the details that the students were expected to work out for themselves.

Example 2

Forced Harmonic Motion. This is represented here as the initial value problem (IVP):

$$y'' + y = \sin \omega t, \quad y(0) = 0, \quad y'(0) = 0$$

- The fundamental solutions for this IVP are $y(t) = \cos t$ and $y(t) = \sin t$ so resonance will occur for $\omega = 1$. Enter this IVP as a matrix:

$y'' + y = \sin \omega t$
$y(0) = 0$
$y'(0) = 0$

- Use Solve ODE Laplace to solve this IVP for $\omega \neq 1$. The solution is given by:

$y'' + y = \sin \omega t$
$y(0) = 0$
$y'(0) = 0$

Laplace solution is:

$$y(t) = \omega \left(\frac{1}{(\omega - 1)(\omega + 1)} \sin t - \frac{1}{(\omega - 1)(\omega + 1)\omega} \sin \omega t \right)$$

Note: You could use Solve ODE Exact to find the solution and then simplify it by use of Maple, Combine, Trig Function.

3. Use New Definition to define the solution of the IVP to be $y(t, \omega)$.
4. Plot $y(t, \omega)$ for $\omega = 0.5, 0.7, 0.9, 1.1, 1.3, 1.5, 5$.
5. Solve the IVP for $\omega = 1$ and plot the solution $y(t)$.
6. Comment on the behaviour of the solution as ω approaches 1 from above and below.

In this way the effect of the forcing frequency ω on the solution of the ODE may be explored.

The graph for $\omega = 1$ is shown in Fig. 6. It shows that, in the case of resonance, the modulus of the solution grows without bound. The graph for $\omega = 1.1$ is shown in Fig. 7. This may be used to illustrate the phenomenon of a beat, where the solution appears as a wave having a periodic variation of amplitude that depends on the sizes of the frequencies $21/10 = 1.1 + 1$ and $1/10 = 1.1 - 1$.

A related problem that may be considered concerns the effect of the initial velocity on the long-term behaviour of the solution. We illustrate this in the case of resonance.

Example 3

Solve the initial value problem:

$$y'' + y = \sin t, \quad y(0) = 0, \quad y'(0) = b$$

Graph the solutions with $b = 0$ and $b = 1$ on the same diagram to determine the long-term effect of the non-zero initial velocity on the solution.

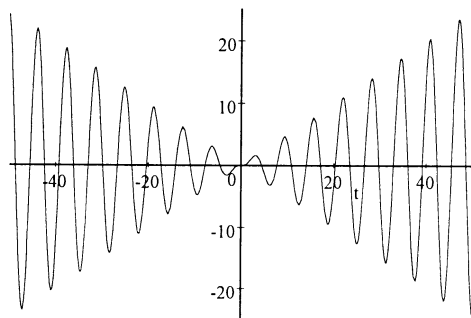


Fig. 6. Graph of $-\frac{1}{2}t \cos t + \frac{1}{2} \sin t$.

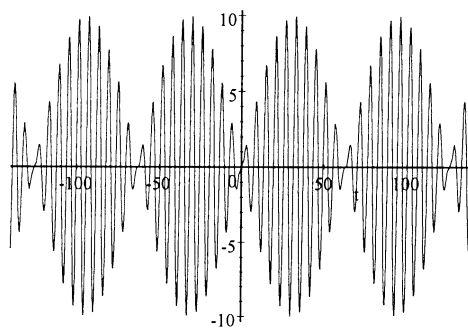


Fig. 7. Graph of $\frac{110}{21} \sin t - \frac{100}{21} \sin \frac{11}{10} t$.

1. Solving the initial value problem via Solve ODE Exact we obtain:

$y'' + y = \sin \omega t$
$y(0) = 0$
$y'(0) = b$

Exact solution is:

$$y(t) = -\frac{1}{2}(\cos t)t + \frac{1}{2} \sin t + b \sin t$$

We can now define:

$$y(t, b) = -\frac{1}{2}(\cos t)t + \frac{1}{2} \sin t + b \sin t$$

and then enter the expression as a new definition. The graphs of $y(t, 0)$ and $y(t, 1)$ on the intervals $[0, 10]$ and $[0, 50]$ are shown in Fig. 8 and Fig. 9 respectively.

2. In Fig. 8 the curve with gradient 1 at the origin represents the graph of $y(t, 1)$ and the curve with gradient 0 at the origin represents $y(t, 0)$. The graphs show that the effect of a non-zero initial velocity is largely felt for values of y close to zero while in the long-term the initial velocity does not play a big role, with the graphs almost coinciding.

SOME POSSIBLE PITFALLS

So simple and natural are most forms of calculations on Scientific Notebook that the rare cases

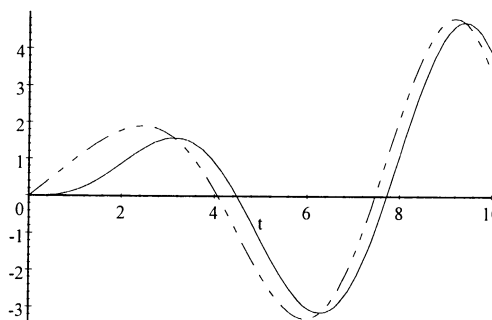


Fig. 8. Graphs of $x(t, 0)$ and $x(t, 1)$ on $[0, 10]$.

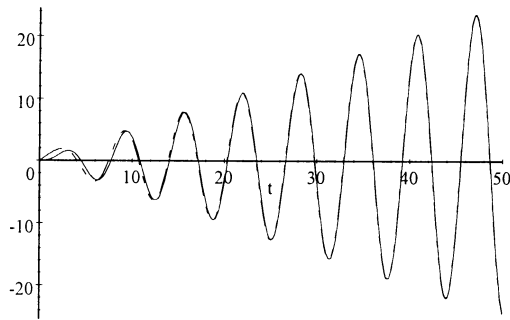


Fig. 9. Graphs of $x(t, 0)$ and $x(t, 1)$ on $[0, 50]$.

where the user’s intuitive feelings for standard mathematical syntax prove to be misleading can be quite perplexing. However, like most software packages, Scientific Notebook has some traps for the unwary and students need to know about these. We draw attention here to some of these pitfalls which are not mentioned in either [3] or [4] or on the Website. Some errors actually arise by carelessness on the part of the user while others need to be attended to by the software writers.

For instance, a very common mistake when re-using the same line for multiple calculations is for the student to forget to delete a relational operator (usually the ‘=’ sign) after the expression being evaluated. This typically leads to obtaining no response from the package. More serious syntactical problems can arise from the improper use of brackets in Scientific Notebook. The rules governing the placement of brackets appear on occasion to be non-standard. For example, although Scientific Notebook reads the expression $\sin(x)/2$ as $\frac{1}{2}\sin(x)$, which is what one would expect, it reads $\sin(-x)/2$ and $\sin(x + 1)/2$ as $-\sin\frac{1}{2}x$ and $\sin(x/2 + 1/2)$ respectively. Similar problems relate to the use of brackets in conjunction with the other trigonometric functions, the hyperbolic functions, and the exponential and logarithmic functions. In general, the only way to ensure that Scientific Notebook is reading an elementary function in the way intended by the user is to place brackets around the entire function. It should be noted that this particular problem is not shared by Maple.

Care also needs to be taken when using Scientific Notebook’s specialised subroutines. The ODE solver, in particular, often has trouble recognising the correct solution to an initial-value problem that involves square roots. For example, if we are looking to solve the initial-value problem $dy/dx = \sqrt{y}$ with $y(1) = 1$ then the command Solve ODE Exact gives:

$\frac{dy}{dx} = \sqrt{y}$
$y(1) = 1$

Exact solution is:

$$y(x) = \frac{1}{4}x^2 + \frac{1}{2}x + \frac{1}{4}$$

which is the correct solution on the domain $x \geq -1$.

However, if the problem is changed slightly to read $dy/dx = -\sqrt{y}$ with $y(1) = 1$ then Solve ODE Exact gives the same solution as before:

$\frac{dy}{dx} = -\sqrt{y}$
$y(1) = 1$

Exact solution is:

$$y(x) = \frac{1}{4}x^2 + \frac{1}{2}x + \frac{1}{4}$$

which is incorrect. The correct solution, valid on the domain $x \leq 3$, is:

$$y(x) = \frac{1}{4}x^2 - \frac{3}{2}x + \frac{9}{4}.$$

What is interesting about this problem is that the same subroutine returns the correct solution to the equation $dy/dx = -\sqrt{y}$ in the absence of the initial condition:

$$\frac{dy}{dx} = -\sqrt{y},$$

Exact solution is: $2y^{3/2}(x) + x = C_1$

Incidentally, Maple when faced with either of the initial value problems mentioned above returns both solutions. It seems that the designers of Scientific Notebook decided for the sake of simplicity to retain only one of the solutions, but unfortunately not always the correct one!

STUDENT EVALUATION

Despite the problems encountered on occasion, *Scientific Notebook* remains an extremely useful package for the non-specialist student; such problems are merely temporary blips in the programming and will be corrected as they are noted. An evaluation survey given to our students a short time after the two workshops confirmed our opinion of the simplicity and efficacy of this particular software package. The response rate to this survey was 94% with seventeen out of the eighteen participants submitting a completed form. The details of the student response now follows. The numbers in square brackets give the number of students who chose the indicated option on the five-point scale.

Student survey

1. My prior experience with Scientific Notebook was:

Non-existent [6] [6] [3] [2] [0] Extensive

2. Scientific Notebook is easy to use:
SD [0] [0] [3] [9] [5] SA
 3. Value of the Differential Equations Workshops to me:
Low [0] [0] [0] [8] [9] High
 4. Workshops Organisation:
Poor [0] [0] [2] [9] [6] Excellent
 5. Overall workload of the Workshops:
Too Little [1] [3] [12] [1] [0] Too Much
 6. Degree of stimulation and satisfaction:
Low [0] [1] [2] [9] [5] High
 7. I would like future Scientific Notebook workshops in my units:
SD [0] [0] [2] [6] [9] SA
 8. I expect to use Scientific Notebook workshops in my assignments:
SD [0] [0] [2] [8] [7] SA
- SD = Strongly Disagree; SA = Strongly Agree

From the student response it can be seen that 71% of the students had no, or very little, previous

exposure to Scientific Notebook, that 82% consider the package easy or very easy to use, that all students found the workshops valuable or very valuable and that 88% of the students expect to use Scientific Notebook in their assignments.

CONCLUSION

We have shown here that Scientific Notebook can be a very useful aid to help the instructor to teach engineering mathematics more effectively. Like any tool, its operation, capabilities and limitations need to be well understood by the user for it to achieve its maximum capabilities. Despite the potential problems indicated earlier, there is no doubt that software such as Scientific Notebook can be of enormous benefit. There is real scope for the instructor to concentrate on concepts and relevant applications without being hampered by the problems of detailed and difficult calculations.

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